Achievable Rate Estimates from Fiber-Optic Transmission Experiment Measurements

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References


Outline

1. Fiber-Optic Transmission Experiment
2. Achievable Information Rates
3. Coded Modulation System Design
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Experimental Setup
Experimental Setup: Interfaces

Input $x^n$

Output $y^n$
Input Sequences

We provide input sequences to the experimentalist:

- $n = 200\,000$ QAM symbols $x^n = x_1, x_2, \ldots, x_n$.
- **Probabilistic shaping:** use outer QAM symbols less often:
Example 64-QAM Distributions $P_X$
Measurement Campaign

- Measurement campaign with provided input sequences $x^n$.
- For each measurement, a noisy output sequences $y^n = y_1, y_2, \ldots, y_n$ is stored.
- We get a data set with the noisy output sequences $y^n$. 
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Reliable Transmission

- The receiver must recover input sequence $x^n$ from output sequence $y^n$.
- An **achievable information rate** (AIR) indicates if recovery is possible.
Channel Coding Theorem (Achievability Part)

- Memoryless channel $p_{Y|X}$.
- Random code $C = \{X^n(1), \ldots, X^n(2^{nR})\}$ with entries iid $\sim P_X$.
- Message $w \in \{1, 2, \ldots, 2^{nR}\}$
- ML decoder
  \[
  \hat{W} = \arg\max_w p_{Y^n|X^n}(Y^n|X^n(w)) = \prod_{i=1}^{n} p_{Y|X}(Y_i|X_i(w))
  \]
- Error probability $\Pr(W \neq \hat{W}) \xrightarrow{n \to \infty} 0$ if
  \[
  R < I(X; Y).
  \]
Estimating Mutual Information

- **Mutual information:**

\[
I(X; Y) = E \left[ \log \frac{p_{Y|X}(Y|X)}{p_Y(Y)} \right].
\]

- **Calculation by Monte-Carlo simulation:** Sample sequences \( x^n, y^n \) of \( n \) independent channel uses.

- **Weak law of large numbers:**

\[
I(X; Y) \approx \hat{I}(x^n; y^n) := \frac{1}{n} \sum_{i=1}^{n} \log \frac{p_{Y|X}(y_i|x_i)}{p_Y(y_i)}.
\]
Measurements

- $\hat{I}$ can be calculated also when $x^n, y^n$ are measurements of some channel.
- The memoryless channels $p_{Y|X}$ is now an auxiliary channel of our choice.
Auxiliary AWGN Channel

- Input distribution is $P_{X^n} = P^n_X$.
- Memoryless auxiliary output channel $p_{Y^n|X^n} = p^n_{Y|X}$
- Auxiliary I/O relation

$$Y = h \cdot X + Z$$

with $Z \sim \mathcal{N}(0, \sigma^2)$.
- Choose $h, \sigma^2$ maximizing $\hat{I}$. 
What is the operational meaning of $\hat{I}$ now?

Assumptions of channel coding theorem not fulfilled:

- We don’t know the channel $p_{Y^n | X^n}$.
- The “true” channel very likely has memory.
Mismatched Decoding


LM Rate

- Random code ensemble $\sim P_{X^n}$.
- Auxiliary channel $q(\cdot|\cdot)$
- Decoder $\hat{W} = \arg\max_w q(\tilde{Y}^n|X^n(w))$.
- Achievable rate

$$R_{LM} = \mathop{\text{p-\lim\,inf}}_{n \to \infty} \frac{1}{n} \log \frac{q(\tilde{Y}^n|X^n)^s r(X^n)}{q_s(\tilde{Y}^n)} =: \hat{R}_{LM}$$

where

- Auxiliary output distribution $q_s(\cdot) = E[q(\cdot|X^n)^s r(X^n)]$
- $s \geq 0$.
- Function $r: \frac{1}{n} \log[r(X^n)] \xrightarrow{n \to \infty} E\{\frac{1}{n} \log[r(X^n)]\}$. 
Mutual Information Estimate

- $\hat{I} = \hat{R}_{LM}$ for $s = 1$, $r(x) = 1$, and
  
  $$q = p^n_{Y|X}$$

$\Rightarrow$ $\hat{I}$ is an achievable rate lower bound for a decoder assuming memoryless channel $q = p^n_{Y|X}$.

- Optimizing over $s, r$ may improve the bound.
Discussion: Signal-to-Noise Ratio

- “Signal-to-noise ratio” is

\[
\text{SNR} = \frac{h^2 \mathbb{E}[|X|^2]}{\sigma^2}
\]

- Depends on our model parameters \( h, \sigma^2 \).
- Can be very different from OSNR measured by the spectral analyzer.
Discussion: Parameter Choice at the Receiver

- MMSE estimate:
  \[ h = \frac{xy^H}{xx^H}, \quad \sigma^2 = \frac{1}{n}(yy^H - |h|^2xx^H) \]

- Problem: the receiver does not know \( x^n \).
Discussion: Scatterplot
Discussion: Blind Estimation of $h, \sigma^2$

- **Blind**: use only $y^n$ to optimize $h$.
- Approach:

  $$D(p_Y \| q_Y) \geq 0 \Rightarrow \mathbb{E} \left[ \log_2 \frac{1}{q_Y(Y)} \right] \geq H(Y).$$

  $$\Rightarrow \text{minimize expectation over } q_Y.$$

- Choose

  $$q_{Y_h}(y) = \sum_{x \in \mathcal{X}} P_X(x) P_{Z_h}(y - h \cdot x)$$

  where $Z_h \sim \mathcal{N}(0, \frac{yy^H}{n} - |h|^2 \text{Var}(X))$,

  $$h_{\text{blind}} = \arg\min_h \frac{1}{n} \sum_{i=1}^n \log_2 \frac{1}{q_{Y_h}(y_i)}$$
AIR Estimates

Loops

$\hat{I}_{\text{awgn}}$ [bit/QAM symbol]

64-QAM ($P_1$)
64-QAM ($P_2$)
64-QAM ($P_3$)
64-QAM ($P_4$)
16-QAM Uniform
64-QAM Uniform

Net data rate (Gbit/s)

480 960 1440 1920 2400 2880 3360 3840 4320 4800

Distance (km)

2 4 6 8 10 12 14 16 18 20 22 24

3
3.5
4
4.5
5
5.5
6

$\hat{I}_{\text{awgn}}$ [bit/QAM symbol]

Loops

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Input Distribution $P_X$

- **Symmetry:**
  \[ P_X(x) = P_X(-x). \]

- **Amplitude-Sign Factorization:**
  \[ P_X(x) = P_A(|x|)P_S(\text{sign}(x)) \]
  where \( A := |X| \) and \( S := \text{sign}(X) \).

- **Uniform sign:**
  \[ P_S(-1) = P_S(1) = \frac{1}{2}. \]
Generation of Amplitude Sequence

Constant Composition Distribution Matching (CCDM) \(^2\):

\[
D^k \xrightarrow{\text{CCDM}} A^n
\]

- Data bits \(D_i\) iid Bernoulli(1/2).
- \(A_i \sim P_A\).
- Rate is \(k/n = H(A)\).
- Invertible: \(D^k\) can be recovered from \(A^n\) with zero error.

Shaping and Channel Coding

Binary systematic rate \( (m-1)/m \) generator matrix \( G = [I|P] \).
Binary amplitude representation \( b(A_i) \in \{0,1\}^{m-1} \).
Binary sign representation \( b(S_i) \in \{0,1\} \).
\( b(S)^n = b(A)^n P \).

Assumption: \( S_i \) is approximately uniformly distributed.

\[ A_i S_i \sim P_X. \]
Uniform Check Bit Assumption: Example
DVB-S2 rate 1/2 LDPC code

- **Data:** empirical distribution \( P_D(1) = 1 - P_D(0) = 0.1082 \).
- **Check bits:** empirical dist. \( P_R(1) = 1 - P_R(0) = 0.4970 \).
Probabilistic Amplitude Shaping (PAS)$^3$

\[ P_{S_i \cdot A_i} = P_X \]

Receiver

- Binary label of $X$ is $b(S)b(A) =: B_1B_2\cdots B_m$
- The demapper calculates bitwise soft-information

\[
L_j = \log \frac{P_{B_j}(0)}{P_{B_j}(1)} + \log \frac{p_{Y|B_j}(Y|0)}{p_{Y|B_j}(Y|1)}, \quad j = 1, \ldots, m.
\]

- No iterative demapping.
Numerical Results: Operating Points FER = “0”

![Graph showing operating points for different modulation schemes and distances.]

- **OP1**: 64-QAM (P1)
- **OP2**: 64-QAM (P2)
- **OP3**: 64-QAM (P3)
- **OP4**: 64-QAM Uniform
- **Ref1**: 64-QAM Uniform
- **Ref2**: 16-QAM Uniform

**Parameters**:
- **Loops**: 2 to 24
- **Distance (km)**: 2 to 24
- **Net data rate (Gbit/s)**: 480 to 5760
- **[bit/QAM symbol]**: 3 to 6
Conclusions

- Achievable Information Rates as interface between fiber-optic transmission experiment and coded modulation system design.
- Probabilistic shaping opportunities: reach & rate increase.
- Experimental work is rewarding.
References I


References II
