Tutorial on Iterative Detection and Decoding

S. ten Brink

Institute of Telecommunications (INÜ), University of Stuttgart, Germany

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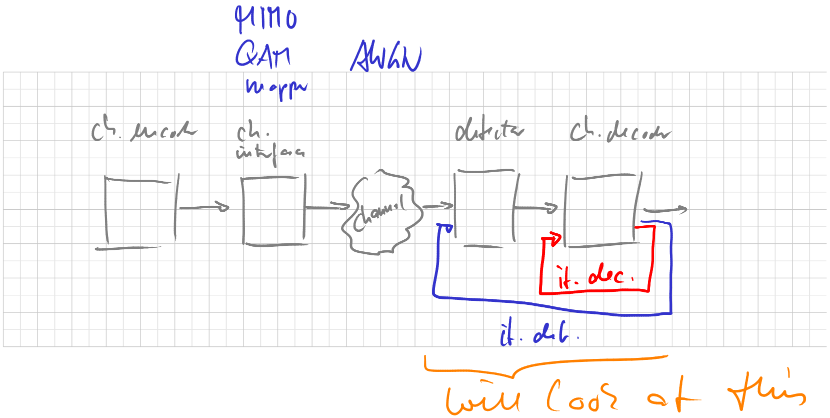
Zandvoort aan Zee
School of Inf. Theory

Outline

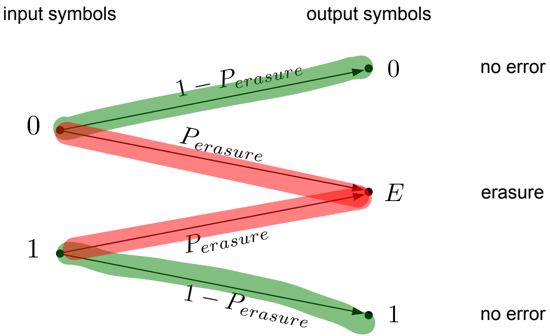
Outline

Intro - Channel Coding



- channel code, channel interface, channel

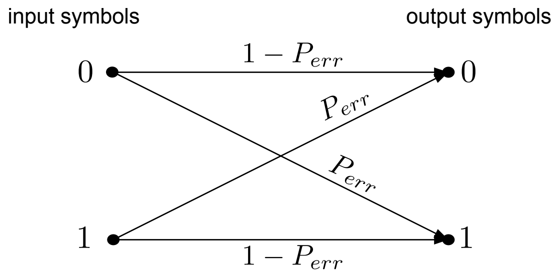
Intro - Simple Channels: BEC



- Binary Erasure Channel (BEC)
- Channel quality parameter: Erasure Probability $P_{erasure}$
- Capacity $C_{BEC} = 1 - P_{erasure}$

↳ used to model prior knowledge (later)

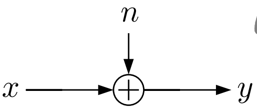
Intro - Simple Channels: BSC



- Binary Symmetric Channel (BSC)
- Channel quality parameter: Error probability P_{err}
- Capacity $C_{BSC}\left(P_{err}\right)=1-H_b\left(P_{err}\right)$
  with $H_b\left(P_{err}\right)=-P_{err}\cdot \mathrm{ld} P_{err}-\left(1-P_{err}\right)\cdot \mathrm{ld}\left(1-P_{err}\right)$

Intro - Simple Channels: AWGN

waveform channel



- Additive White Gaussian Noise (AWGN) channel
- n Gaussian distributed, mean zero, variance σ^2
- Channel quality parameter: $SNR = E_s/2\sigma^2 = E_s/N_0$
- Capacity $C_{AWGN} = \log_2(1 + SNR)$

passband noise

Intro - Simple Codes: Repetition

information bit

- (N,K) block code of rate $R = K/N$
- repetition codes $R = 1/N$, N-fold repetition $\mathbf{c} = (u, \ldots, u)$
- on BEC: can correct $N-1$ erasures
- on BSC: can correct $\lfloor (N-1)/2 \rfloor$ errors
- before Shannon: we have to sacrifice rate to achieve reliability
- Shannon (1948): arbitrarily good reliability achievable as long as the code rate R is below the channel capacity C
- important for coding guys: SNR normalized to energy used per information bit

$$\frac{E_b}{N_0} = \frac{1}{M_b R} \frac{E_s}{N_0} = N \frac{E_s}{N_0}$$

→ LDPC codes

BPSK: $M_b = 1$, $R = \frac{1}{N}$

Intro - Simple Codes: Single Parity Check

- (N,N-1) block code of rate $R = N - 1/N$
- $\mathbf{c} = (u_1, u_2 \ldots, u_{N-1}, u_1 \oplus u_2 \oplus \ldots \oplus u_{N-1})$
- on BEC: can correct one erasure
- on BSC: can not correct errors; only detect odd number of errors

↳ später LDPC

Intro - Simple Codes: Multiple Parity Checks

N=7 K=4

- e.g., (7,4) Hamming code, rate $R = 4/7$
- $\mathbf{c} = (\underbrace{u_1, u_2, u_3, u_4}, \underbrace{u_1 \oplus u_2 \oplus u_3, u_1 \oplus u_2 \oplus u_4, u_1 \oplus u_3 \oplus u_4}_{N-K \text{ parity bits}})$
- on BEC: can correct two erasures
- on BSC: can correct one error; detect two errors
- extended Hamming code: additional parity bit $u_2 \oplus u_3 \oplus u_4$

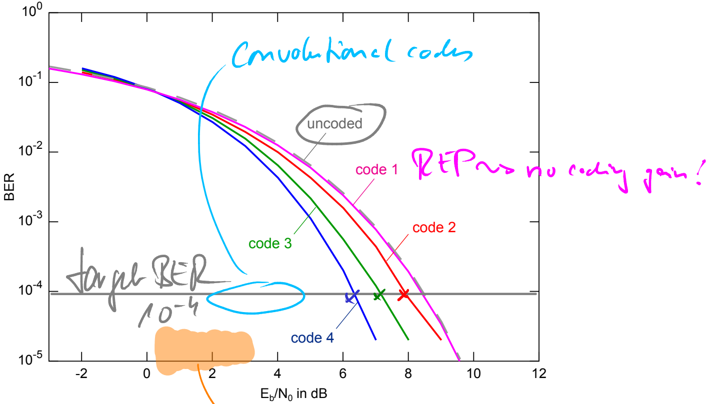
Intro - Comparison of Simple Codes

- An Experiment: Comparison of four $R = 1/2$ codes
- code 1: $\mathbf{c} = (u, u)$ Rep. Code
- code 2: $\mathbf{c} = (u_1, u_2, u_1 \oplus u_2, u_1)$ SPC + REP
- code 3: $\mathbf{c} = (u_1, u_2, u_3, u_1 \oplus u_2, u_1 \oplus u_3, u_2 \oplus u_3)$ multiple parity check
- code 3:
  $\mathbf{c} = (u_1, u_2, u_3, u_4, u_1 \oplus u_2 \oplus u_3, u_1 \oplus u_2 \oplus u_4, u_1 \oplus u_3 \oplus u_4, u_2 \oplus u_3 \oplus u_4)$

extended Hamming
$(7,4) \rightarrow (8,4)$

Intro - Simple Codes, BER Chart

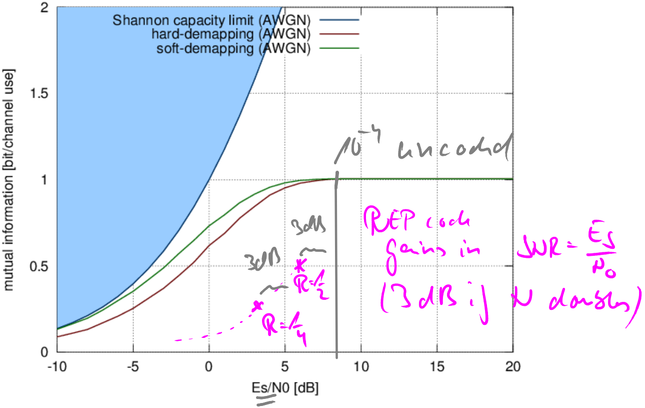
$R = \frac{1}{2}$



iterative decoding

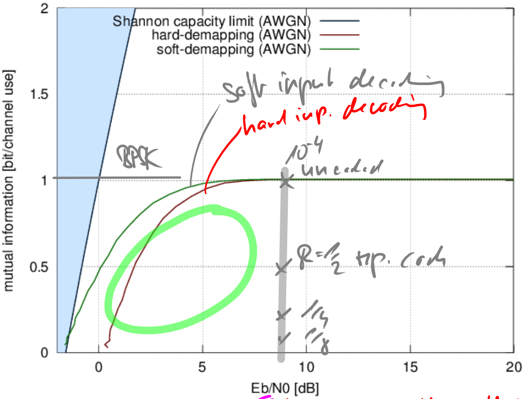
- code 1 (repetition) has no coding gain
- code 4 has highest coding gain

Intro - Mutual Information Chart vs E_s/N_0



- AWGN-channel: repetition code has some gain...?
- http://webdemo.inue.uni-stuttgart.de/webdemos/02_lectures/communication_3/performance_measures/

Intro - Mutual Information Chart vs E_b/N_0



null, nothing, nada, nient, nichts ...

- Now versus E_b/N_0: repetition code has no coding gain!

Intro - Need for Coding

- long, random-like codes (with structure) suffice
- various coding schemes emerged over the past 50 years
- and: channel interface should make channel "look nice" (Gaussian-like) for channel code
- soft-input decoding seems important
- soft-output decoding...?

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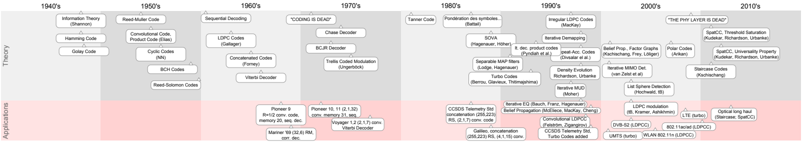
Intro - Channel Coding Timeline, 1940-2020



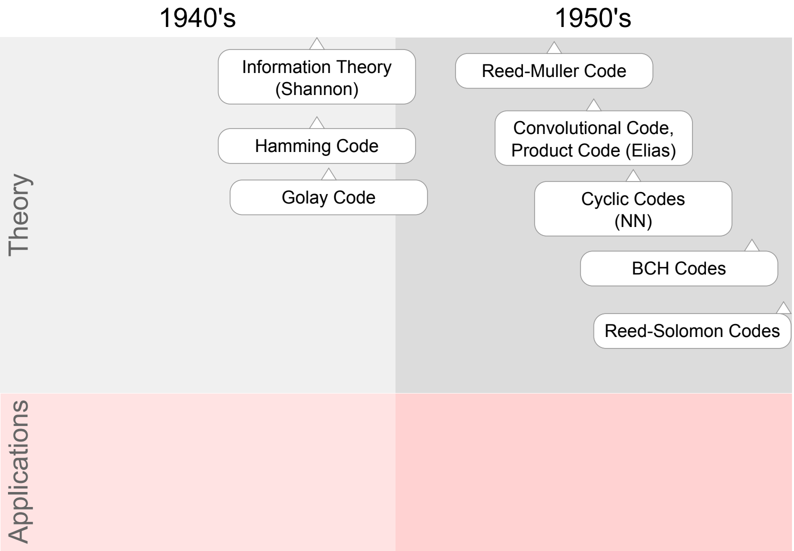
Note: this is an attempt of a coding timeline with perspective on it. decoding, it. detection

→ incomplete, biased, ...

send me email with improvements

tenbrink@ieee.org thanks

Intro - Channel Coding Timeline, 1940-1960



Intro - Channel Coding Timeline, 1960-1980

Really? ....

1960's

- Sequential Decoding
- LDPC Codes (Gallager)
- Concatenated Codes (Forney)
- Viterbi Decoder

RS + conv.

1970's

- "CODING IS DEAD"
- Chase Decoder
- BCJR Decoder
- Trellis Coded Modulation (Ungerböck)

Soft in/out

Applications

- Pioneer 9 R=1/2 conv. code, memory 20, seq. dec.
- Mariner '69 (32,6) RM, corr. dec.
- Pioneer 10, 11 (2,1,32) conv. memory 31, seq.
- Voyager 1,2 (2,1,7) conv. Viterbi Decoder

Intro - Channel Coding Timeline, 1980-2000

it-detection

irregular

1980's

Tanner Code

Pondération des symboles... (Battail)

SOVA (Hagenauer, Höher)

Separable MAP filters (Lodge, Hagenauer)

Turbo Codes (Berrou, Glavieux, Thitimajshima) IT+

CCSDS Telemetry Std concatenation (255,223) RS, (2,1,7) conv. code

Galileo, concatenation (255,223) RS, (4,1,15) conv.

1990's

Irregular LDPC Codes (MacKay)

Iterative Demapping

It. dec. product codes (Pyndiah et al.)

epeat-Acc. Codes (Divsalar el al.)

Density Evolution Richardson, Urbanke

Iterative MUD (Moher)

extrinsic

Iterative EQ (Bauch, Franz, Hagenauer)

Belief Propagation (McEliece, MacKay, Cheng)

Convolutional LDPCC (Felström, Zigangirov)

CCSDS Telemetry Std, Turbo Codes added

Intro - Coding History Timeline, 2000-2020

yeah yeah... of course...

2000's

- Belief Prop., Factor Graphs (Kschischang, Frey, Löliger)
- Polar Codes (Arikan)
- Iterative MIMO Det. (van Zelst et al)
- List Sphere Detection (Hochwald, tB)
- LDPC modulation (tB, Kramer, Ashikhmin)
- LTE (turbo)
- DVB-S2 (LDPCC)
- 802.11ac/ad (LDPCC)
- UMTS (turbo)
- WLAN 802.11n (LDPCC)

"THE PHY LAYER IS DEAD"

2010's

- SpatCC, Threshold Saturation (Kudekar, Richardson, Urbanke)
- SpatCC, Universality Property (Kudekar, Richardson, Urbanke)
- Staircase Codes (Kschischang)
- Optical long haul (Staircase; SpatCC)

X your innovation here

spatial coupling

↳ b > 200000

Notes

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Outline

Soft Decoding - Log-Likelihood Ratio Values

$x \rightarrow \oplus \xrightarrow{y} L(x|y)$, n

- Discrete-time channel model for binary, antipodal signaling

$$y = x + n;\; x \in \{\pm\sqrt{E_s}\}$$

- realizations of n, e.g., i.i.d. Gaussian; in the following $E_s = 1$
- A posteriori L-value ("soft channel output") given by the ratio of the a posteriori probabilities $P(x = \pm 1|y)$

a posteriori

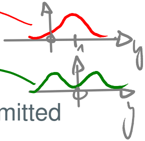
$$L(x|y) = \ln \frac{P(x = +1|y)}{P(x = -1|y)}.$$

0 - hypothesis
1 - hypothesis

- Applying Bayes' rule yields

a priori

$$P(x = +1|y) = \frac{p(y|x = +1)}{p(y)} \cdot P(x = +1)$$



  - a priori probability $P(x = +1)$ that $x = +1$ was transmitted,
  - channel output PDF $p(y|x = +1)$ conditioned on the transmitted symbol $x = +1$
  - and $p(y) = P(x = -1) \cdot p(y|x = -1) + P(x = +1) \cdot p(y|x = +1)$

Soft Decoding - Log-Likelihood Ratio Values

if we had redundancy or coupling

- Finally, we obtain the a posteriori L-value as

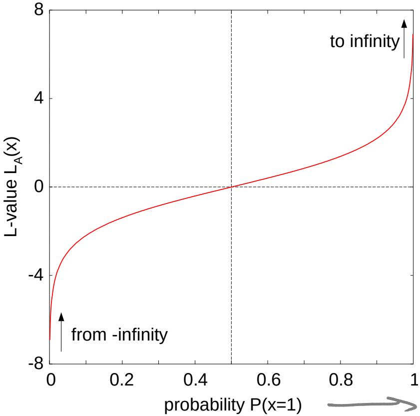
$$L(x|y) = \underbrace{\ln \frac{P(x=+1)}{P(x=-1)}}_{L_A(x)} + \underbrace{\ln \frac{p(y|x=+1)}{p(y|x=-1)}}_{L_{ch}(x|y)} + \underbrace{L(x|y)}_{E}$$

soft channel output

- Sign of L-value: hard decision
- Absolute value $|L(x|y)|$: reliability of the decision
- A priori L-value $L_A(x)$ accounts for available prior knowledge on x
- Channel L-value $L_{ch}(x|y)$: knowledge on x based on y
- Additions/subtractions rather than multiplications/divisions

Soft Decoding - Log-Likelihood Ratio Values

L-values



prob

- Connection between a priori probability $P(x=1)$ and corresponding L-value $L_A(x) = \ln \frac{P(x=+1)}{P(x=-1)} = \ln \frac{P(x=+1)}{1-P(x=+1)}$

Soft Decoding - Log-Likelihood Ratio Values

- For the AWGN channel we have

$$p(y|x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left[-\frac{(y-x)^2}{2\sigma^2}\right]$$

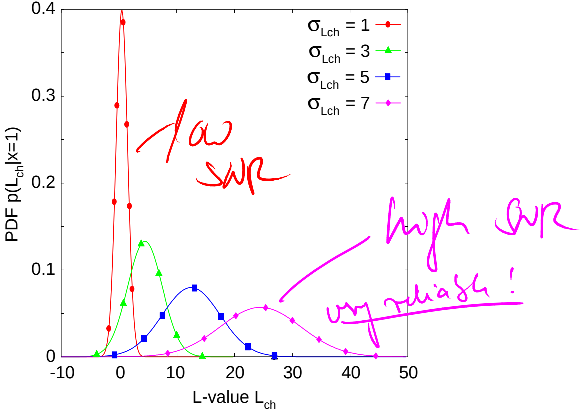
and obtain

$$L_{ch}(x|y) = \ln \frac{\exp\left[-\frac{(y-1)^2}{2\sigma^2}\right]}{\exp\left[-\frac{(y+1)^2}{2\sigma^2}\right]} = \frac{2}{\sigma^2} \cdot y$$

- Channel L-values are simply weighted versions of the channel observations $y = x + n$
- Can be easily included into a metric for soft input decoding (e.g. Viterbi algorithm)

Soft Decoding - Log-Likelihood Ratio Values

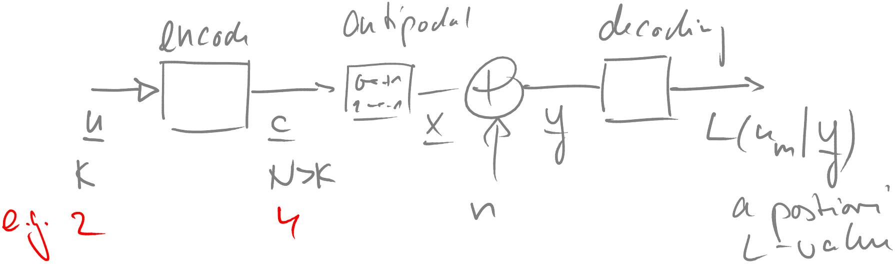
- Channel L-values $L_{ch}(x|y)$ for transmitted symbol $x = 1$



- It can be shown that the mean value $\mu_{L_{ch}}$ and the variance $\sigma^2_{L_{ch}}$ satisfy $\mu_{L_{ch}} = \sigma^2_{L_{ch}}/2$

Soft Decoding - Difference of ML versus APP Decoding

- Encoder: maps K information bits $\underline{u}$ into the sequence of N coded bits $\underline{c} = \mathrm{map}(\underline{u})$
  (or antipodal sequence $\underline{x}(\underline{u} \in \{\pm 1\})$ respectively)
- Mapping is one-to-one and thus invertible $\underline{u} = \mathrm{map}^{-1}(\underline{c})$
- Received signal from the channel: $\underline{y} = \underline{x} + \underline{n}$.
- Realizations of $\underline{n}$ are i.i.d. Gaussian, mean zero, variance σ^2
- Channel decoder: obtain estimate $\hat{\underline{u}}$ of transmitted bit sequence $\underline{u}$



Soft Decoding - ML and APP Decoding

1 2

- Two decoding rules based on different optimization criteria:
  - 1 Minimizing the sequence error probability P_{seq}
  - 2 Minimizing the bit error probability P_b
- Minimizing P_{seq}: maximum likelihood sequence estimation (MLSE, or ML)
- Minimizing P_b: take hard decision on a posteriori probabilities (APP) of information bits (APP, or MAP decoding)

↳ compute a posteriori prob.

Viterbi decoding

Soft Decoding - ML-Decoding

- ML decoding: Receiver generates all possible 2^K transmitted codeword hypotheses $\underline{c}$ (message hypotheses $\underline{u} = \mathrm{map}^{-1}(\underline{c})$) and selects that one which maximizes the sequence a posteriori probability

$$\max_{\forall \underline{u}} P\left(\underline{u} \middle| \underline{y}\right)$$

- Assuming that all transmitted message vectors $\underline{u}$ are equally likely, we obtain a maximization of the likelihood function

$$\max_{\forall \underline{u}} p\left(\underline{y} \middle| \underline{u}\right)$$

- Complexity of direct approach (testing all 2^K message hypotheses) grows exponentially in the message length K
- Viterbi algorithm: Exploits trellis structure of convolutional codes; complexity grows only linearly in K

Soft Decoding - ML-Decoding

- For the AWGN channel, the likelihood computation reduces to

$$\tilde{p}\left(\underline{y}\,|\underline{x}\,(\underline{u})\right) = \sum_{i=0}^{N-1} x_i \cdot y_i = \underline{x} \cdot \underline{y}^T$$

- A simple correlation is sufficient to find the best matching hypothesis $\underline{x}$ (codeword $\underline{c}$, message $\underline{u}$ respectively)

Soft Decoding - APP Decoding

- Maximum a posteriori probability criterion
  - optimizes the bit (symbol) error probability rather than the sequence error probability
- Provides soft output values, expressed in terms of a posteriori log-likelihood ratio values (L-values)
  - based on a posteriori probabilities with respect to the bits (not the sequence, as opposed to MLSE)
- Maximum A Posteriori Probability decoding is abbreviated as “APP decoding”, or “MAP decoding”

Soft Decoding - APP Decoding

$K=2$ $\quad$ $N=4$

$\underline{u}=(u_0,u_1) \longrightarrow \square \longrightarrow \underline{c}$ $\quad\ldots$

- Example: Compute a posteriori L-values for an arbitrary rate $1/2$ block code with $K=2$ and $N=4$
- E.g., the a posteriori L-value of bit u_0 conditioned on the received vector $\underline{y}=(y_0,y_1,y_2,y_3)$ is

$\underline{y}=\underline{x}+\underline{n}$

a posteriori L-value:

$$L_D\left(u_0\,|\underline{y}\right)=\ln\frac{P\left(u_0=0\,|\underline{y}\right)}{P\left(u_0=1\,|\underline{y}\right)}$$

noisy c.w. after channel

The probability

$$P\left(u_0=0\,|\underline{y}\right)=P\left(u_0=0,u_1=0\,|\underline{y}\right)+P\left(u_0=0,u_1=1\,|\underline{y}\right)$$

is the a posteriori probability that the transmitted information bit was $u_0=0$

Soft Decoding - APP Decoding

zu bestimmen

- Next we apply Bayes' rule

$$P\left(u_0, u_1 \middle| \underline{y}\right) = \frac{p\left(\underline{y} \middle| u_0, u_1\right)}{p\left(\underline{y}\right)} \cdot P\left(u_0, u_1\right)$$

- The information bits u_0, u_1 (e. g. output of a memoryless source) can be assumed to be independent, and we have

$$P\left(u_0, u_1\right) = P\left(u_0\right) \cdot P\left(u_1\right)$$

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Soft Decoding - APP Decoding

- We obtain

$$L_D\left(u_0\,|\underline{y}\right) = \ln \frac{p\left(\underline{y}|u_0{=}0,u_1{=}0\right)\cdot P(u_0{=}0)\cdot P(u_1{=}0) + p\left(\underline{y}|u_0{=}0,u_1{=}1\right)\cdot P(u_0{=}0)\cdot P(u_1{=}1)}{p\left(\underline{y}|u_0{=}1,u_1{=}0\right)\cdot P(u_0{=}1)\cdot P(u_1{=}0) + p\left(\underline{y}|u_0{=}1,u_1{=}1\right)\cdot P(u_0{=}1)\cdot P(u_1{=}1)}$$

$$= \ln \frac{P(u_0{=}0)}{P(u_0{=}1)} + \ln \frac{p\left(\underline{y}|u_0{=}0,u_1{=}0\right)\cdot \frac{P(u_1=0)}{P(u_1=1)} + p\left(\underline{y}|u_0{=}0,u_1{=}1\right)}{p\left(\underline{y}|u_0{=}1,u_1{=}0\right)\cdot \frac{P(u_1=0)}{P(u_1=1)} + p\left(\underline{y}|u_0{=}1,u_1{=}1\right)}$$

$$= L_A\left(u_0\right) + \underbrace{\ln \frac{p\left(\underline{y}\,|u_0=0,u_1=0\right)\cdot \exp L_A\left(u_1\right) + p\left(\underline{y}\,|u_0=0,u_1=1\right)}{p\left(\underline{y}\,|u_0=1,u_1=0\right)\cdot \exp L_A\left(u_1\right) + p\left(\underline{y}\,|u_0=1,u_1=1\right)}}_{L_{E'}\left(u_0|\underline{y}\right)}$$

- A posteriori L-value $L_D(u_0|\underline{y})$ is composed of its a priori L-value $L_A(u_0)$ and the channel-and-“extrinsic” L-value $L_{E'}(u_0|\underline{y})$
- For systematic codes: can separate $L_{E'}(u_0|\underline{y})$ into channel observation $L_{ch}(u_0|y_0)$ and “pure” extrinsic L-value $L_E(u_0|\underline{y}_{[0]})$
- The extrinsic L-value: captures all information we learn about bit u_0 based on code redundancy and observation of bit u_1

Soft Decoding - APP Decoding

- Suppose that the first $K = 2$ bits of the codeword $\underline{c}$ are the systematic bits, $\underline{c} = (c_0 = u_0, c_1 = u_1, c_2, c_3)$
- Then we can factorize the PDF for the systematic bits u_i, $0 \leq i < 2$, into

$$p\left(\underline{y}\,|\underline{u}\right) = p\left(\underline{y}\,|\underline{c} = \mathrm{map}\left(\underline{u}\right)\right) = p\left(y_i\,|c_i\right) \cdot p\left(\underline{y}_{[i]}\,\middle|\underline{c}_{[i]}\right) = p\left(y_i\,|c_i\right) \cdot p\left(\underline{y}_{[i]}\,|\underline{u}\right)$$

and find for u_0

$$L_D\left(u_0\,\middle|\underline{y}\right) = L_A\left(u_0\right) + \underbrace{\ln \frac{p\left(y_0\,|u_0 = 0\right)}{p\left(y_0\,|u_0 = 1\right)}}_{L_{ch}\left(u_0|y_0\right)}$$

$$+\underbrace{\ln \frac{p\left(\underline{y}_{[0]}\,|u_0 = 0, u_1 = 0\right) \cdot \exp L_A\left(u_1\right) + p\left(\underline{y}_{[0]}\,|u_0 = 0, u_1 = 1\right)}{p\left(\underline{y}_{[0]}\,|u_0 = 1, u_1 = 0\right) \cdot \exp L_A\left(u_1\right) + p\left(\underline{y}_{[0]}\,|u_0 = 1, u_1 = 1\right)}}_{L_E\left(u_0\middle|\underline{y}_{[0]}\right)}$$

Soft Decoding - APP Decoding

- Notation: $\underline{y}_{[i]}$ is the vector $\underline{y}$ where the ith element is omitted, i.e. $\underline{y}_{[i]} = (y_0, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_{N-1})$
- For a general number of K information bits we find

$$L_D\left(u_i \middle| \underline{y}\right) = L_A\left(u_i\right) + \underbrace{\ln \frac{\sum\limits_{\underline{u}\in\mathbb{U}_{i,0}} p\left(\underline{y}\middle|\underline{u}\right) \cdot \exp \sum\limits_{j\in\mathbb{J}_{i,\underline{u}}} L_A\left(u_j\right)}{\sum\limits_{\underline{u}\in\mathbb{U}_{i,1}} p\left(\underline{y}\middle|\underline{u}\right) \cdot \exp \sum\limits_{j\in\mathbb{J}_{i,\underline{u}}} L_A\left(u_j\right)}}_{L_{E'}\left(u_i\middle|\underline{y}\right)}$$

with $\mathbb{U}_{i,0}$ being the set of 2^{K-1} bit vectors $\underline{u}$ having $u_i = 0$

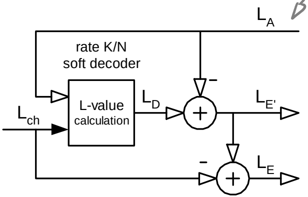
$$\mathbb{U}_{i,0} = \left\{\underline{u}\middle|u_i = 0\right\},\ \mathbb{U}_{i,1} = \left\{\underline{u}\middle|u_i = 1\right\} \text{ respectively}$$

and $\mathbb{J}_{i,\underline{u}}$ being the set of indices j with

$$\mathbb{J}_{i,\underline{u}} = \left\{j\middle|0 \le j < K,\ j \ne i \wedge u_j = 0\right\}$$

Soft Decoding - APP Decoding

coming from another component dec.



non-systematic codes
SCC

systematic codes
PCC

$\ln \frac{P(u_j=0)}{P(u_j=1)}$

- For systematic codes with $\underline{c} = (c_0 = u_0, \ldots, c_{K-1} = u_{K-1}, c_K, \ldots, c_{N-1})$ we can extract the "pure" extrinsic information for u_i, $0 \leq i < K$

0, or: from another decoding block

separately into:

$$L_D\left(u_i \middle| \underline{y}\right) = L_A(u_i) + L_{ch}(u_i) + \underbrace{\ln \frac{\sum_{\underline{u}\in\mathbb{U}_{i,0}} p\left(\underline{y}_{[i]} \middle| \underline{u}\right) \cdot \exp \sum_{j\in\mathbb{J}_{i,\underline{u}}} L_A(u_j)}{\sum_{\underline{u}\in\mathbb{U}_{i,1}} p\left(\underline{y}_{[i]} \middle| \underline{u}\right) \cdot \exp \sum_{j\in\mathbb{J}_{i,\underline{u}}} L_A(u_j)}}_{L_E\left(u_i \middle| \underline{y}_{[i]}\right)}$$

u_0 noisy c.w.

noisy version of u_0

↳ extrinsic information only depends on code constraints

Notes

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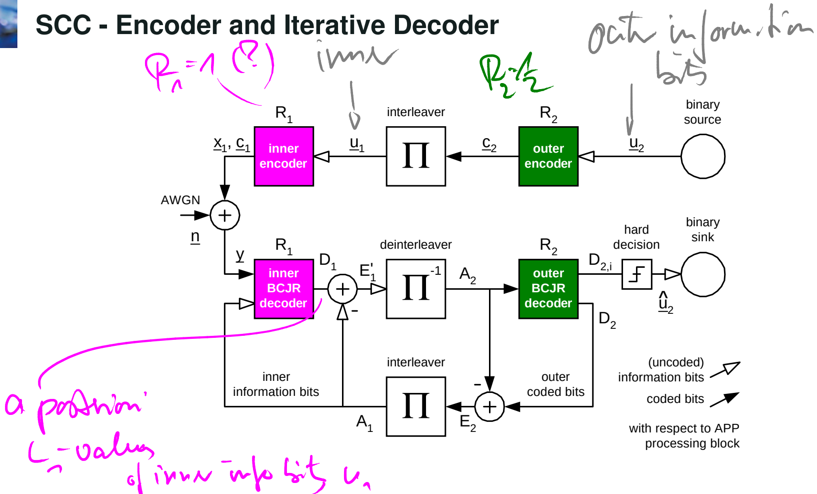
Outline

SCC - Encoder and Iterative Decoder



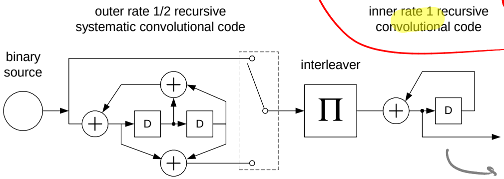
- Index "1": Elements belonging to inner encoding/decoding
- Index "2": Elements belonging to the outer encoding/decoding

SCC - BER Chart

but adds memory

rate 1 = only mapping

↳ no redundancy added



↳ accumulator (diff. enc.)

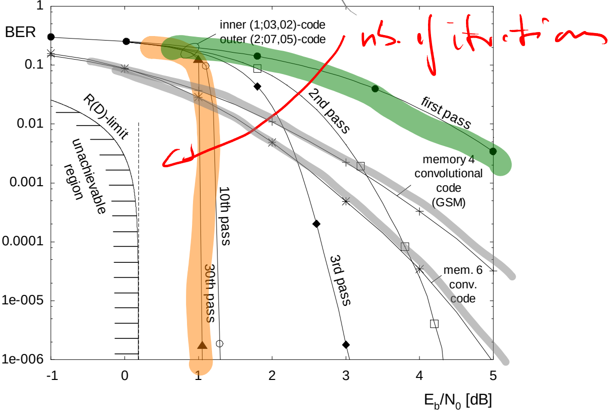
$\nu = 2$

$\nu = 1$

- Serially concatenated code consisting of outer rate $1/2$ memory 2 recursive systematic convolutional code $(G_r, G) = (07, 05)$ and inner rate 1 memory 1 (differential) code $(G_r, G) = (03, 02)$

SCC - BER Chart

(R=1/2)



- Bit error rate curves for serially concatenated code example; interleaver size $4 \cdot 10^5$ coded bits

how to design?

SCC - BER Chart

- We identify three typical regions of the BER chart:
  - The region of low $E_b/N_0 < E_b/N_0|_{\text{cliff}}$ with negligible iterative BER reduction
  - The turbo cliff region at about $E_b/N_0 \approx E_b/N_0|_{\text{cliff}}$ with persistent iterative BER reduction over many iterations
  - The BER floor region for moderate to high E_b/N_0-values in which a rather low BER can be reached after just a few number of iterations
- Property of the particular concatenation used

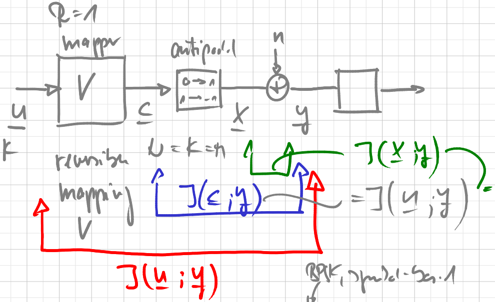
SCC - Inner Transfer Characteristics

- Experiment: Rate 1 mappings
- Compact definition of M-bit rate 1 mappings: regard vector realizations $\underline{u}$ and $\underline{c}$ as integer values v, $0 \leq v < 2^M$, with $v_{\underline{u}} = \sum_{m=0}^{M-1} u_m \cdot 2^m$, and $v_{\underline{c}}$ correspondingly
- The value $v_{\underline{u}}$ serves as an index to the elements of a vector $\mathbf{V}$ which contains the definition of the mapping

$$\mathbf{V} = \left(v_{\underline{c}}\left(v_{\underline{u}} = 0\right), v_{\underline{c}}\left(v_{\underline{u}} = 1\right), \ldots, v_{\underline{c}}\left(v_{\underline{u}} = 2^M - 1\right)\right)$$

- Example: The most simple mapping is the identity mapping $\underline{c} = \underline{u}$, given by the vector $\mathbf{V}_{id} = (0, 1, 2, \ldots, 2^M - 1)$; an arbitrary 2-bit mapping is defined by $\mathbf{V}_2 = (0, 3, 1, 2)$, an arbitrary 3-bit mapping by $\mathbf{V}_3 = (0, 4, 1, 2, 3, 5, 7, 6)$
- There are $(2^M)!$ different M-bit mappings possible; but many are equivalent

Notes



id. mapping: $I(\underline{u};\underline{y}) = N \cdot C_G$

$$I(\underline{u};\underline{y}) = I(\underline{c};\underline{y}) = I(\underline{x};\underline{y})$$

Mapping is information preserving (reversible)

SCC - Inner Transfer Characteristics

- Conditional Mutual Information and Discrete A Priori Knowledge:
- Assumption: AWGN channel, BPSK
- (Average) mutual information between transmitted information bit vector $\underline{u}$ and the noise-corrupted channel output vector $\underline{y} = \underline{x} + \underline{n}$:

$$I(\underline{U};\underline{Y}) = \sum_{\forall \underline{u}} P(\underline{U} = \underline{u}) \cdot \underbrace{\int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} p\left(\underline{\xi}\,|\underline{U} = \underline{u}\right) \cdot \mathrm{ld} \frac{p\left(\underline{\xi}\,|\underline{U} = \underline{u}\right)}{p\left(\underline{\xi}\right)} d\xi_0 \ldots d\xi_{M-1}}_{M-\text{fold integration}} .$$

- $p(\underline{\xi}|\underline{U} = \underline{u})$ is the PDF of the AWGN channel
- The bits u_m are independent, $P(U_m = 0) = P(U_m = 1) = 1/2$
- Consequently, the a priori probability of a vector realization $\underline{u}$ is $P(\underline{U} = \underline{u}) = 1/2^M$

SCC - Inner Transfer Characteristics

- Chain rule of mutual information: write $I(\underline{U};\underline{Y})$ as a sum of M *bitwise* conditional mutual informations I_L

Info-sequence

$$I\left(\underline{U};\underline{Y}\right) = \sum_{L=0}^{M-1} I_L = M \cdot C_G\left(\frac{E_b}{N_0}\right) \leq M$$

ch. output

- I_L is a short-hand notation of

$$I_L = \overline{I\left(U_m;\underline{Y}\,|L \text{ other bits known}\right)}$$

discard

$$0 \leq I_L \leq 1$$

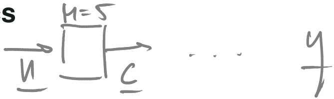
"prior knowledge"

- The bar indicates that I_L is averaged
  - over bitwise mutual information with respect to all M bits
  - all possible $\binom{M-1}{L}$ combinations to choose L known bits out of the total of $M-1$ other bits
  - and over all 2^L bit vector realizations thereof

SCC - Inner Transfer Characteristics

- Vector channel $0 \leq I(\underline{U};\underline{Y}) \leq M$ can be viewed as being composed of M parallel sub-channels with mutual information $0 \leq I_L \leq 1$ each
- Allows interesting interpretation:
  - The mapping only influences the partitioning of the total amount of mutual information $M \cdot C_G$ among the different conditional sub-channels I_L,
  - whereas the sum $\sum I_L$ always adds up to the constant value $M \cdot C_G$, independently of the applied mapping
- Quantities I_L are well suited for characterizing different mappings
- Partitioning of mutual information among the sub-channels I_L has a strong impact on the behavior of the particular mapping in an iterative decoding scheme

SCC - Inner Transfer Characteristics



- Example: 5-bit mappings
- Two randomly chosen examples ($E_b/N_0 = 1\text{dB}$ assuming $R = 1/2$)

5-bit	$M = 5$ parallel sub-channels I_L					
mappings	I_0	I_1	I_2	I_3	I_4	$\sum I_L = M \cdot C_G$
$V_{5,id}$	0.562	0.562	0.562	0.562	0.562	2.81
$V_{5,1}$	0.416	0.487	0.562	0.638	0.708	2.81
$V_{5,2}$	0.249	0.410	0.574	0.730	0.848	2.81

- The mappings are defined as

$$\mathbf{V}_{5,id} = (0, 1, 2, \ldots, 30, 31)$$

$$\mathbf{V}_{5,1} = (17, 16, 8, 0, 4, 20, 12, 28, 18, 26, 2, 7, 6, 22, 14, 30, 1, 9, 25, 24, 5, 21, 13, 29, 3, 19, 11, 27, 23, 15, 31, 10)$$

$$\mathbf{V}_{5,2} = (29, 10, 31, 27, 21, 17, 8, 22, 30, 6, 9, 12, 1, 13, 14, 26, 19, 24, 5, 16, 28, 2, 7, 15, 25, 3, 20, 23, 18, 4, 0, 11)$$

permutation

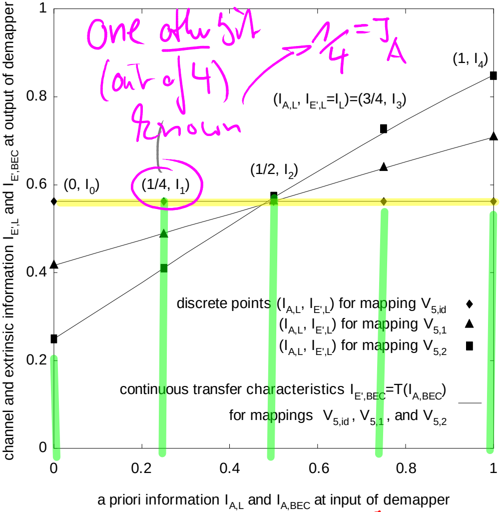
- For the identity mapping we find $I_L = C_G$
- For other mappings: I_L increases with L, $I_{L-1} < I_L$, $1 \leq L < M$
- Chain rule: the I_L sum up to constant $M \cdot C_G$ for all mappings

SCC - Inner Transfer Characteristics

INNER MAPPING!

M=5

$J_{E'}$



$\rightarrow J_A$

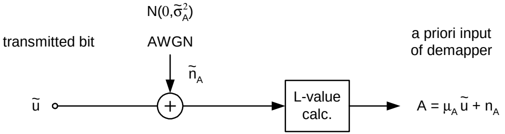
Part I

SCC - Inner Transfer Characteristics

Part II

BEC

- Gaussian (Continuous) A Priori Knowledge:
- From simulations of actual decoder: Extrinsic L-values E_2 tend to be Gaussian-like distributed



- Model a priori input A as an independent Gaussian random variable n_A with variance σ_A^2 and mean zero
- In conjunction with the known transmitted inner information bits $\tilde{u} = 1 - 2u$, $\tilde{u} \in \{\pm 1\}$, we write

$$A = \frac{2}{\tilde{\sigma}_A^2} \cdot (\tilde{u} + \tilde{n}_A) = \mu_A \cdot \tilde{u} + n_A$$

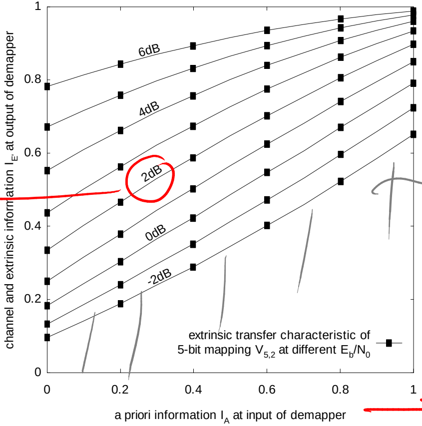
with $\mu_A = 2/\tilde{\sigma}_A^2$ and $\sigma_A^2 = 4/\tilde{\sigma}_A^2$

SCC - Inner Transfer Characteristics

$I_{E'}$

$\frac{E_b}{N_0}$ ($@R=\frac{5}{2}$)

$V_{5,2}$



approx. of $4 \cdot C_a$

I_A (Gaussian)

- Transfer characteristics for Gaussian distributed a priori knowledge
- Note: $I_{E'}(0)$ and $I_{E'}(1)$ independent of a priori distribution

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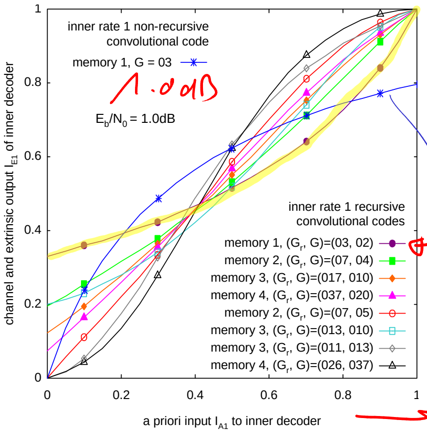
SCC - Inner Transfer Characteristics

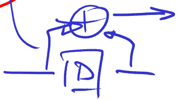
all Rate $R_i = 1$

recursive



I_{E_1}

I_{A_1}



non-recursive

- Extrinsic transfer characteristics of some inner rate 1 codes
- Note: Non-recursive codes do not go up to $I_{E_1}(1) \approx 1$

SCC - Outer Transfer Characteristic

- In the same way as for inner codes, we can derive transfer characteristics of the outer code
- We consider the a priori information $I_{A_2} = I(C_2; A_2)$, and the extrinsic information $I_{E_2} = I(C_2; E_2)$ of the decoder output, to get the transfer characteristic

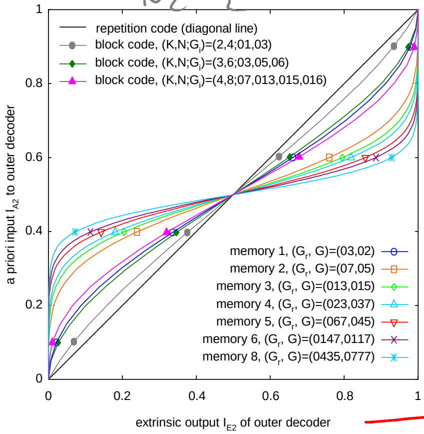
$$I_{E_2} = T_2(I_{A_2})$$

- For the computation we assume A_2 to be Gaussian distributed, and measure histograms $p_{E_2}(\xi|C_2 = 0)$, $p_{E_2}(\xi|C_2 = 1)$
- The outer transfer characteristics are independent of the E_b/N_0-value
- Note that the axes are swapped: Input I_{A_2} is on ordinate, output I_{E_2} on abscissa
- This is in preparation of the design tool where we connect both inner and outer transfer characteristic in a single diagram (EXIT chart)

SCC - Outer Transfer Characteristic

$R_2 = 1/2$

ch. inner outer deinterleaver J_{A_2} J_{E_2}



J_{A_2} input

Output J_{E_2}

- Extrinsic transfer characteristics of some outer rate $1/2$ codes
- Note: Rate $1/2$ repetition code is just diagonal line $I_{E_2} = I_{A_2}$

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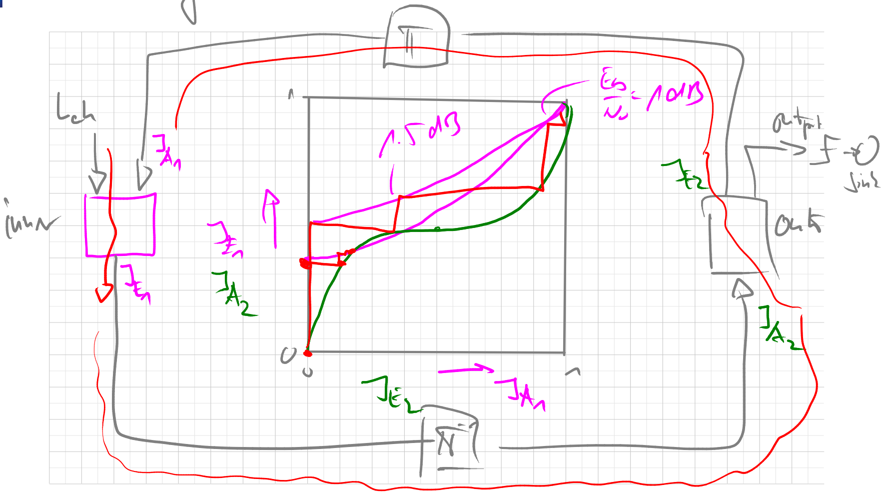
Notes

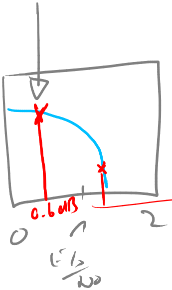
a first EXIT chart:

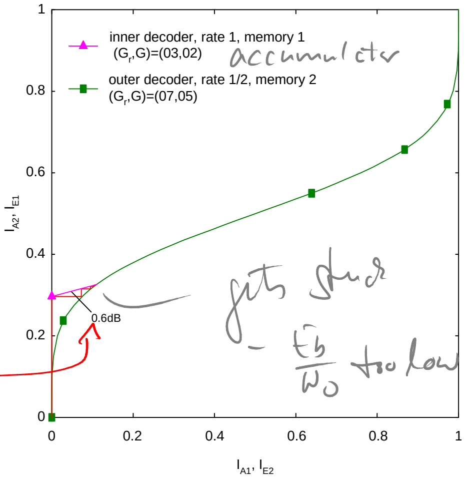


SCC - Extrinsic Information Transfer Chart

- To account for the iterative nature of the sub-optimal decoding algorithm, both decoder characteristics are plotted into a single diagram; for second decoder, axes are swapped
- This diagram is referred to as extrinsic information transfer chart (EXIT chart) since the exchange of extrinsic information can be visualized as a decoding trajectory
- Provided that independence (large interleaver) and Gaussian assumptions hold for modelling extrinsic information (a priori information respectively), the decoding trajectory that can be graphically obtained by simply drawing a zigzag-path into the EXIT chart (bounded by the decoder transfer characteristics) should match with the trajectory computed by simulations
- for simulations (next), interleaver size $4 \cdot 10^5$ coded bits used

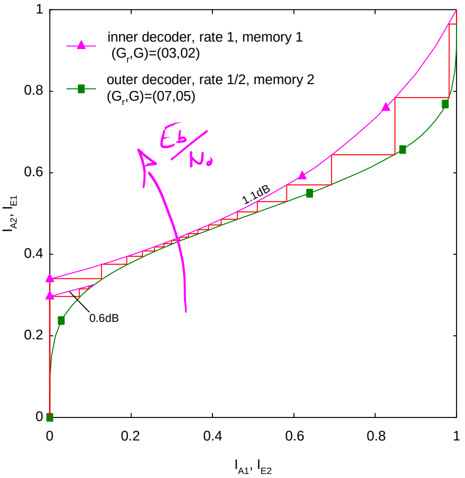
SCC - EXIT Chart





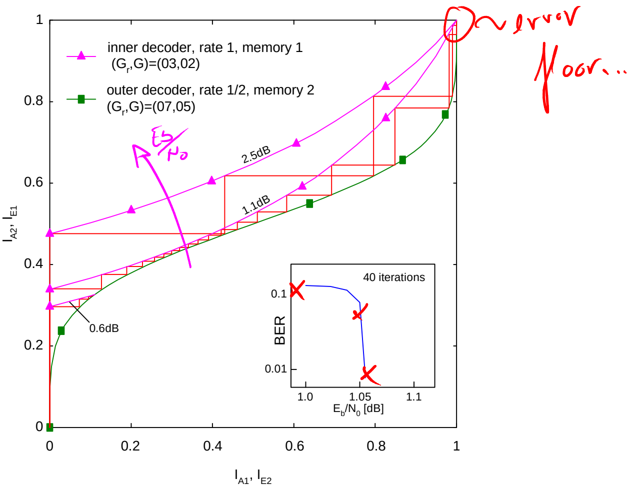
- $E_b/N_0 = 0.6$dB, trajectory gets stuck

SCC - EXIT Chart



- $E_b/N_0 = 1.1$dB, convergence to low BER through narrow tunnel

SCC - EXIT Chart

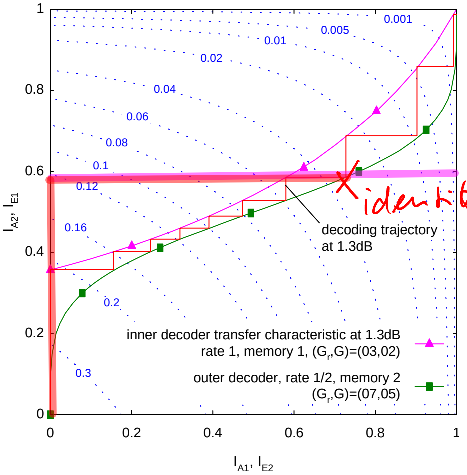


- $E_b/N_0 = 2.5$dB, convergence tunnel wide open

SCC - EXIT Chart

- For $E_b/N_0 = 0.6$dB the trajectory gets stuck
- For $E_b/N_0 = 1.1$dB the inner transfer characteristic has been raised just high enough to open a narrow tunnel ("bottleneck") for the trajectory to "sneak through" and to converge towards low BER ($\approx 10^{-6}$, depending on interleaver size)
- At $E_b/N_0 = 2.5$dB, less iterations are needed to get down to low BER
- For short interleavers the trajectory tends to diverge from the characteristics towards smaller extrinsic output after a few iterations, owing to increasing correlation of extrinsic information
- Main advantage of EXIT chart:
  - Only simulations of individual component decoders are required
  - Transfer characteristics can be used in any combination

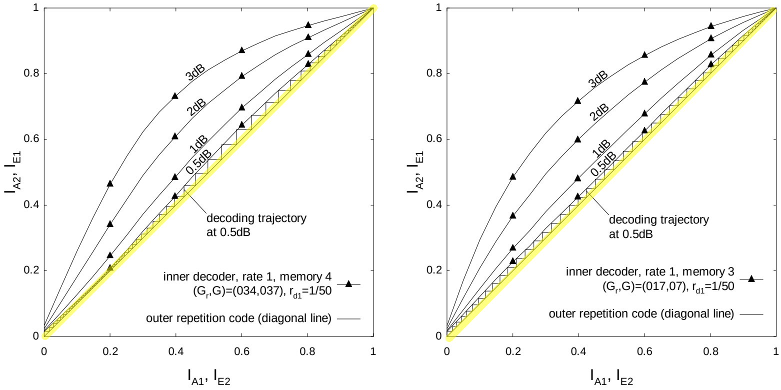
SCC - BER from EXIT Chart



identity mapping
→ no iteration
decoding possible

- Note: BER contour lines are independent of the E_b/N_0-value

SCC - Code Design Examples



Pinch-off 0.41dB

Pinch-off 0.27dB

- Outer repetition codes and systematic doping yields early turbo cliffs

Notes

Notes

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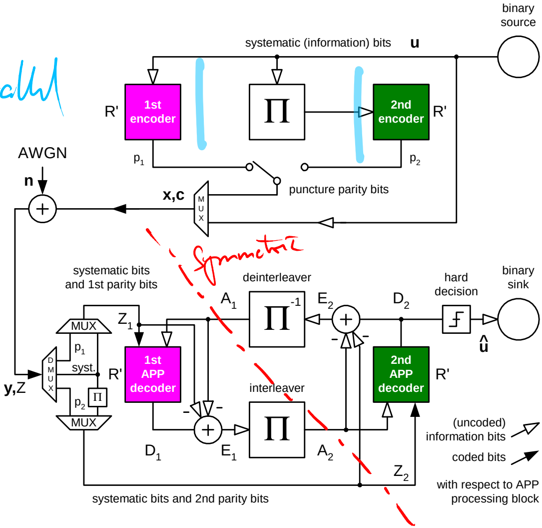
Outline

PCC - Encoder and Iterative Decoder

|| ≙ parallel



PCC - Encoder and Iterative Decoder

- Iterative “turbo” decoding was introduced by C. Berrou, A. Glavieux in 1993
- First scheme to make use of extrinsic information
- Typically, PCC are systematic
- The classic turbo codes of 1993 consist of memory 4 constituent codes with polynomials $(G_r, G) = (037, 021)$.

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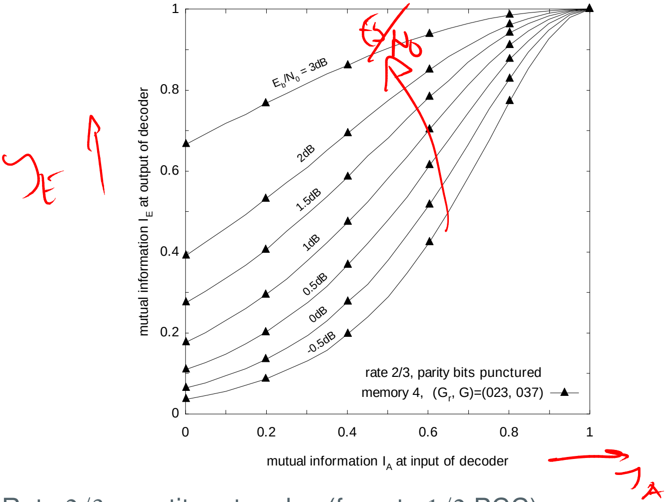
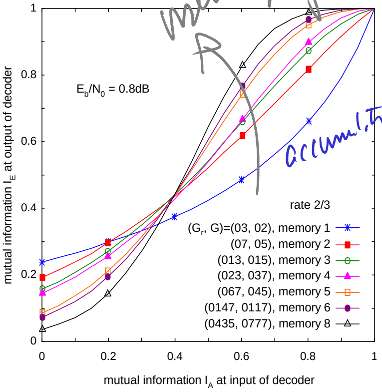
PCC - Transfer Characteristics

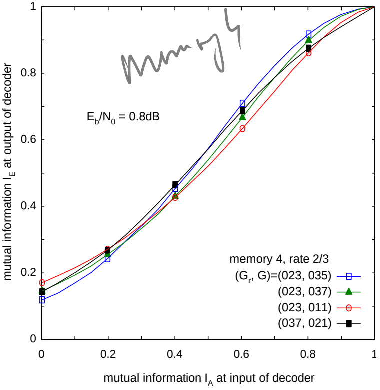


- Rate $2/3$ constituent codes (for rate $1/2$ PCC)

PCC - Transfer Characteristics

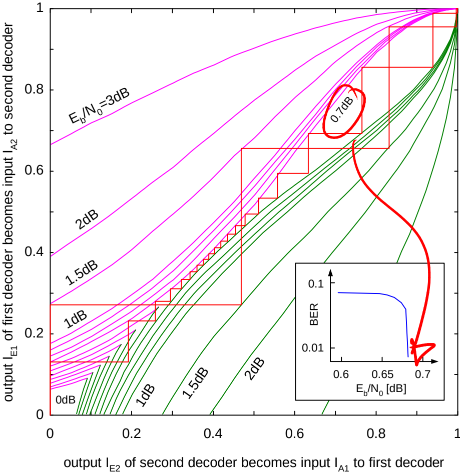
memory of shift register





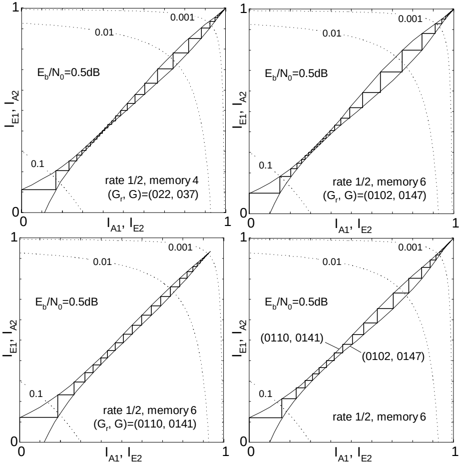
- Different code memory (left), different code polynomials for memory fixed to 4 (right); $E_b/N_0 = 0.8$dB

PCC - EXIT Chart



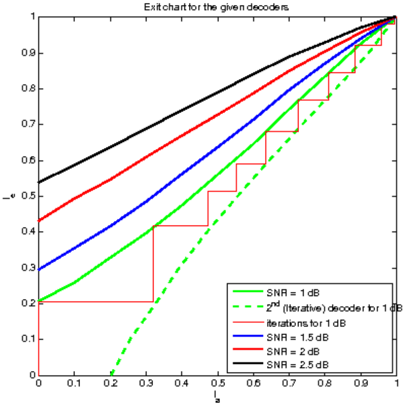
- PCC rate $1/2$, memory 4, $(G_r, G) = (023, 037)$; interleaver 10^6 bits

PCC - Code Design Examples



- Decoding trajectories for rate $1/2$ PCC with turbo cliff below 0.5dB

PCC - Webdemo



- for your edutainment, turbo code webdemo
- http://webdemo.inue.uni-stuttgart.de/webdemos/03_theses/turboCodingM/

Notes

Notes

Outline

LDPCC - Definitions

- Invented in 60's, regained popularity in 1997 (irregular LDPCC)
- A (d_v, d_c)-regular LDPC code is a binary linear block code that has a parity-check matrix H
  - with d_v ones in each column
  - with d_c ones in each row

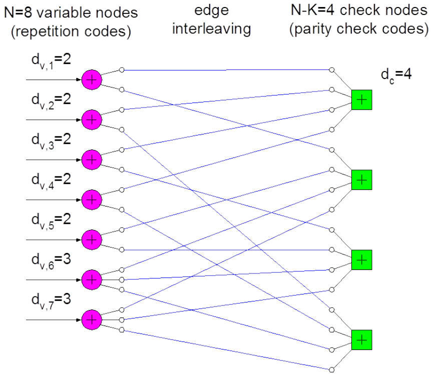
$H \cdot \underline{c}^T = \underline{0}$

- Example: $(d_v = 2, d_c = 4)$-regular LDPC code
- Described by $(N-K) \times N$ matrix $\mathbf{H}$ (parity-check matrix) $\mathbf{H}\mathbf{c}^T = 0$

$$\mathbf{H} = \underbrace{\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}}_{N=8} \left.\vphantom{\begin{pmatrix}1\\1\\1\\1\end{pmatrix}}\right\} N-K$$

- $\mathbf{H}$ is sparse

LDPCC - Irregular Code

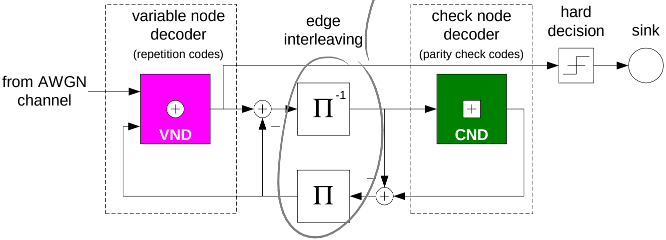


- Example of an irregular code; parity check matrix

$$\mathbf{H} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

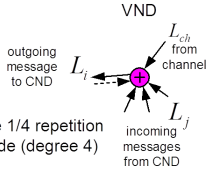
LDPCC - Iterative Decoder

H parity check matrix

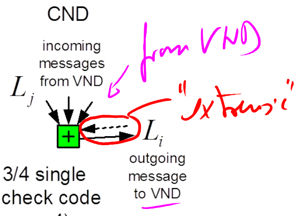


- Variable node decoder: viewed as inner code, typically irregular
- Check node decoder: viewed as outer code, typically regular

LDPCC - Component Decoders



rate 1/4 repetition code (degree 4)



rate 3/4 single parity check code (degree 4)

"soft majority vote"

$$L_i = L_{ch} + \sum_{j\neq i} L_j$$

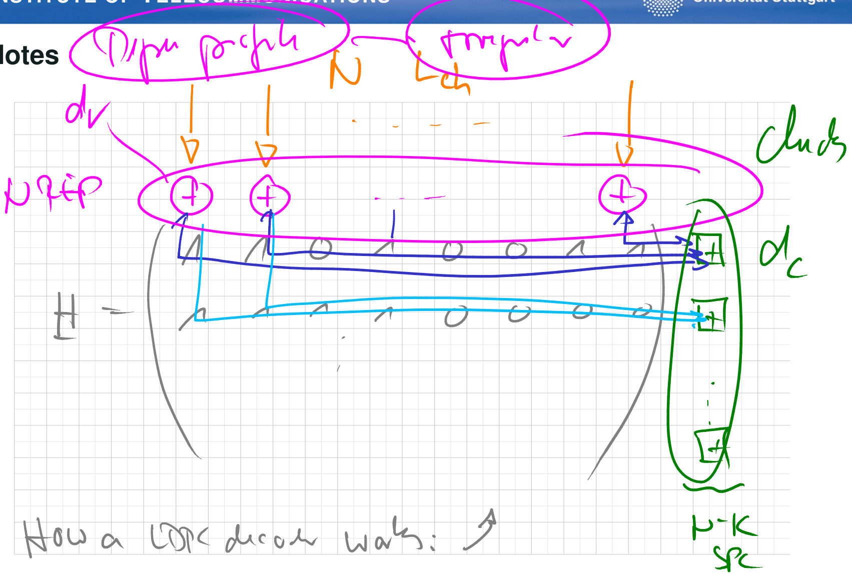
add

REP sum

$$L_i = \sum_{j\neq i} \boxplus L_j \approx \underbrace{\prod_{j\neq i} \operatorname{sgn} L_j}_{\text{parity check}} \cdot \underbrace{\min_{\forall j\neq i} (|L_j|)}_{\text{weakest link dominates}}$$

SPC max

Notes



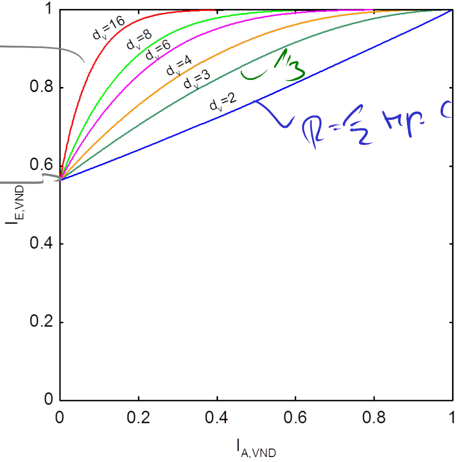
How a LDPC decoder works:

LDPCC - VND EXIT curves

VND

Rep d_v

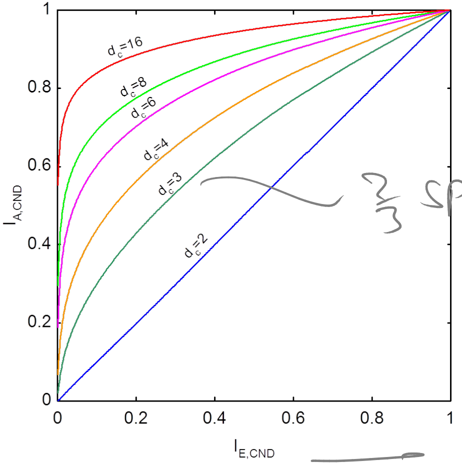
channel input



- repetition codes of rate $1/d_v$ (here: $E_b/N_0 = 1\text{dB}$ at $R = 1/2$)

LDPCC - CND EXIT curves

input



SPC

check

output

- single parity check codes of rate $(d_c - 1)/d_c$

LDPCC - Superposition of EXIT Curves

- Code design by mixing nodes of different degree (irregular LDPCC)
- Transfer characteristic of resulting curve is a linear combination of individual curves of respective degrees
- E.g., variable node curve (mixture of D different degrees)

$$I_{E,VND}\left(I_A\right) = \sum_{i=1}^{D} b_i I_{E,VND}\left(I_A, d_{v,i}\right)$$

with edge fractions b_i,

$$\sum_{i=1}^{D} b_i = 1$$

- LDPC code design boils down to curve fitting in EXIT chart!

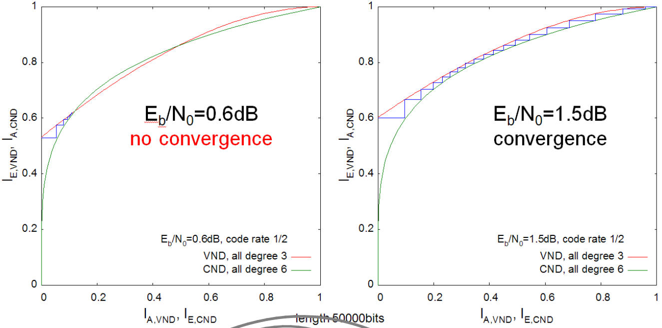
LDPCC - Example, Regular

$d_v = 3$ 3REP

$d_c = 6$ 5/6 SPC

$R = 1/2$

(3,6) regular LDPCC



regular

- Example of a regular LDPC code over AWGN channel
- Convergence to low bit error rate at 1.3dB

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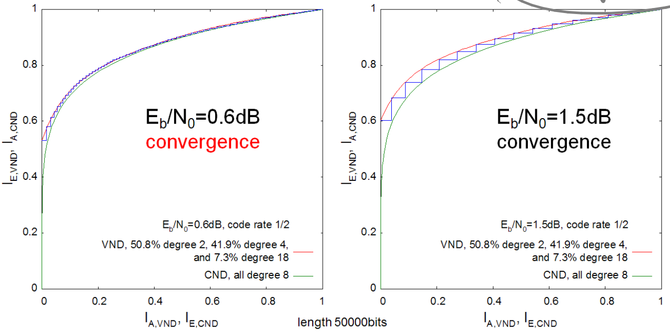
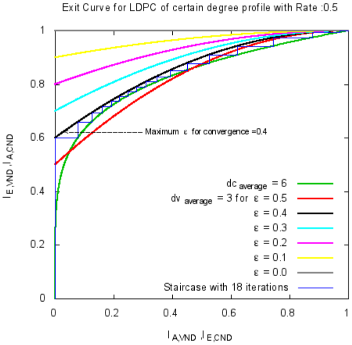
LDPCC - Example, Irregular

R = 1/2 irregular



- Example of an irregular LDPC code over AWGN channel
- Convergence to low bit error rate at 0.6dB (issue: degree 2 variables...)

LDPCC - Webdemo



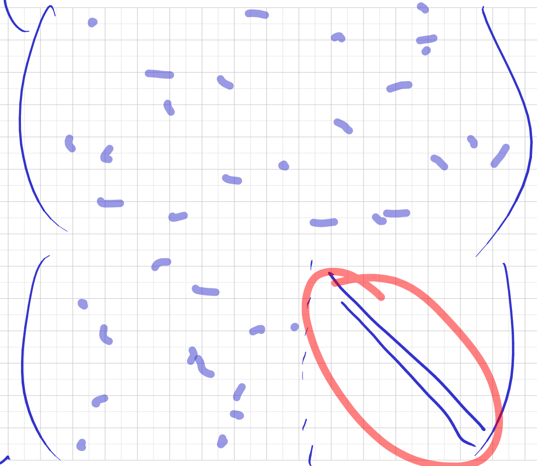
- for your edutainment, LDPC code webdemo
- http://webdemo.inue.uni-stuttgart.de/webdemos/03_theses/ldpcExit/

Notes

random ensemble $\rightsquigarrow \mathcal{O}(N^2)$ encoding complexity

$H =$

$H_{RA} =$



Encoding in practice

$\rightsquigarrow \mathcal{O}(N)$

repeat accumulate code

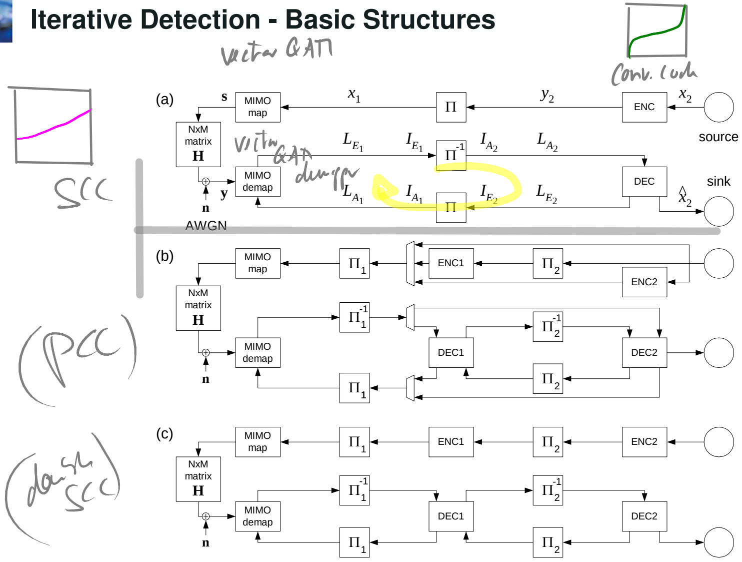
$\rightsquigarrow$ linear encoding complexity

Notes

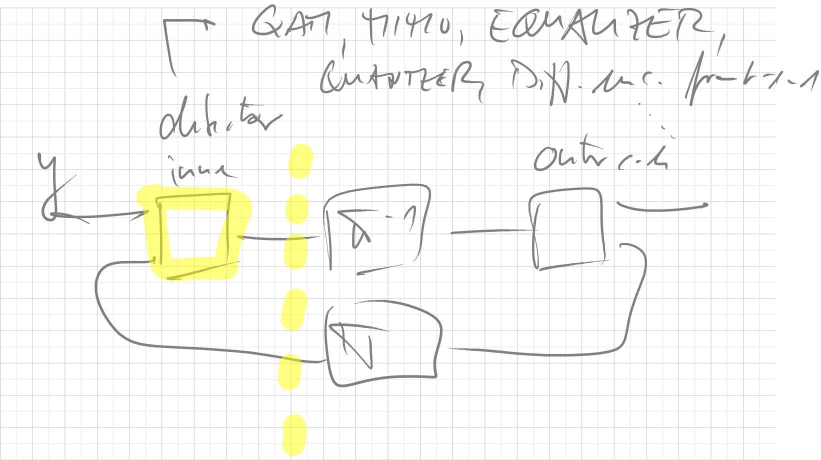
Outline

Iterative Detection - Basic Structures

vector QAM



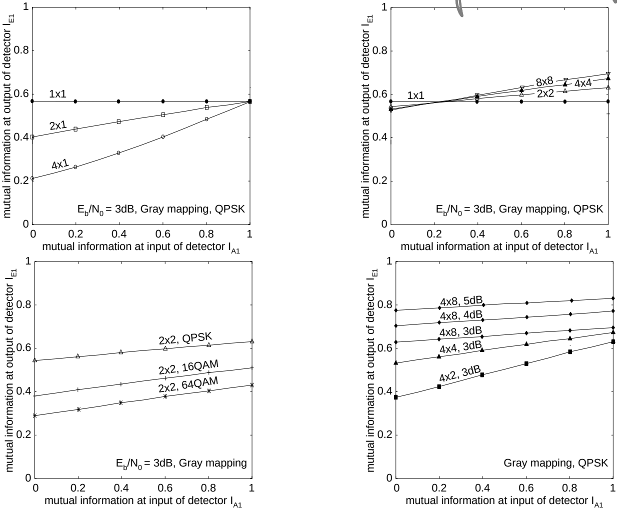
Notes



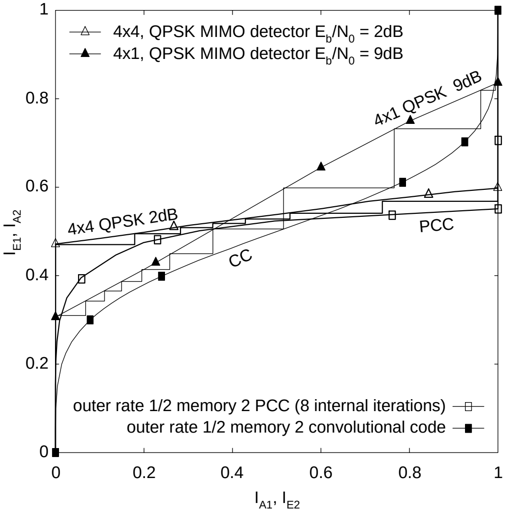
Iterative Detection - Inner MIMO Detector

inner MIMO detector a posteriori prob...
L-values

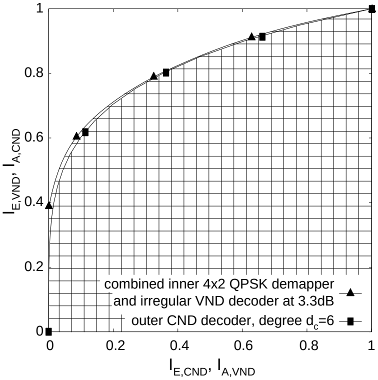
BPSK

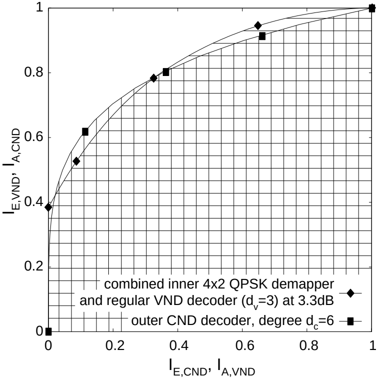


Iterative Detection - EXIT Chart



Iterative Detection - Outer LDPC Codes

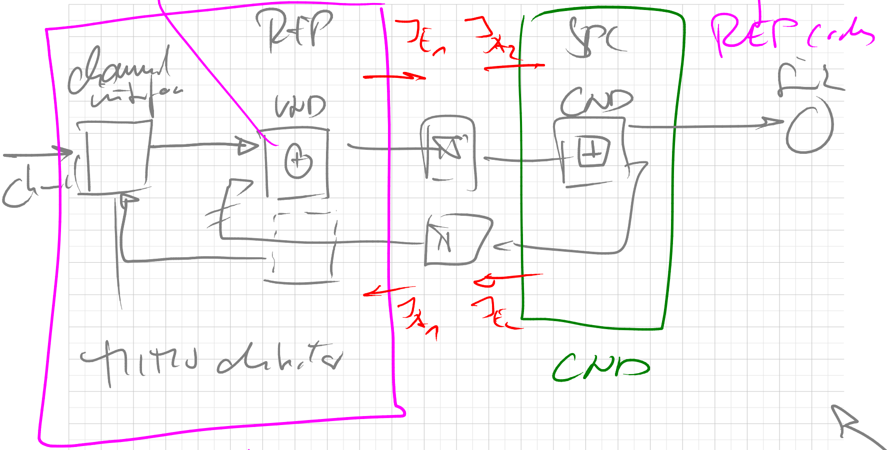




- irregular vs regular LDPC codes, 4×2 MIMO, $E_b/N_0 = 3.3\text{dB}$

Notes

curve fitting: find best matching mixture of REP codes



new resulting inner decoder ~ DET & VND decoder

iterative detection & LDPC decoding

www.inue.uni-stuttgart.de

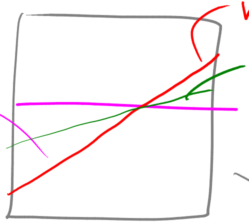
Outline

lots of multipath

no multipath

little multipath

$I_{E,DET}$



$I_{A,DET}$

1. Introduction
2. Soft Output Decoding
3. Serially Concatenated Codes (SCC)
4. Parallel Concatenated Codes (PCC)
5. Low-Density Parity Check (LDPC) Codes
6. Iterative Detection
7. **Future Trends**

3 different channel profiles needed ... inconvenient

Future Trends

- recently, it was shown that spatially coupled codes (e.g. convolutional LDPC codes) are "universal"
  - can universally achieve capacity over binary-input memoryless symmetric-output channels
  - good for various detection front-ends...
- specific degree distribution design becomes obsolete
  - 1960s: Gallager, LDPC codes, regular
  - 1990s: MacKay, Richardson, Urbanke et al: make them irregular to get close to capacity
  - 2010s: Kudekar, Richardson, Urbanke, universality of spatially coupled (LDPC) codes: regular codes suffice

time

→ back to regular ... (?)

Future Trends - Spatial Coupling

- Degree profile optimization for dedicated channel detector, e.g.
  - MIMO detector (multiple antennas)
  - equalizer (multipath channel)
  - QAM mapping (QPSK, 16QAM. . .)
  - differential coding

possible, all with same code?

- For each detector (or each channel), different degree profile needed!
- Thus, with spatially coupled codes as “universally good codes”...
  - should be no matching to channel interface needed anymore!

Notes

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Future Trends - Spatial Coupling

- Pariy check matrix a terminated convolutional LDPC code,

$$\mathbf{H}_\mathrm{L} = \begin{pmatrix} \mathbf{H}_0 & & & \\ \mathbf{H}_1 & \mathbf{H}_0 & & \\ \vdots & \mathbf{H}_1 & \ddots & \\ \mathbf{H}_m & \vdots & \ddots & \ddots \\ & \mathbf{H}_m & & \ddots & \mathbf{H}_0 \\ & & \ddots & & \mathbf{H}_1 \\ & & & \ddots & \vdots \\ & & & & \mathbf{H}_m \end{pmatrix}_{(L+m)M' \times LN'}$$

at edge

- slight irregularities (in check node degrees) kick off decoding wave below BP threshold (but above MAP threshold)

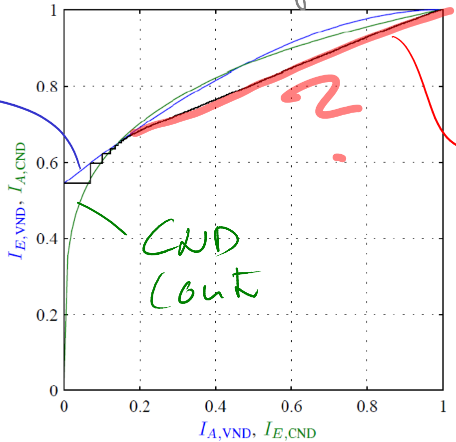
Future Trends - Spatial Coupling

just a Spat. CC. (no DET yet)

irreg VND

regular (3,6)-code

1.2 dB...



what's that?

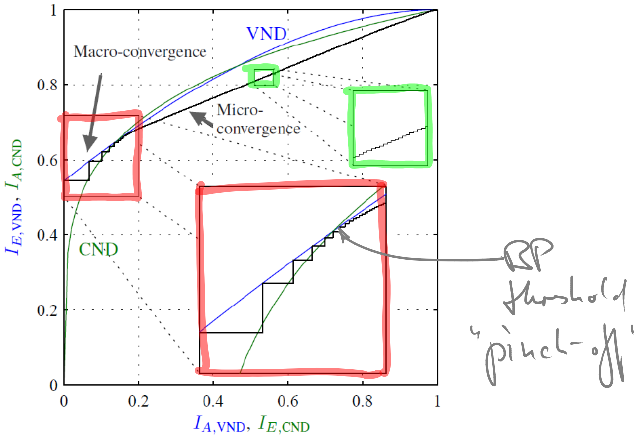
↳ "micro-convergence"

wave like convergence below BP thresh.!!

- Regular (3,6) code, extended into an conv. LDPC code $L = 100$; $E_b/N_0 = 0.85\text{dB}$

using BP decoding...

Future Trends - Spatial Coupling



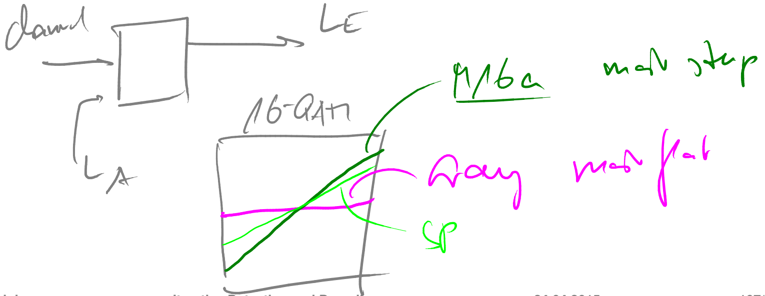
- Tunneling through the pinch-off... against conventional wisdom; better than BP threshold

[„Modulation/Detection with Spatially Coupled Codes“, L. Schmalen, tB, IEEE/ITG Conf. on. SCC, Jan. 2013]

Future Trends - Spatial Coupling, Detection Experiment

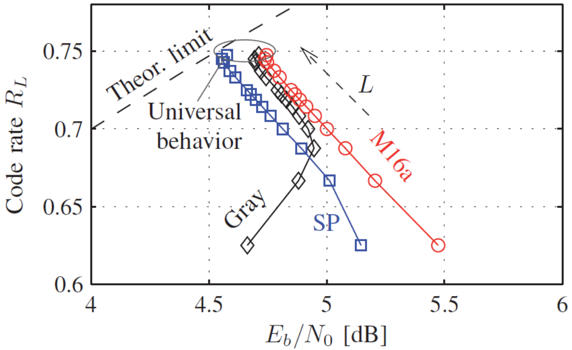
- experiment: spat. coupled code with BICM-detector
- three different labelings (16-QAM)
  - Gray labeling (most flat detector EXIT curve)
  - set partitioning (SP)
  - M16a (from PhD thesis, F. Schreckenbach): steepest slope

QAM demapper 2007, TUM



Notes

Future Trends - Spatial Coupling, Detection Experiment



- density evolution on protograph; replication factor up to $L = 100$
- capacity of 16-QAM with rate $R \to 3/4$-code at $E_b/N_0 \approx 4.53$dB

[„Modulation/Detection with Spatially Coupled Codes“, L. Schmalen, tB, IEEE/ITG Conf. on. SCC, Jan. 2013]

Future Trends - Spatial Coupling, Detection Experiment

3 different detector behaviors

DE THRESHOLDS FOR THE SPATIALLY COUPLED AND THE CORRESPONDING REGULAR ENSEMBLES AND OPTIMIZED IRREGULAR ENSEMBLES FOR BICM-ID; CAPACITY LIMIT $\frac{E_b}{N_0}\Big|_{\min} = 4.528$ dB

Code	Gray	SP	M16a
Spatially coupled code	4.712 dB	**4.577 dB**	**4.745 dB**
Opt. irregular (for Gray)	**4.600 dB**	*6.217 dB*	*6.344 dB*
Opt. irregular (for SP)	*> 7 dB*	4.739 dB	*4.983 dB*
Opt. irregular (for M16a)	*> 7 dB*	*4.729 dB*	4.959 dB

gute

Conventional LDPC codes

→ degree profile design, Opt.

- simulation results: $L = 50$, codeword length $N = 200000$ bits, 16-QAM with rate $R \to 3/4$-code
- spatially coupled codes (regular) can be *universally* good
  - avoids degree 2 variable nodes, ...

Could also be 3 different multipath channels and eqn. detector

[„Modulation/Detection with Spatially Coupled Codes“, L. Schmalen, tB, IEEE/ITG Conf. on. SCC, Jan. 2013]

Summary

- Iterative decoding: to approach capacity
- Iterative detection and decoding: include channel interface in iterative decoding loop
- LDPC codes: Degree profile matching
- Spatially Coupled Codes: Can be universally good, regular codes suffice
- Most communication problems in practice can benefit from iterative detection and decoding

thank you, it was fun!

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