Achievable Rate Regions for the Broadcast Channel With Cognitive Relays

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Summary

- We investigate the broadcast channel with two cognitive relays (BCCR) from an information theoretic perspective.
- New coding schemes based on rate splitting, Gel'fand-Pinsker coding, Marton’s binning, and superposition coding are proposed.
- New achievable rate regions are derived, and are shown to include the existing ones.
- Specialized one of achievable rate regions to the case with a single cognitive relay.
- A new achievable rate region is obtained for the well-studied cognitive radio channel (CRC).
- Developed a Gaussian example to demonstrate the dominance of the proposed coding schemes over other existing coding schemes.

Motivations

- Interference channel with a cognitive relay [Sahin & Erkip 07; Sridharan et al. 08]
- Cognitive radio channel (also called the interference channel with one cognitive transmitter) [Mard et al. 08, etc.]
- Only the Gaussian case has been studied.
- The generality of the channel model is largely ignored.
- The channel model in fact includes the CRC as a special case.

New Achievable Rate Regions

- Most of the existing works on CRC view the channel as a variant of the interference channel.
- Most existing coding strategies are constrained by an encoding order: 1) primary user massage first, 2) then the cognitive user message with the Gel’fand-Pinsker coding scheme.

A Gaussian Example

- Dirty paper coding is applied twice in a unique order: encode $W_1$ against $V_1$, and then encode $W_2$ against $V_2$, which is impossible with any other existing coding schemes for the CRC or BCCR.
- The obtained rate region is the capacity region for the channel when the interference link from $X_2$ to $Y_2$ is removed.

Future work: 1) to determine whether the above region is the capacity region for the example setting; 2) also for the general Gaussian setting.

New Coding Scheme

- Rate splitting: $M_1 = (M_{11}, M_{12}), M_2 = (M_{21}, M_{22})$
- Joint PDF of the RVs for codebook generation:

\[
\begin{align*}
\phi(u_1(u_1), u_2(u_2), u_3, \ldots, u_N) & = \\
& \prod_{i=1}^{N} \phi(u_i(u_i))
\end{align*}
\]

Thm. 1: For a fix joint PDF in (1), any non-negative rate pair $(R_{11}, R_{12}, R_{21}, R_{22})$ is achievable for the BCCR with $R_{11}, R_{12}, R_{21}, R_{22}$ satisfying

\[
\begin{align*}
& \sum_{i=1}^{2} R_i - \sum_{i=1}^{2} I(U_i; F_1, F_2) \\
& \sum_{i=1}^{2} R_i - \sum_{i=1}^{2} I(U_i; V_i) \leq \sum_{i=1}^{2} R_i - \sum_{i=1}^{2} I(U_i; V_i) \leq \sum_{i=1}^{2} R_i - \sum_{i=1}^{2} I(U_i; V_i)
\end{align*}
\]

Set $V_1$ and $U_1$ as constants in (2):

Thm. 3: Any non-negative rate pair $(R_{11}, R_{12}, R_{21}, R_{22})$ is achievable for the CRC with $R_{11}, R_{12}, R_{21}, R_{22}$ satisfying

\[
\begin{align*}
& \sum_{i=1}^{2} R_i - \sum_{i=1}^{2} I(U_i; F_1, F_2) \\
& \sum_{i=1}^{2} R_i - \sum_{i=1}^{2} I(U_i; V_i) \leq \sum_{i=1}^{2} R_i - \sum_{i=1}^{2} I(U_i; V_i) \leq \sum_{i=1}^{2} R_i - \sum_{i=1}^{2} I(U_i; V_i)
\end{align*}
\]

Future work: 1) to determine whether the above region is the capacity region for the example setting; 2) also for the general Gaussian setting.

Evaluate

\[
\begin{align*}
& R_1 \leq \log_2(1 + \alpha \gamma_1) \\
& R_2 \leq \log_2(1 + \alpha \gamma_2)
\end{align*}
\]

Set $V_1$, $U_1$, and $U_2$ as constants in (2):

\[
\begin{align*}
& R_1 \leq \log_2(1 + \alpha \gamma_1) \\
& R_2 \leq \log_2(1 + \alpha \gamma_2)
\end{align*}
\]

Gel’fand-Pinsker Binning
- Marton Binning

This new rate region is shown to include the best existing rate region for the BCCR in [Sridharan et al. 08].

A further improved rate region with simplified description is obtained using successive superposition encoding with a modified joint PDF.