Matrix Completion from a Few Entries
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What is Matrix Completion?

Problem: How many revealed entries \(|E|\) do we need to get

\[
\frac{1}{mn} \| M - \hat{M} \|_F^2 \leq \delta
\]

Algorithm [OptSpace]

Trim : Trim \( M^E \) to \( M^E \);

Project : Project \( M^E \) onto \( \text{Tr}(M^E) \);

Clean : Minimize Cost \( F(X,Y) \), s.t. \( X,Y \) orthogonal.

SVD : \( \text{Tr}(M^E) = \frac{mn}{|E|} \sum_{i=1}^{r} \sigma_i X_i^T Y_i \)

Solution : Trimming

Main Results

Theorem (Keshavan, Montanari, Oh, 2009 [1])

Assume \( r = O(1) \), and let \( M \) be an \( n \times n \) matrix satisfying \( (\mu_0, \mu_1) \)-incoherence with \( \sigma_1(M) / \sigma_r(M) = O(1) \). If \( |E| \geq C'n \log n \), then OptSpace returns, whp., the matrix \( M \).

Theorem (Keshavan, Montanari, Oh, 2009 [2])

Let \( N = M + Z \) with \( M \) as above and \( Z \) any \( n \times n \) matrix. If \( |E| \geq C'n \log n \), then (under appropriate technical conditions) OptSpace with input \( N^E \) returns \( \hat{M} \) such that whp.

\[
\frac{1}{\sqrt{mn}} \| M - \hat{M} \|_F \leq C \frac{n^{\alpha r}}{|E|^r} \| Z^E \|_2
\]

Implementation

References
