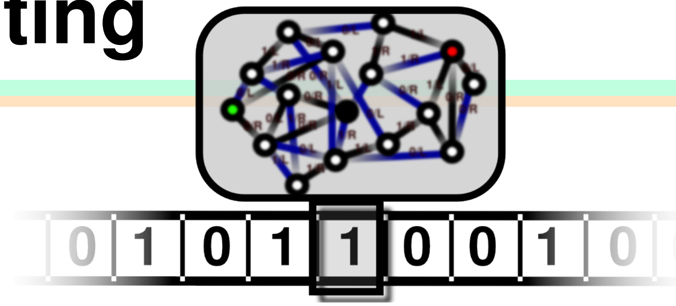
Building Quantum Computers

Produced with pdflatex and inkscape

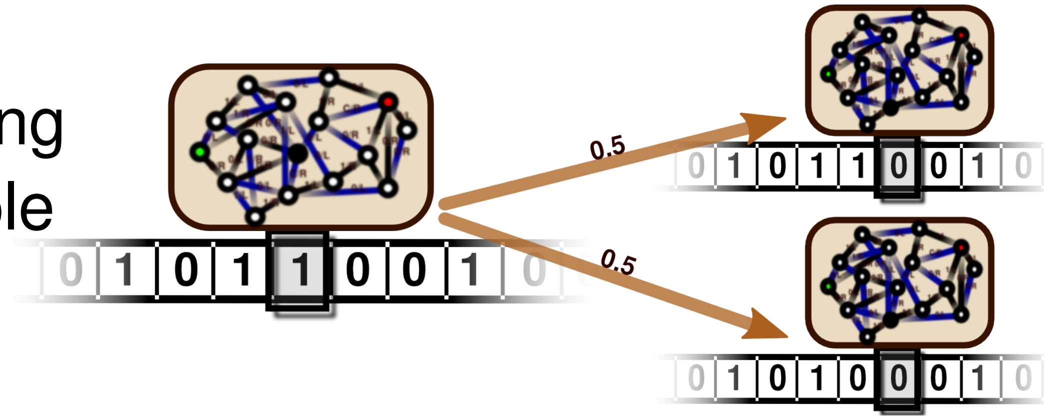
- Why quantum compute?
- How does it work?
- Why is it difficult?
- How much can we do today?
- What are the prospects?

E. “Manny” Knill: knill@boulder.nist.gov

Models of Computing

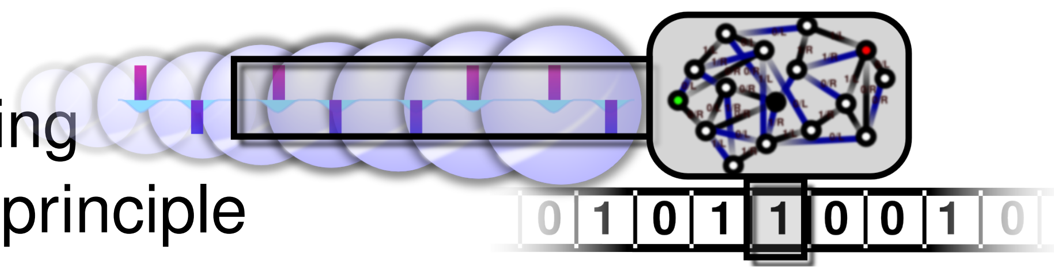


- Classical (deterministic) computing.



- Probabilistic computing

$$= \begin{cases} \text{classical computing} \\ + \text{ mixture principle} \\ + \text{ coin flip} \end{cases}$$

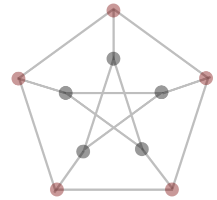


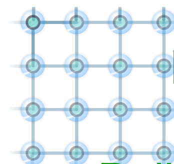
- Quantum computing

$$= \begin{cases} \text{classical computing} \\ + \text{ superposition principle} \\ + \text{ an interference gate} \end{cases}$$

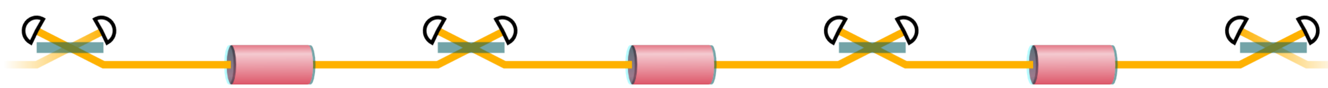
Why Quantum Compute?

- Algorithmic speedups.
  - Efficient quantum factoring. $N=pq$ Shor '94 [1]
  - Quadratic speedups of combinatorial search and Monte Carlo algorithms. Grover '95 [2], . . .





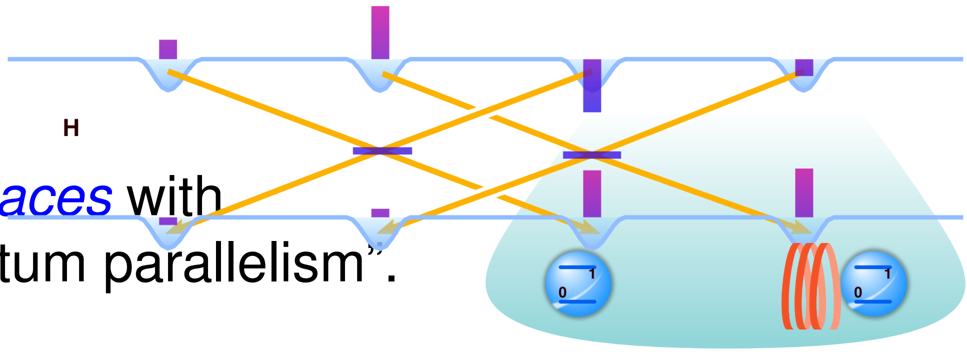
  - Efficient physics simulations. Feynman '82 [3], . . .
  - Quadratic improvements in measurement precision. Bollinger&al. '96 [4], . . .
- Cryptographic protocols.
  - Remove distance limitations of quantum key exchange. Briegel&al. '98 [5]
  - Quantum digital signatures. Gottesman&Chuang '01 [6]
  - Extensions of classical crypto to quantum information.



- Tests of quantum mechanics.
  - Validity of the superposition principle.
  - Ability to preserve many-system "entanglement".

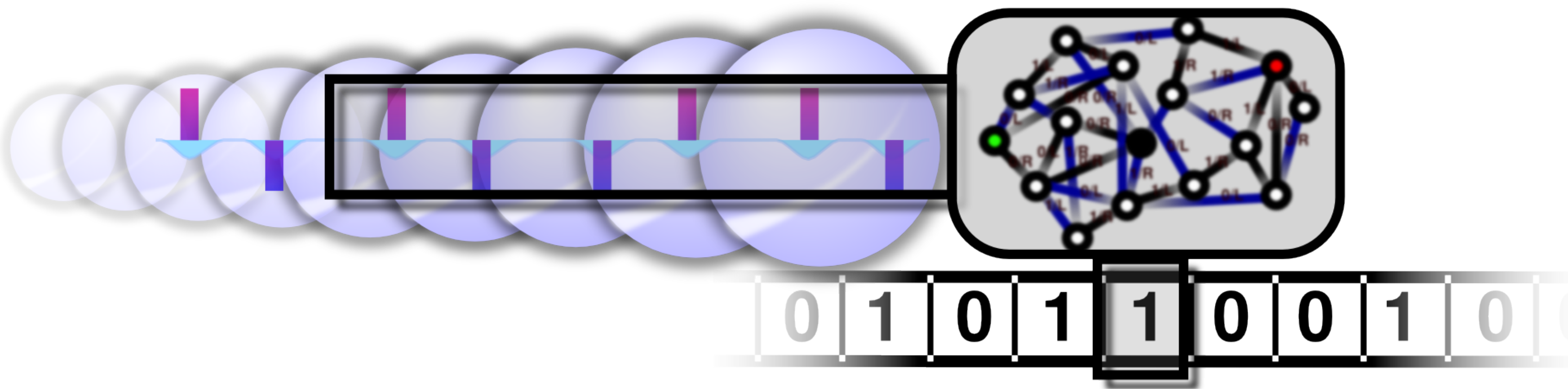
Preview

- QCs control
  - *wavefunctions* on
  - efficiently realized *configuration spaces* with
  - *interference* gates exploiting “quantum parallelism”.



Quantum Computers

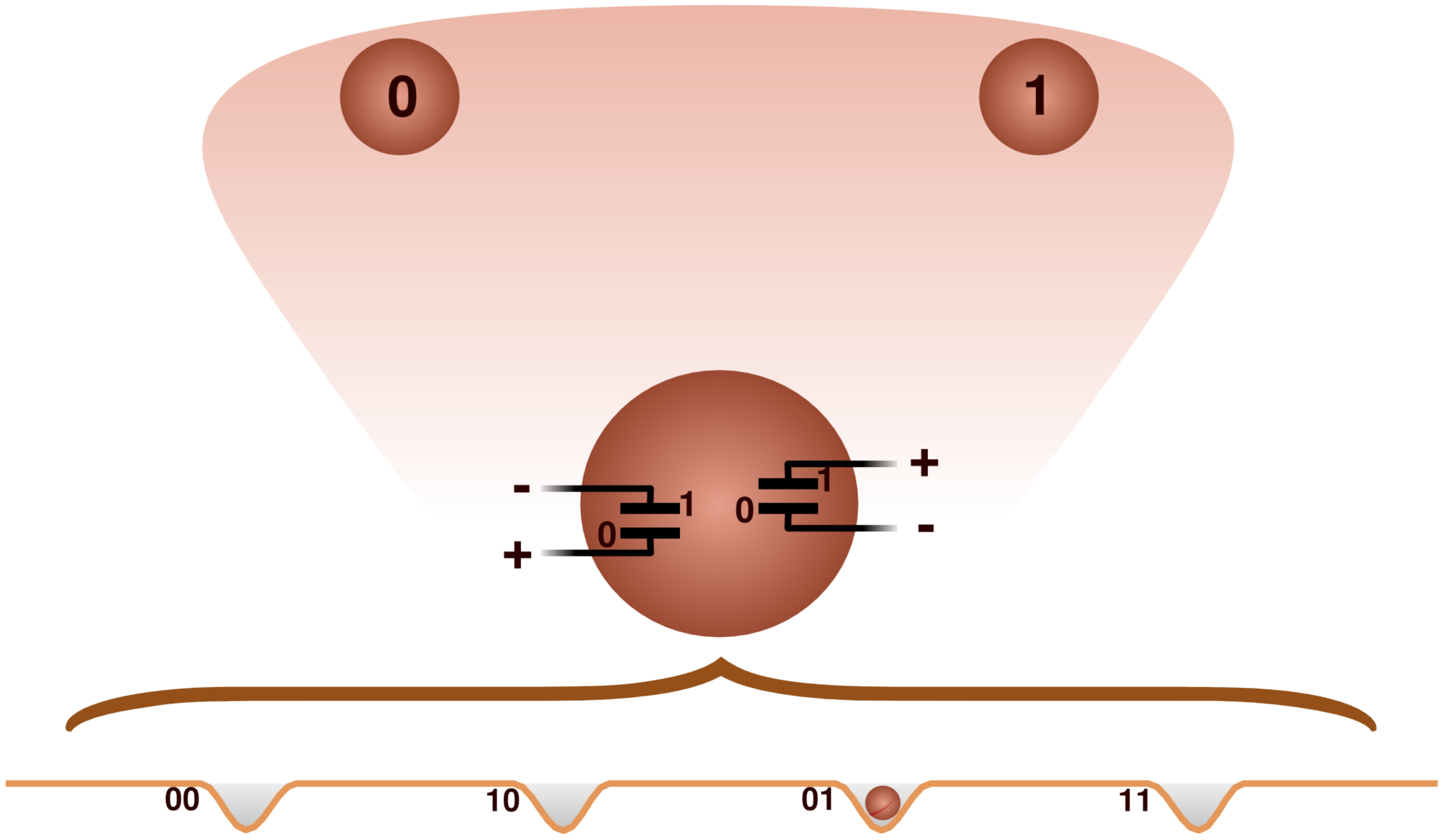
- Classical computer + quantum state machine.
  - Advanced programming constructs provided classically.



- State machine specified by:
  - State space.
  - State-transition operators.
  - Initial state.
  - Readout.

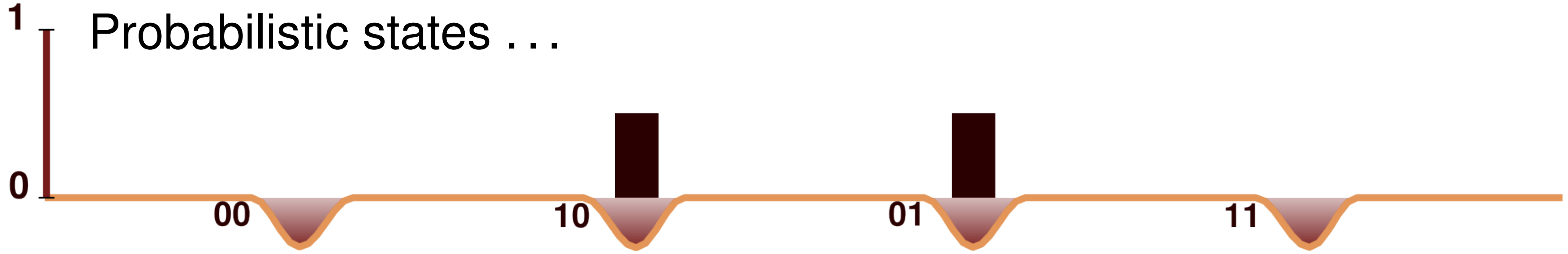
How Does It Work?

Classical states . . .



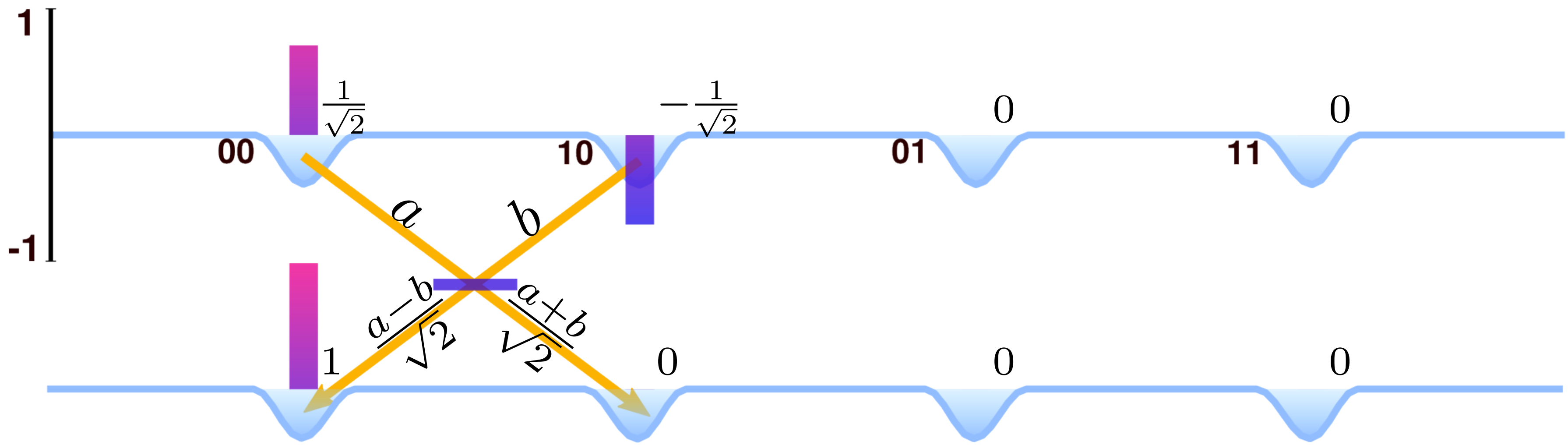
How Does It Work?

Probabilistic states . . .



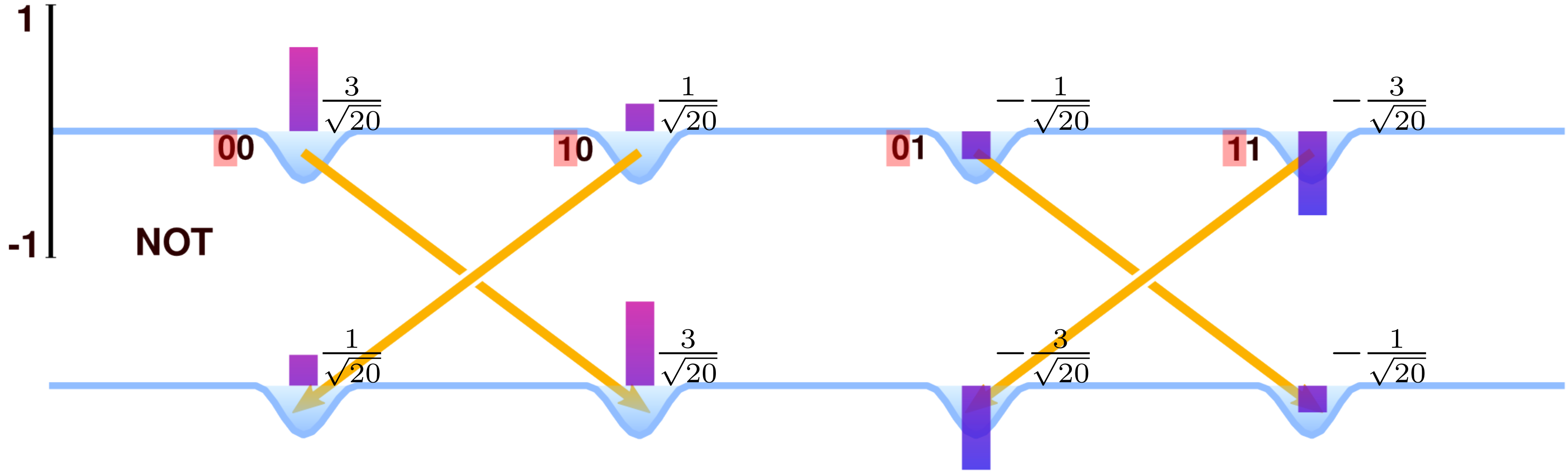
How Does It Work?

Quantum states . . . Interference . . .



How Does It Work?

Quantum states . . . Interference . . . Q. Parallelism . . .



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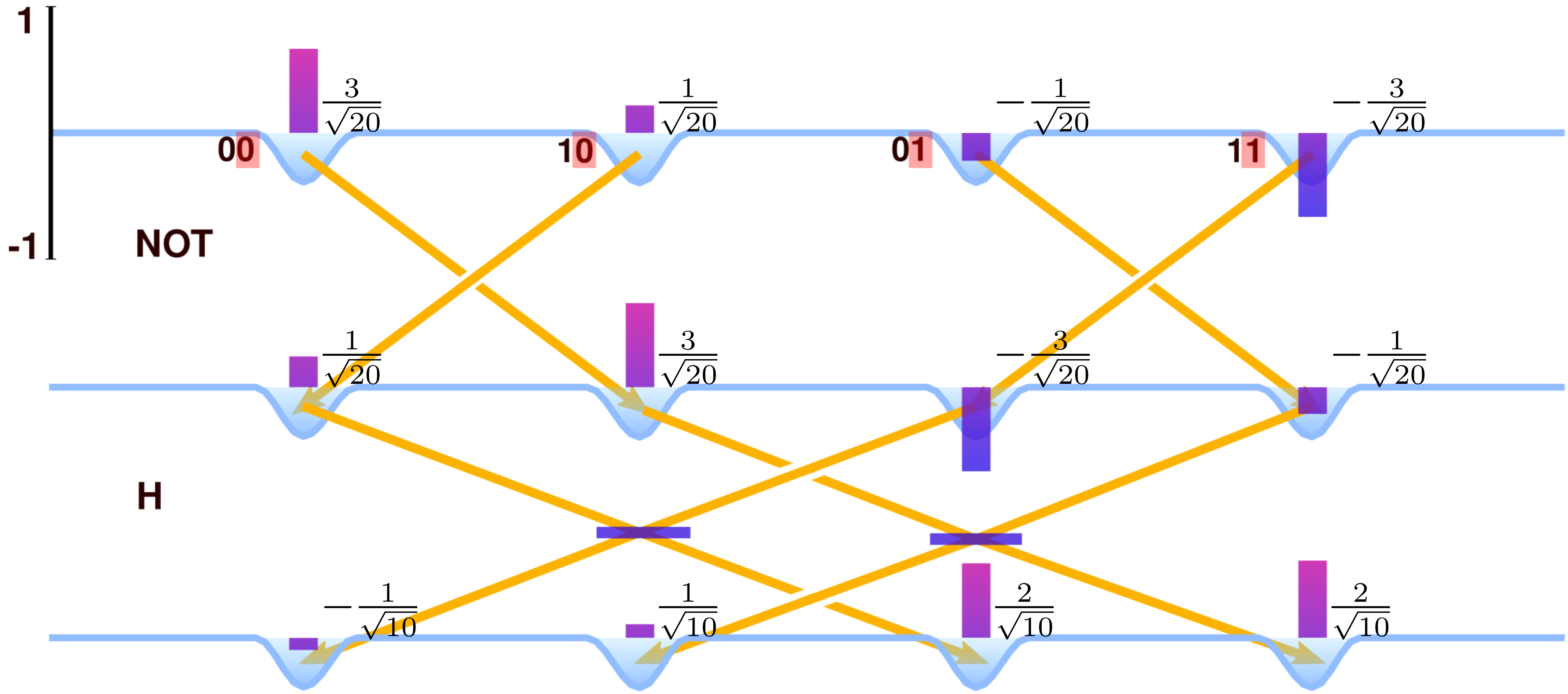
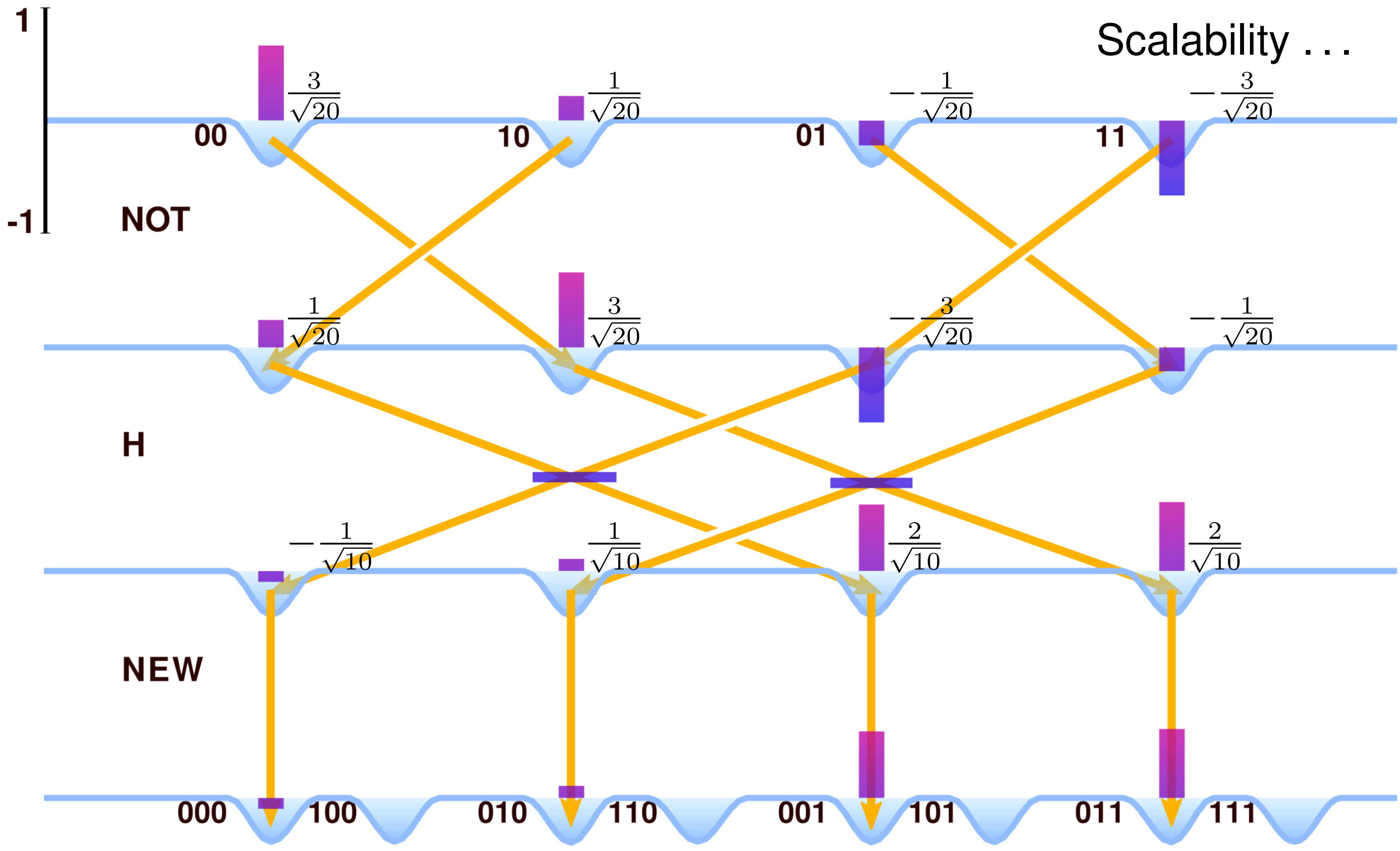


How Does It Work?

Quantum states . . . Interference . . . Q. Parallelism . . .

Scalability . . .



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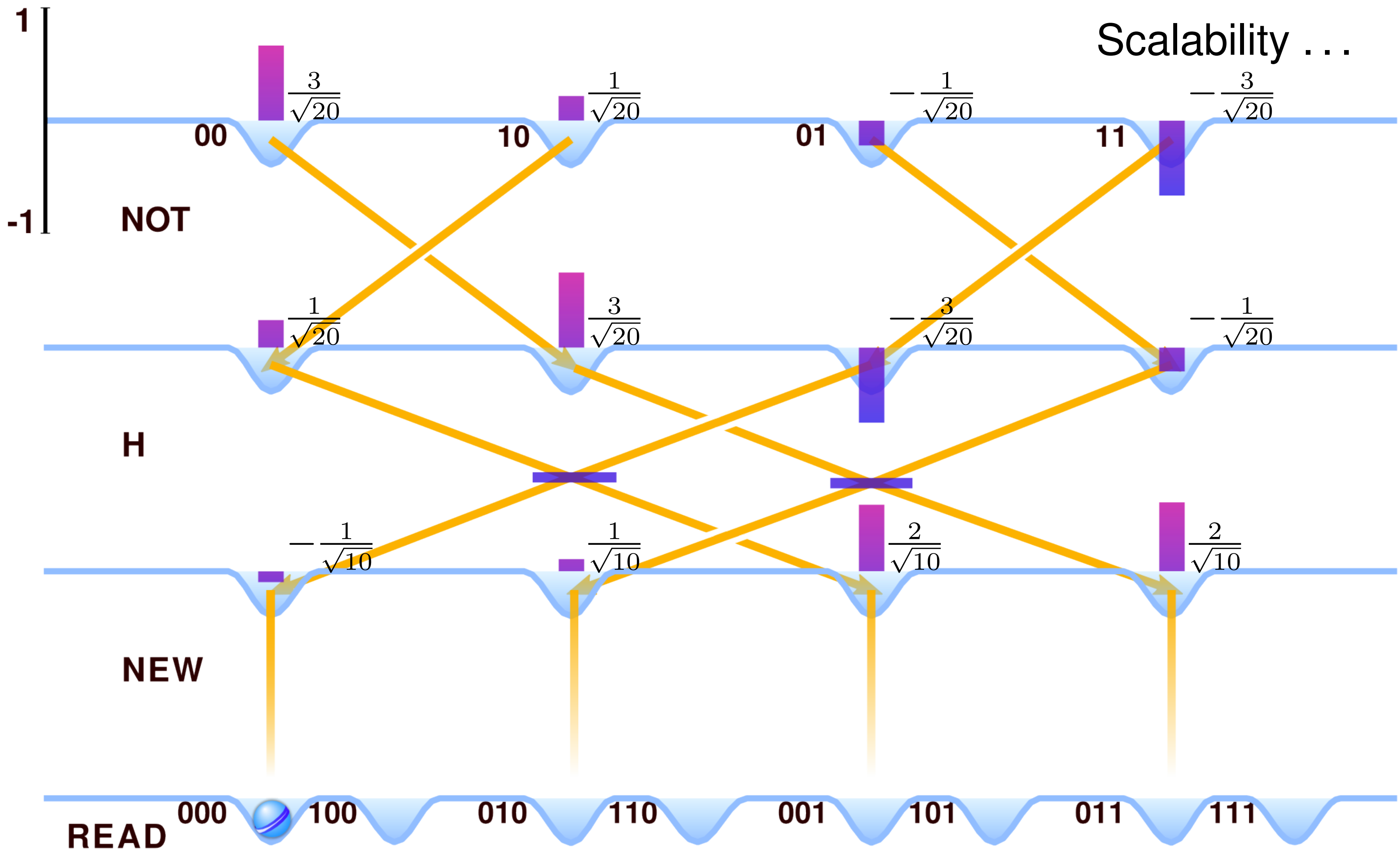
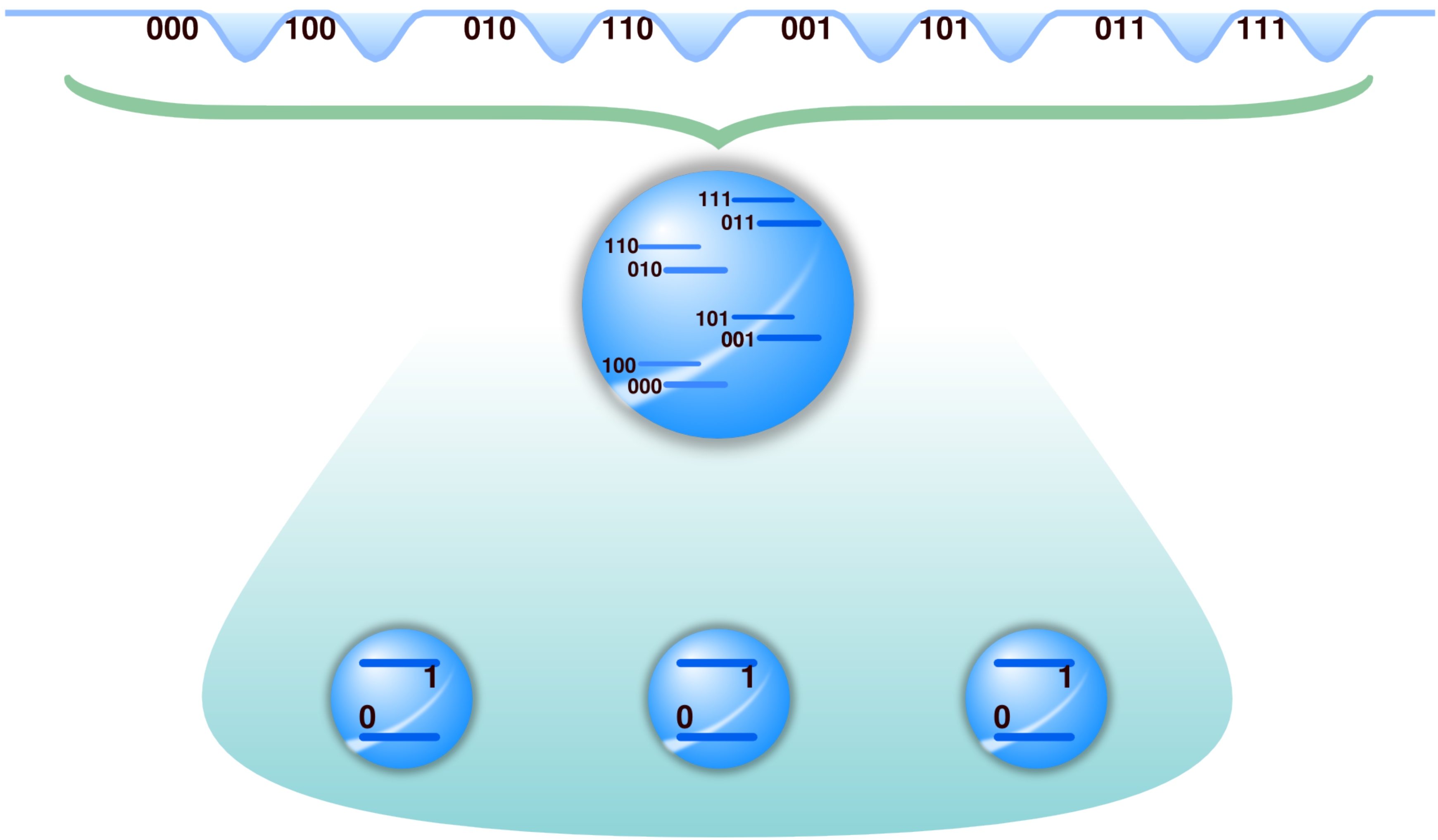


How Does It Work?

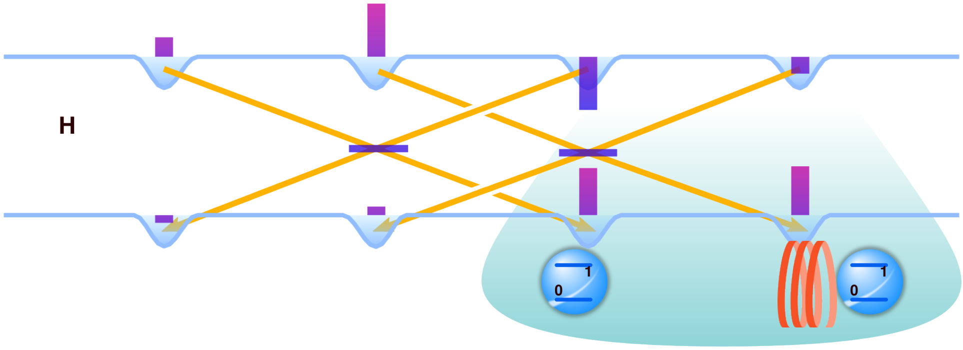
Quantum states . . . Interference . . . Q. Parallelism . . .

Efficient Scalability . . .

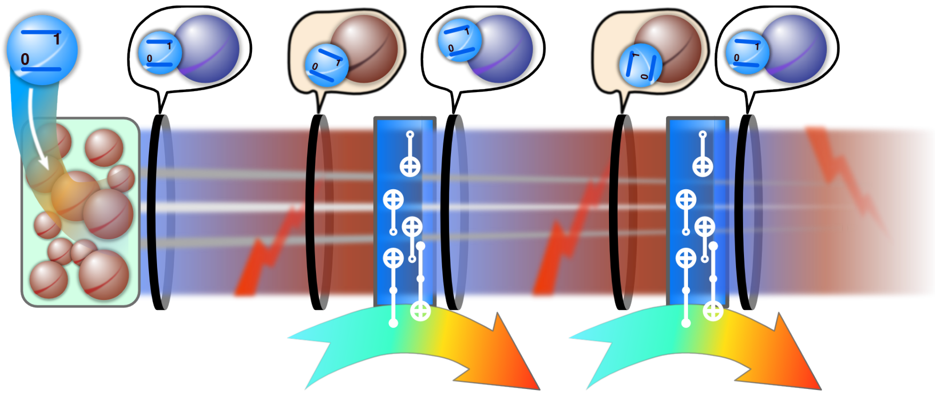


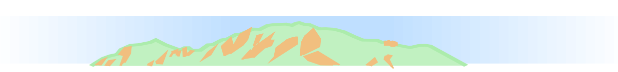
Preview

- QCs:
  - *wavefunctions*
  - *configuration spaces*
  - *interference*



- QCs can be realized in the presence of noise because of
  - the *threshold theorem*, whose implementation requires
  - *quantum error control*, which is based on
  - the *subsystems principle*, the most general way of “encoding” information.





Building QCs: Challenges

- DiVincenzo's requirements: DiVincenzo '00 [7]

  1. Independent QI-carrying quantum systems.
     (Demonstrated, many candidates.)
  2. Initializability of these quantum systems.
     (Demonstrated.)
  3. States are subject to sufficiently low noise.
     (Not demonstrated.)
  4. Universal control.
     (Demonstrated on a few qubits in a few systems.)
  5. Read-out.
     (Demonstrated in a few systems.)

- Challenges:
  - Reducing the effects of quantum noise.
    Low noise requires isolation
    but read-out and gates require strong coupling.
  - Satisfying all requirements in one device.

Decoherence

Definition: *Decoherence* is the loss of phase relationships between amplitudes.

- *Decoherence* often refers to any quantum noise leading to errors.

- Some sources of decoherence:
  - Interactions with the environment.
  - Noise in quantum control fields.
  - Systematic, calibration errors.

> **Fault-Tolerance Threshold Theorem.** *Given: Noisy qubits and gates. If the error rates are sufficiently low, then it is possible to efficiently process quantum information arbitrarily accurately.*

Shor '95 [8, 9], Kitaev '96 [10], Aharonov&Ben-Or '96 [11], Knill&Laflamme&Zurek '96 [12], Gottesman&Preskill '99, Steane '02 [13], Knill '04 [14, 15], Reichardt '04 [16], Aliferis&Gottesman&Preskill '05 [17]

Classical Versus Quantum Error Control

Conceptual difficulties:

- Error control must not “see” stored information.
  - “No cloning” theorem.
  - Nontrivial generalization of repetition codes.
- There is a continuity of error models.
  - Error models have many parameters.
  - Repetition codes are insufficient.

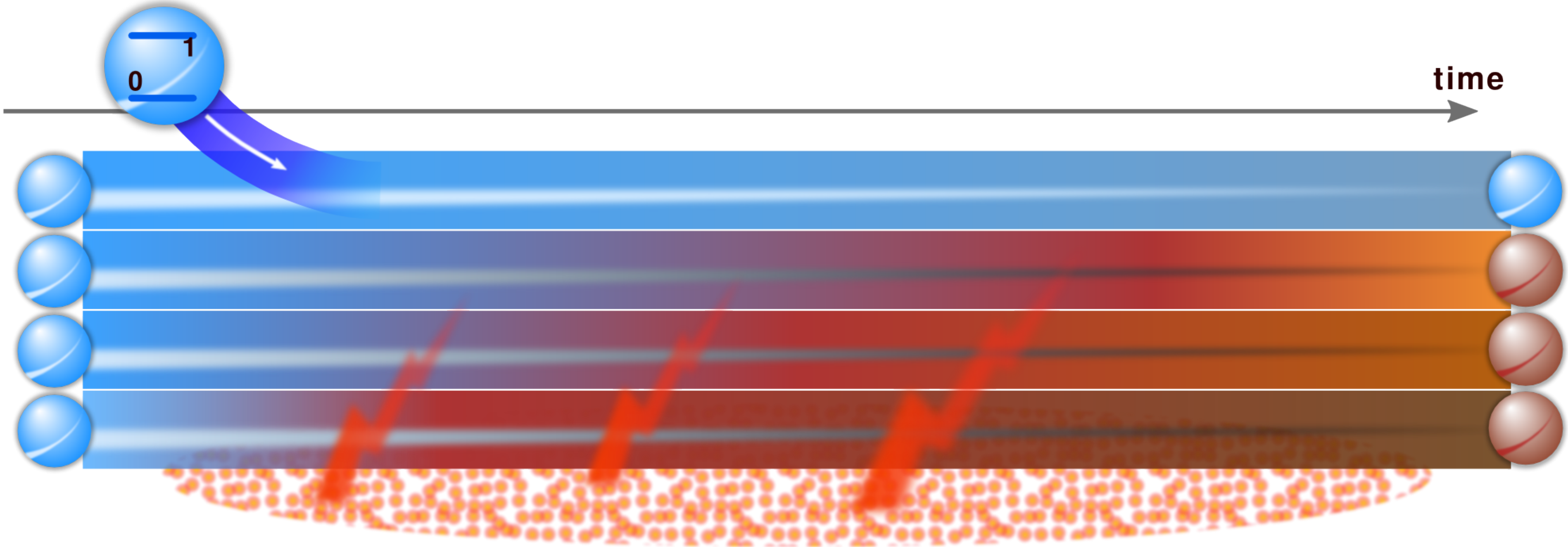
A sample of coping strategies:

- Quantum stabilizer codes generalize classical linear codes.
  Calderbank&al. ’96 [18], Gottesman ’96 [19]
- Adopt the subsystems principle. Knill&Laflamme&Viola ’99[21, 22]

Overcoming Decoherence

- **Goal:** Realize accurate QI in noisy physical systems.
  - Noise must be local in space and time (independence assumptions).

Trivial QI realizations:



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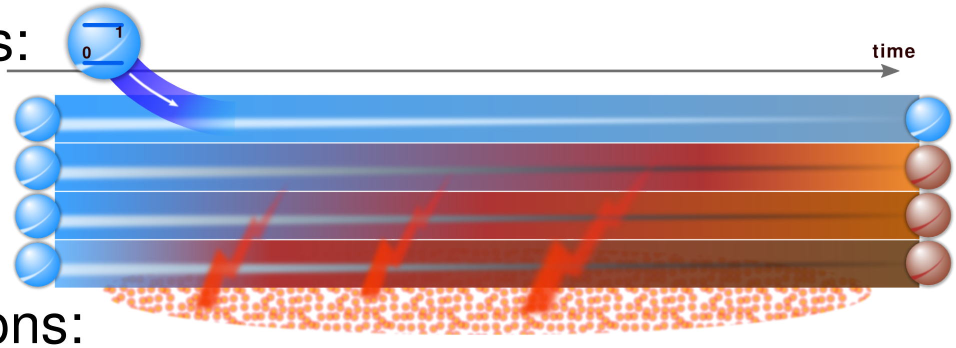
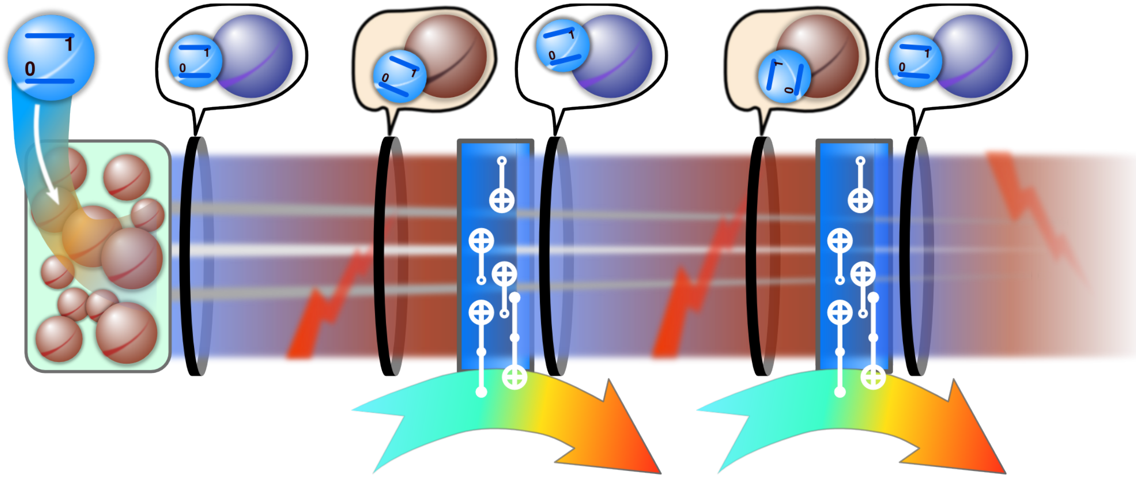
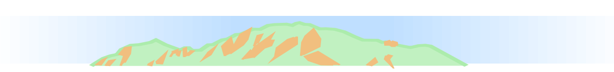


General QI realizations:

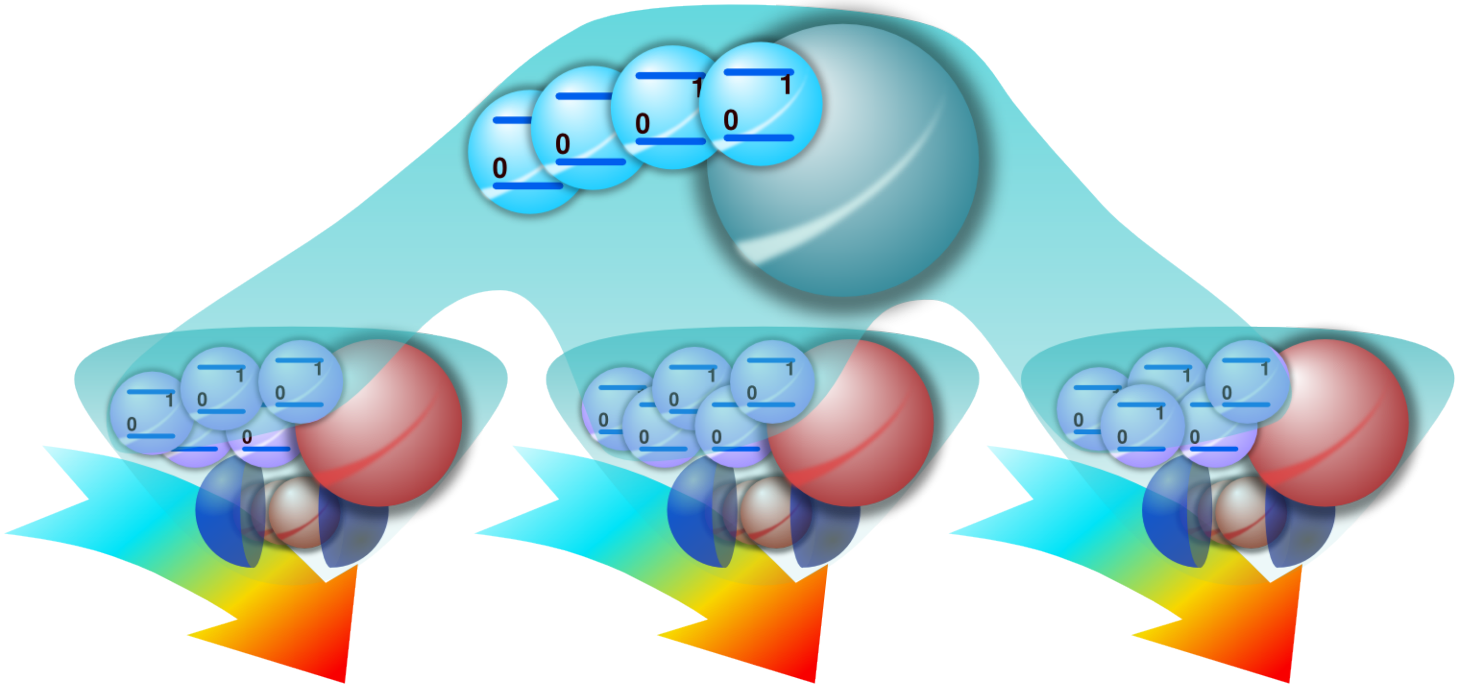


- Information should never be “decoded”.
- The subsystems principle applies to and enhances classical information theory.



A Path to Large-Scale QCs

- Well defined physical quantum systems.
- Protectable quantum subsystems.
- Error entropy sink.
- Concatenation may help.



A Path to Large-Scale QCs

- Well defined physical quantum systems.
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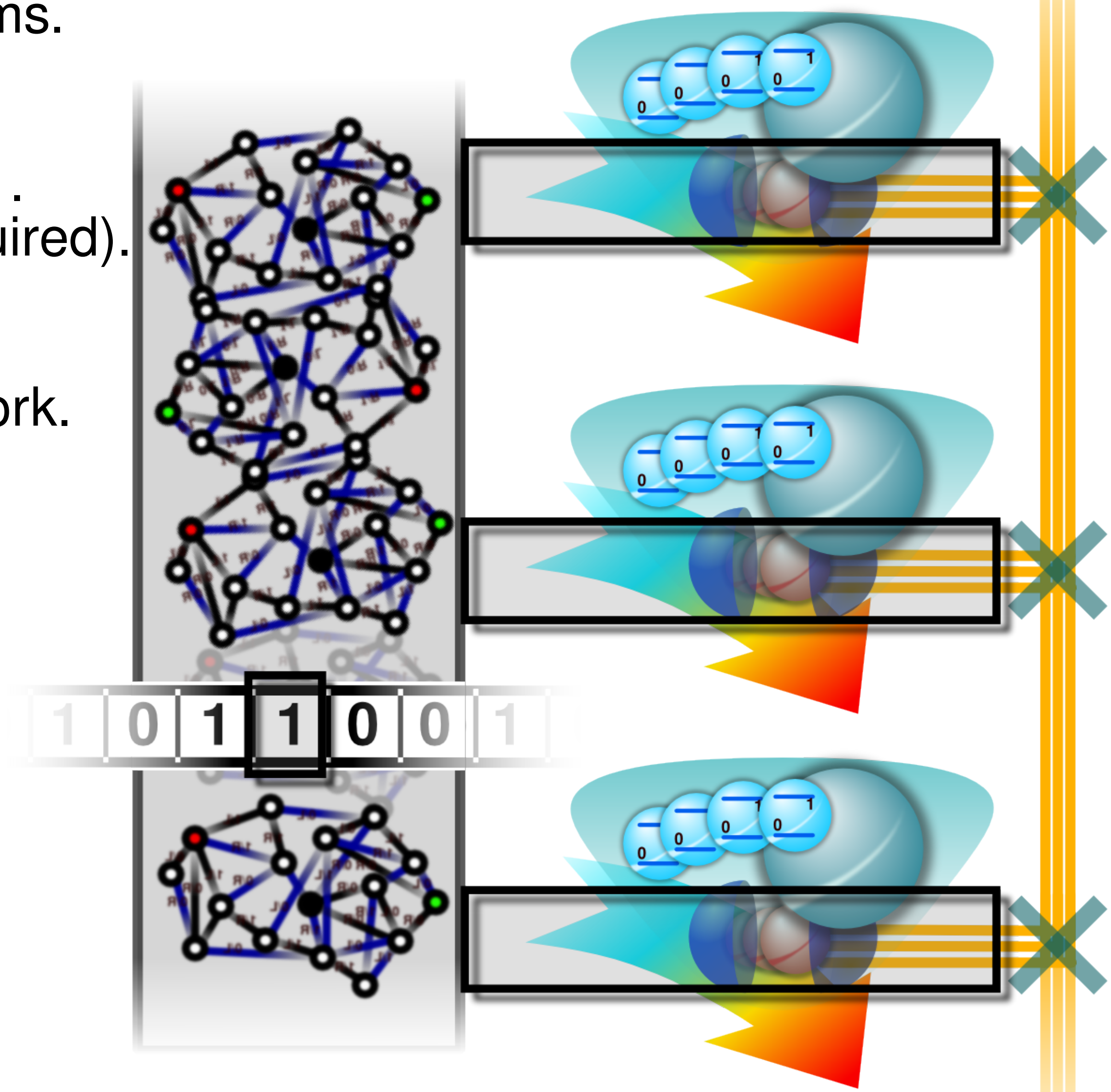
- A small quantum register with. . .
  external control (high ||ism required).
- Replicate quantum register. . .
  and control.
- Quantum communication network.
  ⇒ quantum computer.



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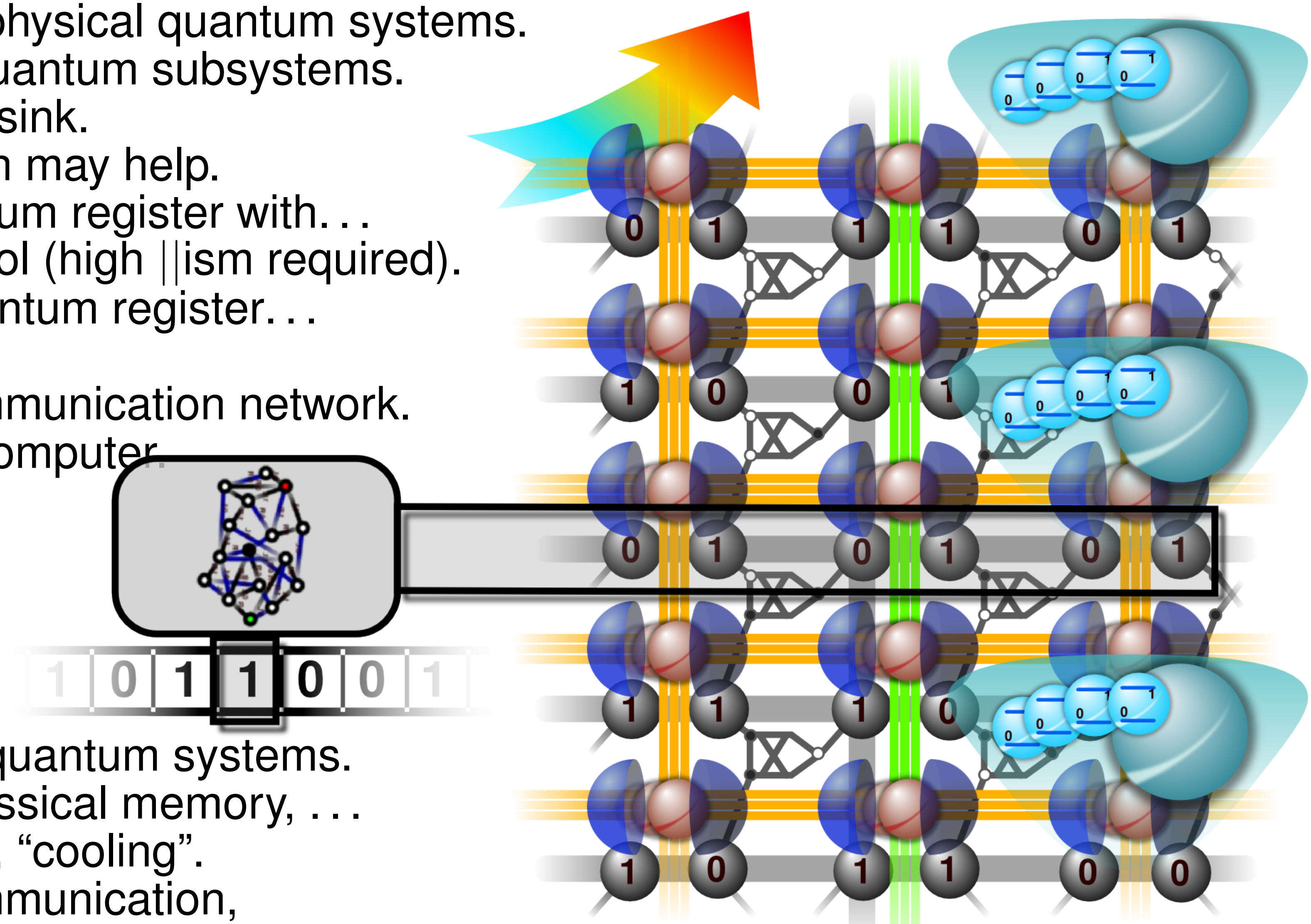
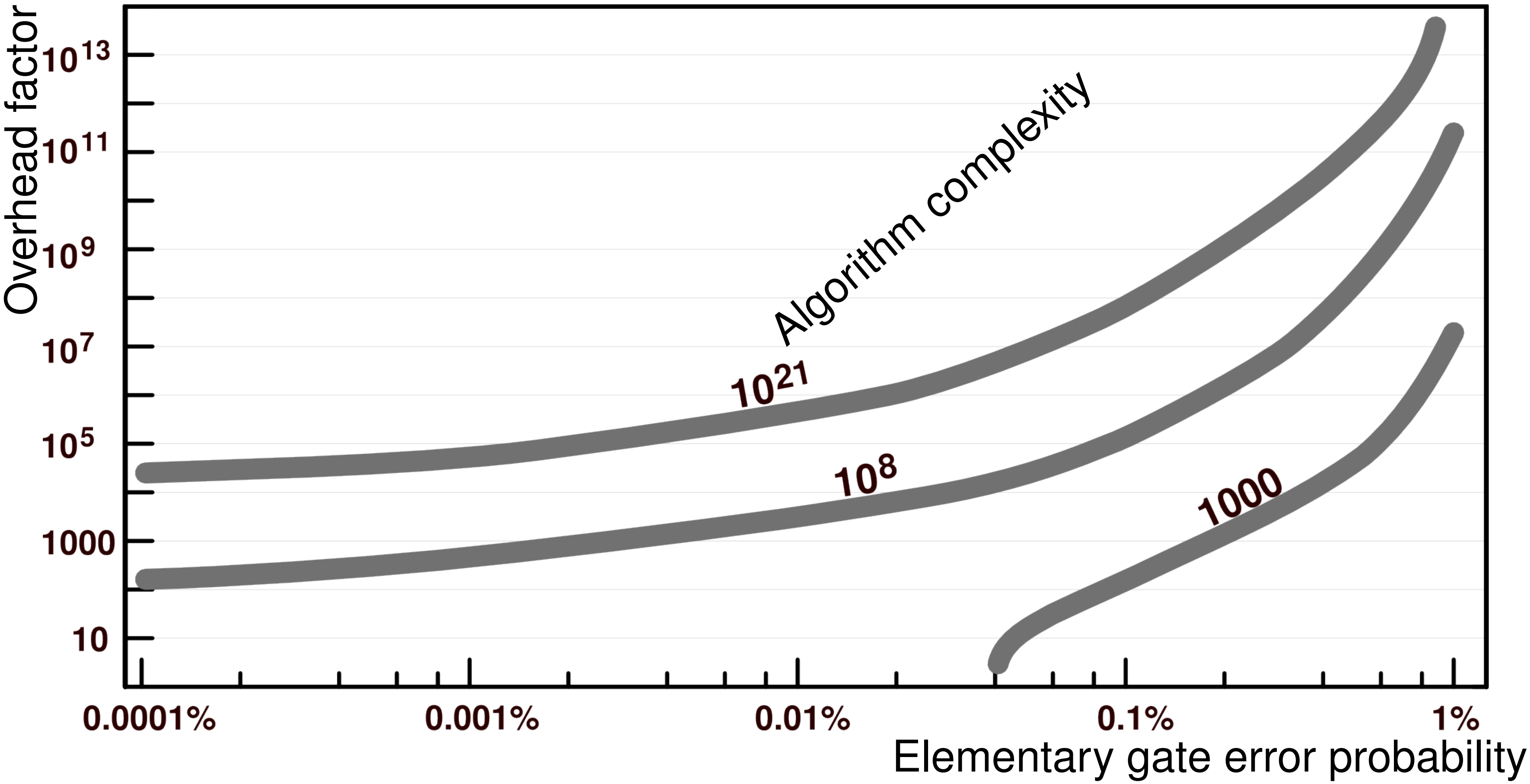


- High density quantum systems.
- Integrated classical memory, . . .
  gates, control, “cooling”.
- Quantum communication,
  state prep. factories.
- Interface to classical control.

Goal: Physical computation is quantum, minimal overhead for FTQC over FTCC.

Resource Requirements



- Schematic resource overheads, depending on algorithm complexity and gate error.

Loosely based on Knill '05 [15]

Error Guidelines?

- How low error rates should be depends on:
  - Error parameters/error model.
  - Error tradeoffs for different gates/operations.
  - Architectural constraints.
  - How much resource overhead is acceptable.
  - The specific computation to be implemented.
  - …

- Rough error guidelines for practical scalability:

Gottesman&Preskill ’98, ’00 [27, 28]

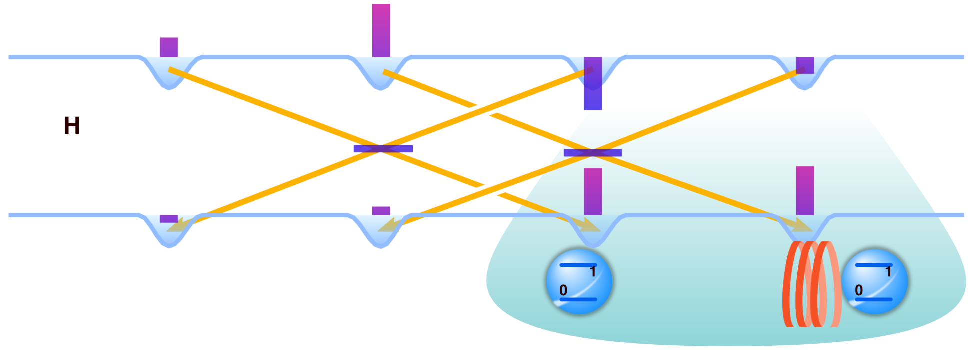
$<10^{-4}$
per gate

$<10^{-2}$
per preparation

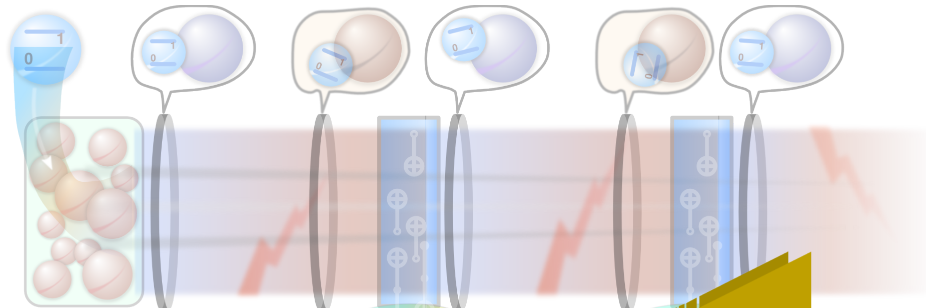
$<10^{-2}$
per measurement

Preview

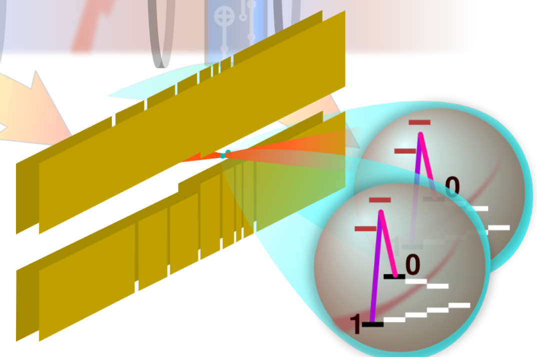
- QCs:
  - *wavefunctions*
  - *configuration spaces*
  - *interference*



- QCs can be realized:
  - *threshold theorem*
  - *quantum error control*
  - *subsystems principle*



- Experimental QC is close to achieving
  - *two-qubit registers*, with
  - *ion trap* devices being the current front runners, but
  - many other systems catching up rapidly.
  - *Decoherence* and *technology integration* are among the challenges.

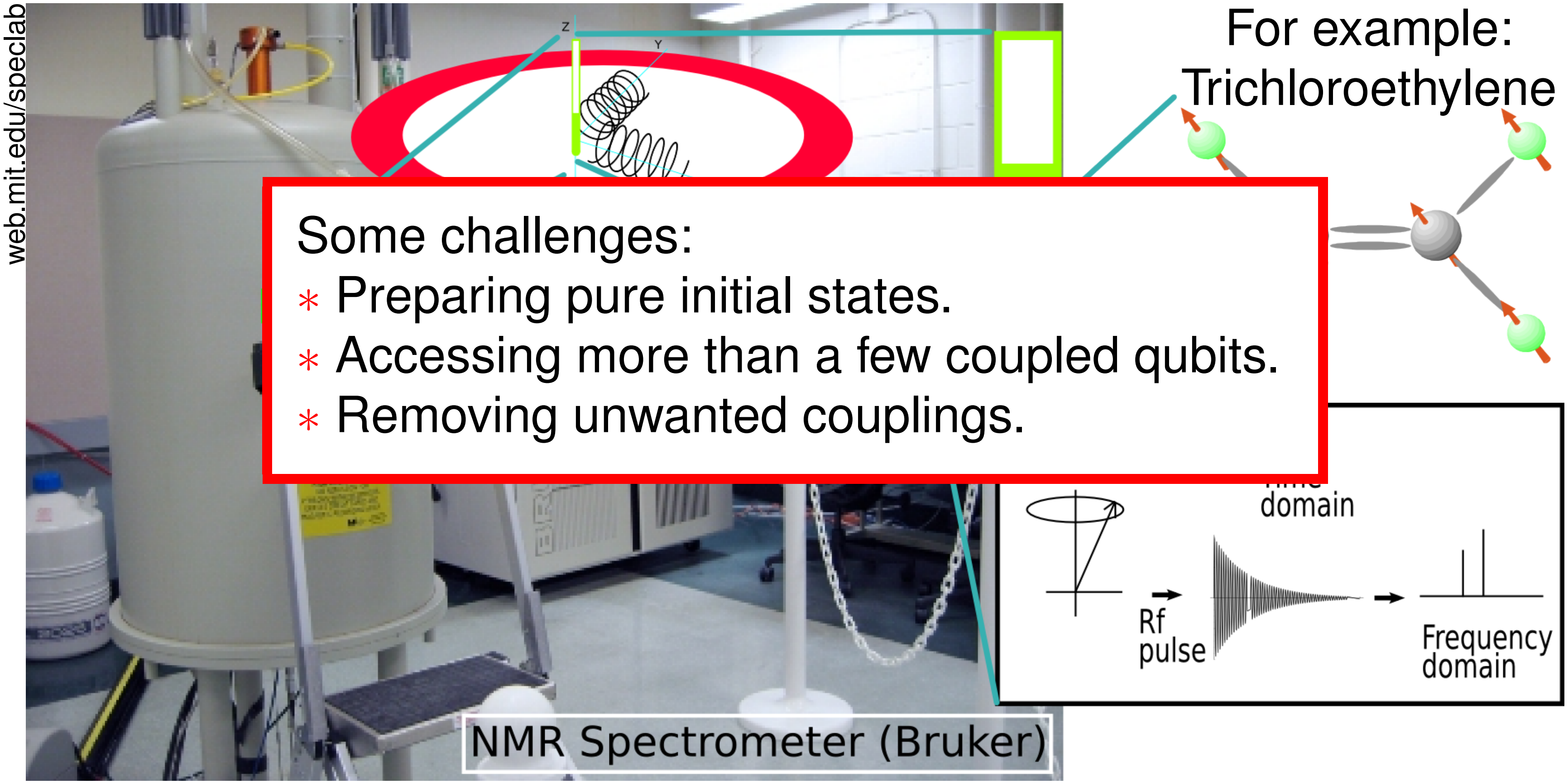


Available and Required QC Resources

- Available resources:
  - Approaching 2 computationally useful qubits.
  - For exploring quantum control:
    Up to ~ 8 qubits and ~ 16 control steps.
- Required resources:

Application	Order of minimum useful	
	qubits	gates
Secret key exchange, short distance	10^0	10^0
Secret key exchange, long distance	10^1	10^1
Factoring, other number theory algorithms	10^3	10^{10}
Unstructured search, optimization	10^2	10^8
Physics simulation	?	?

NMR QC



Some challenges:

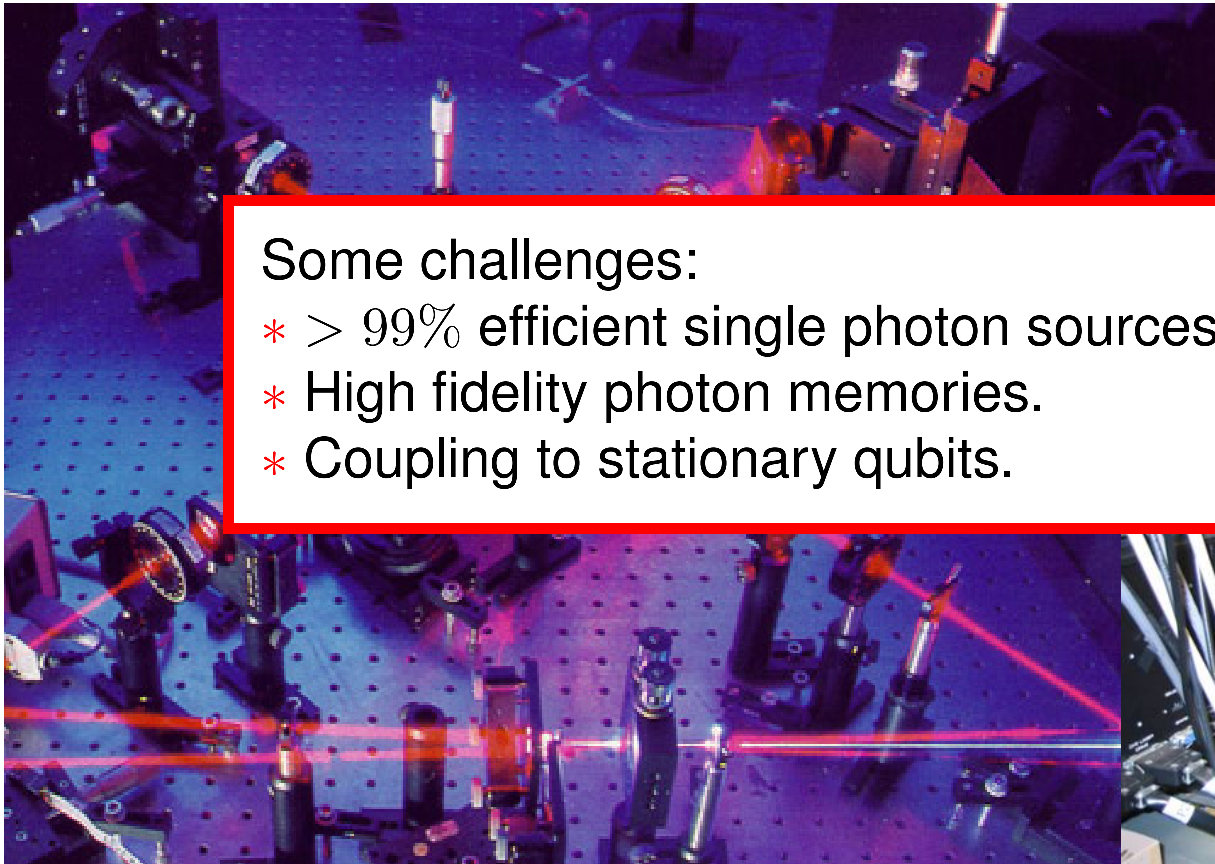
- ∗ Preparing pure initial states.
- ∗ Accessing more than a few coupled qubits.
- ∗ Removing unwanted couplings.

- ≈ 10 qubits, ≈ 20 two-qubit gates are realizable.
- Errors per two-qubit gate are $\gtrsim 1\%$.
- Not realistically scalable due to lack of “purity”. ...scalable in theory.

Reviews: Cory&al. ’00 [29], Laflamme&al. ’02 [30]

Photonic QC, Postselected

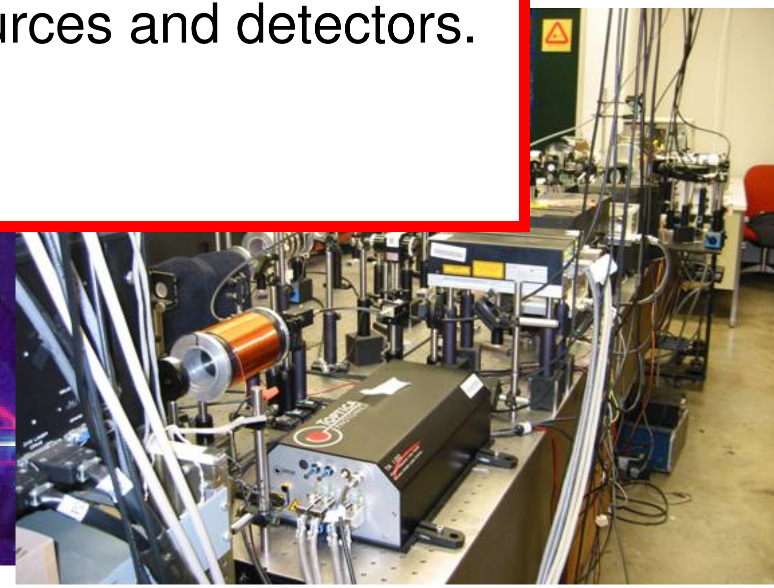
www.quantum.at



Quantum teleportation,
Bouwmeester&al., U. Wien

Some challenges:
* $>99\%$ efficient single photon sources and detectors.
* High fidelity photon memories.
* Coupling to stationary qubits.

s lab, FIU



http://www.fiu.edu/yifuzhu

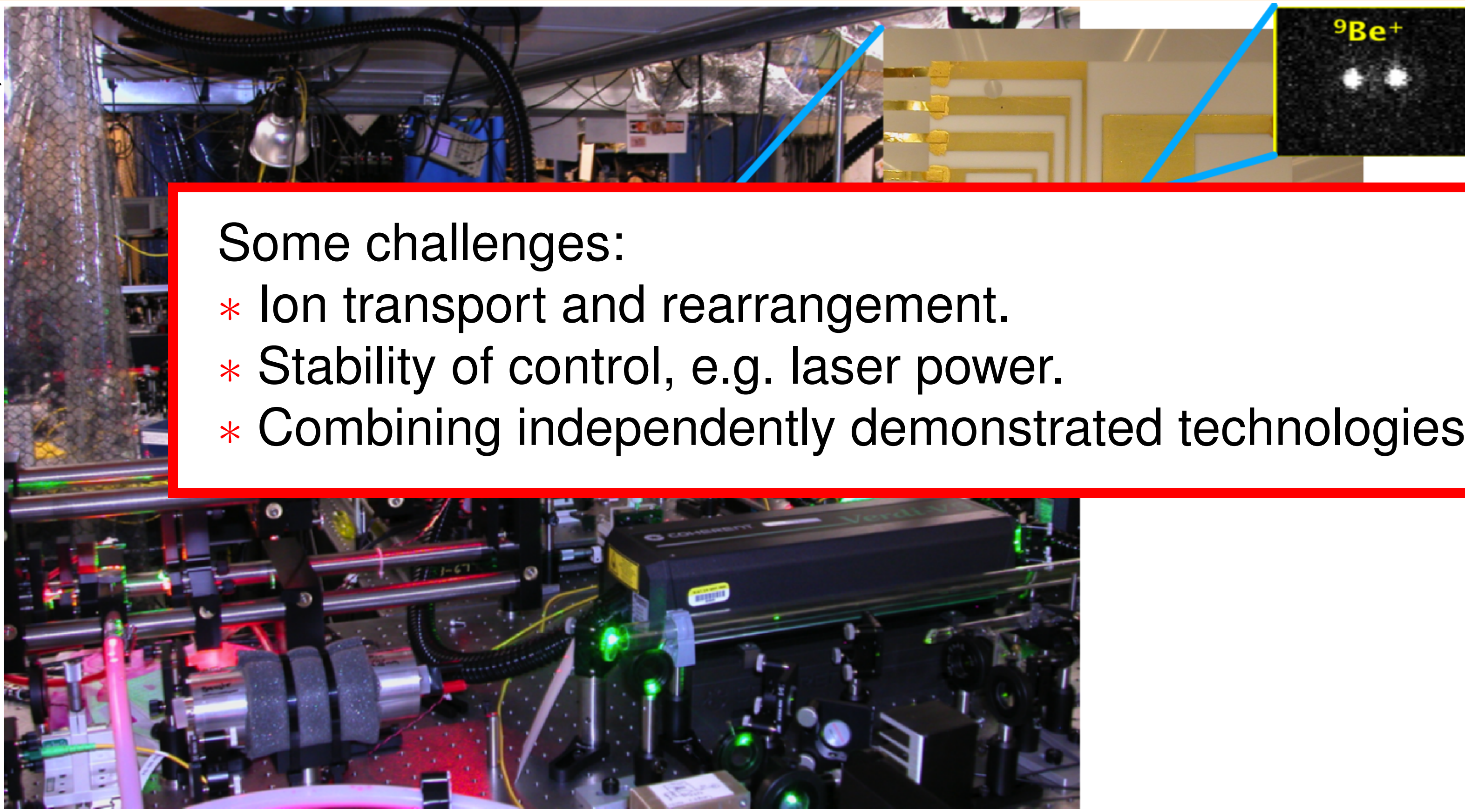
- ≈ 5 qubits, ≈ 3 two-qubit gates are realizable.
- Error per two-qubit gate $\gtrsim 3\%$ plus $>50\%$ loss.
- Not realistically scalable due to lack of determinism.

Review: Kok&al. '07 [31]

...optical QC is scalable in principle.

Ion Trap QC

Joe Britton, NIST



Some challenges:

* Ion transport and rearrangement.
* Stability of control, e.g. laser power.
* Combining independently demonstrated technologies.

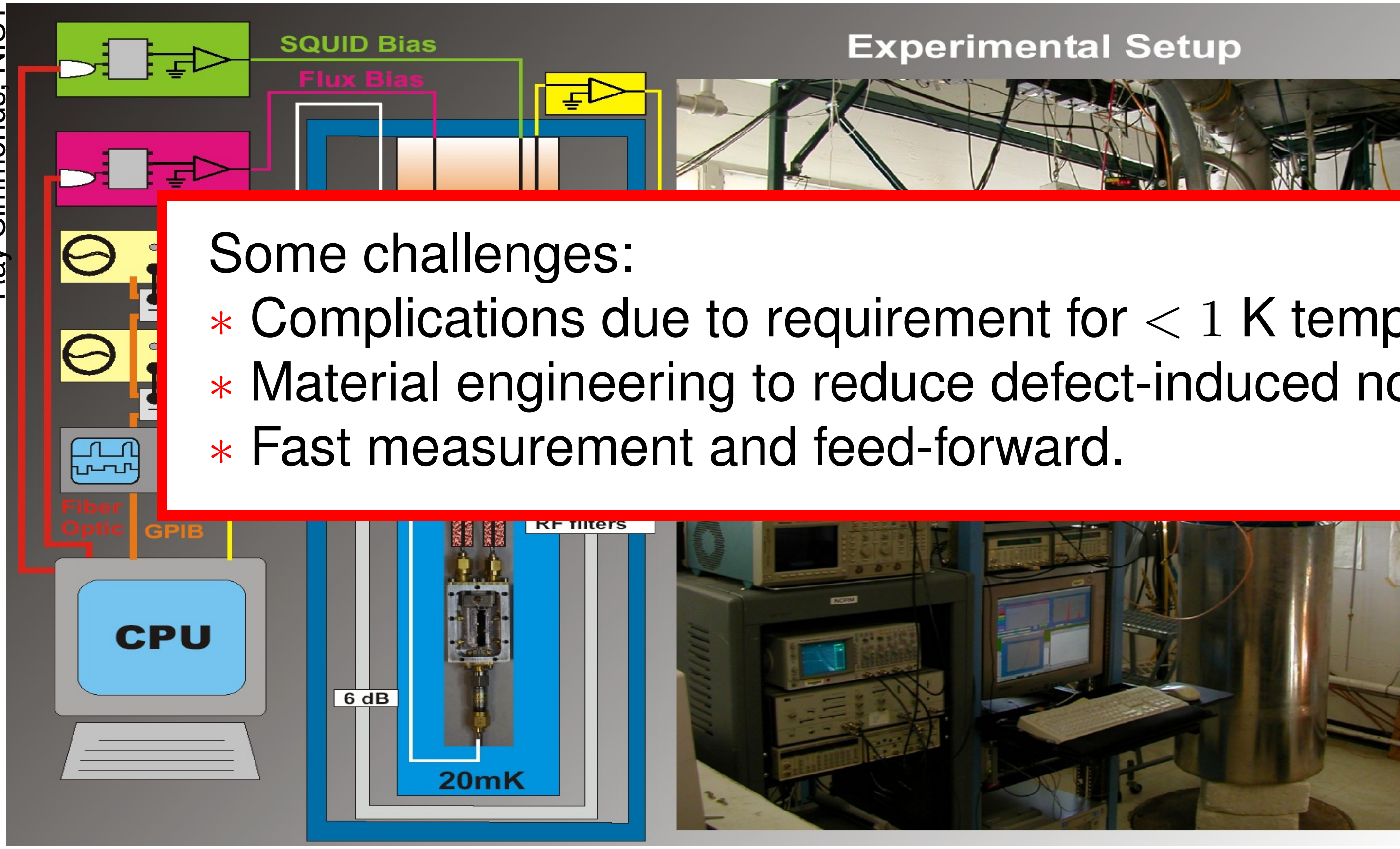
NIST

- Up to 8 qubits (2 routinely), $\gtrsim 5$ gates are realizable.
- $\lesssim 1\%$ achieved for two-qubit gate error.
- Currently closest to realistic scalability.

Reviews: Wineland&al. ’98 [32], Kielpinski ’08 [33]

Superconducting QC

Ray Simmonds, NIST



R. Simmond's lab, NIST

Some challenges:
* Complications due to requirement for < 1 K temperatures.
* Material engineering to reduce defect-induced noise.
* Fast measurement and feed-forward.

- Approaching 2 qubits, $\gtrsim 2$ gates.
- $\approx 15\%$ achieved for two-qubit gate error.
- Should be scalable with sufficient engineering.

Review: Devoret&al. '04 [34]

Experimental QC: Summary

System	Physical qubits	Number of qubits	Number of gates	Error per gate	Scal-able?
NMR	Nuclear spins in a molecule	$\lesssim 10$	$\lesssim 20$	$\gtrsim 1\%$	No
Photonic	Polarization of photons	$\lesssim 5$	$\lesssim 4$	$\gtrsim 3\%$ $> 50\%$	No
Ion trap	Energy levels of trapped ions	$\lesssim 8$	$\lesssim 4$	$\sim 1\%$ $\sim 10\%$	Yes
Superconducting	Collective states of superconducting circuits	< 2	$\lesssim 2$	$>15\%?$	Yes?
Cold atom	Energy levels of trapped atoms	< 2	$\lesssim 2$	?	?
Atomic impurities	Localized states of impurities	< 3	$\lesssim 4?$	?	?
Quantum dot	Localized states at dots	≈ 1	?	?	?
Topological	Delocalized states associated with excitations	0	0	??	??

. . . as of 2008.

Evaluating and Comparing Quantum Devices

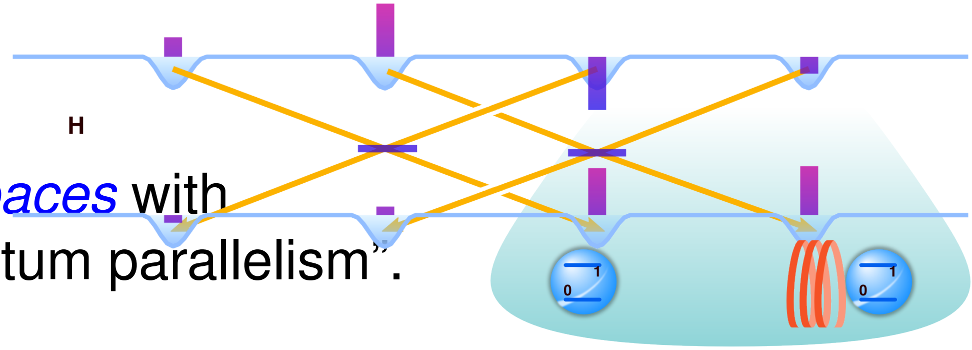
- How much quantum computing has been demonstrated?
  - How to measure "number of qubits used"?
  - How to measure "number of gates realized"?

- Log Quantum Speedup (LQS):

  L_2QS of 1.58 (implicitly) demonstrated with ion qubits on "unique quantum search with random target".

  Schaetz&al. '04 [37], Brickman&al. '05 [38]

Summary and Prospects



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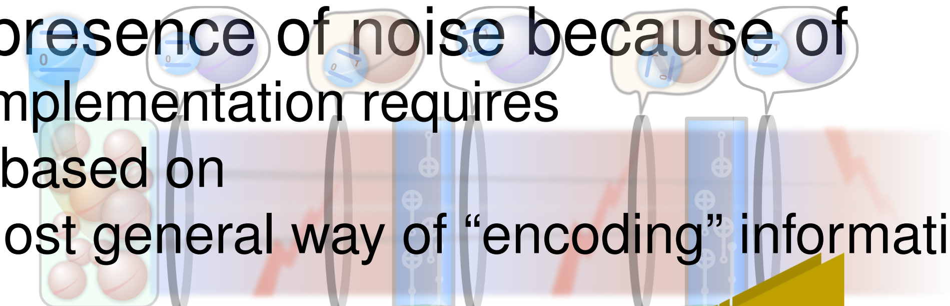

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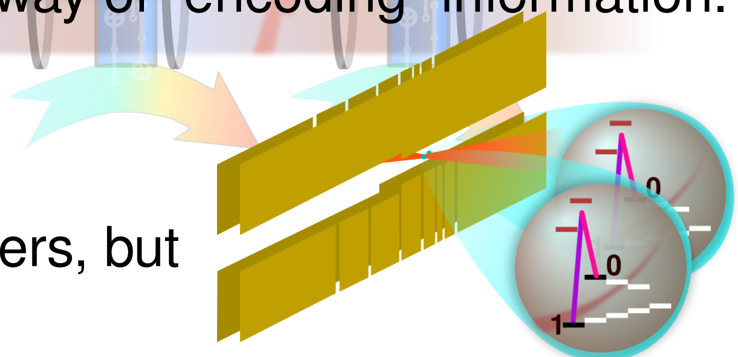

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Quantum computing or new physics!

Contents

References

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