MONASH University

Lattice Index Coding

Part III - Constructing Codes

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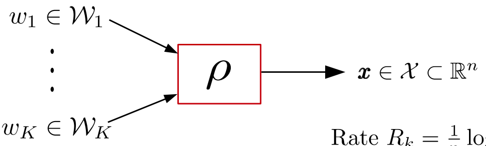
Recap: Channel Model

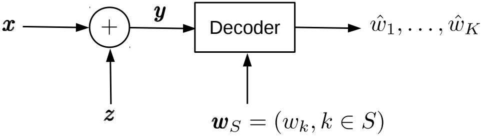
Broadcast $(w_1, \ldots, w_K)$ to multiple receivers $\{(\mathsf{SNR}, S)\}$ where $S \subset \{1, \ldots, K\}$ denotes the available side information

Transmitter



Rate $R_k = \frac{1}{n} \log_2 |\mathcal{W}_k|$

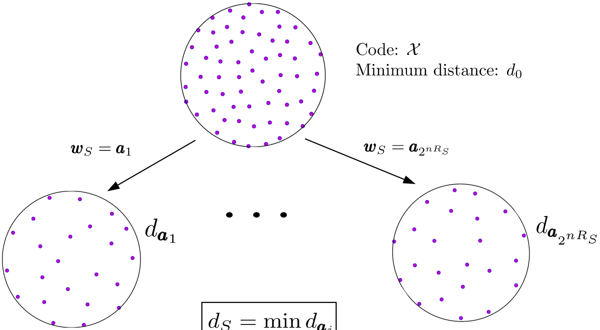
A receiver (SNR, S)



Variance $\frac{1}{\mathsf{SNR}} \cdot \frac{\mathbb{E}\,\|\boldsymbol{x}\|^2}{n}$

Side information rate $R_S = \sum_{k \in S} R_k$

Recap: Decoding and Side Information Gain



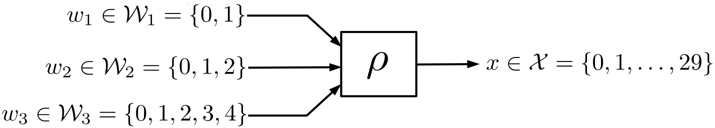
$$\boxed{d_S = \min_i d_{\boldsymbol{a}_i}}$$

smallest min distance among the 2^{nR_S} codebooks

$$\text{Side Information Gain } \Gamma(\mathcal{X}) = \min_{S \subset \{1,\ldots,K\}} \frac{10\log_{10}\left(\frac{d_S^2}{d_0^2}\right)}{R_S} \text{ dB/b/dim}$$

Design Objective: maximize d_0 and $\Gamma(\mathcal{X})$

Algebraic Construction: Main Idea



- Label 30-PAM with elements of the ring $\mathbb{Z}_{30} = \{0, 1, \ldots, 29\}$.
  - Addition and multiplication in $\mathbb{Z}_{30}$ are performed modulo 30.
- Encode messages to $\mathcal{X} = \mathbb{Z}_{30}$ using

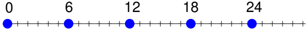
$$x = \rho(w_1, w_2, w_3) = 15w_1 + 10w_2 + 6w_3 \bmod 30.$$

- Chinese remainder theorem $\Rightarrow$ ρ is bijective.
- Dimension of the codebook is $n = 1$.
- $R_1 = 1$, $R_2 = \log_2 3$, $R_3 = \log_2 5$ b/dim.

$$x = 15w_1 + 10w_2 + 6w_3 \bmod 30$$

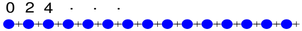
$$S = \{1, 2\}$$

$$\mathcal{X}_{a_S} = 6w_3 + \text{constant}$$



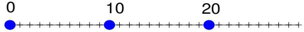
$$S = \{1\}$$

$$\mathcal{X}_{a_S} = 10w_2 + 6w_3 + \text{constant}$$



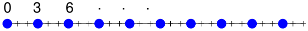
$$S = \{1, 3\}$$

$$\mathcal{X}_{a_S} = 10w_2 + \text{constant}$$



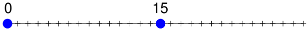
$$S = \{2\}$$

$$\mathcal{X}_{a_S} = 15w_1 + 6w_3 + \text{constant}$$



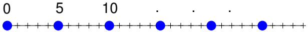
$$S = \{2, 3\}$$

$$\mathcal{X}_{a_S} = 15w_1 + \text{constant}$$



$$S = \{3\}$$

$$\mathcal{X}_{a_S} = 15w_1 + 10w_2 + \text{constant}$$



$$S = \varnothing$$

$$\mathcal{X}_{a_S} = \mathcal{X} = \{0, 1, \ldots, 29\}$$

$$\boxed{d_S = 2^{R_S}}$$

Side information gain

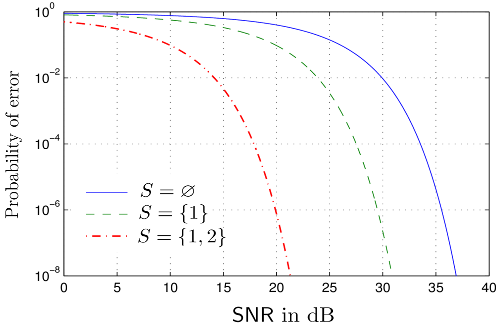
- The code guarantees: $d_S = 2^{R_S}$ for all $\varnothing \subsetneq S \subsetneq \{1,2,3\}$
  - Min distance improves with the amount of available side information

- Side information gain

$$\Gamma(\mathcal{X}) = \min_S \frac{10\log_{10}\left(d_S^2/d_0^2\right)}{R_S} = \min_S \frac{10\log_{10}\left(d_S^2\right)}{\log_2 d_S} = 20\log_{10} 2$$
$$\approx 6 \text{ dB/b/dim.}$$

- Uniform Side Information Gain:
  Identical normalized distance gain for all receivers

$$\frac{10\log_{10}\left(d_S^2/d_0^2\right)}{R_S} = 20\log_{10} 2 \approx 6 \text{ for all } S \subset \{1,2,3\}$$



SNR gain over $S = \varnothing$ at $\mathrm{P}_e = 10^{-4}$

S	Actual gain	Predicted $\Gamma \times R_S$ dB
$\{1\}$	6	6
$\{1,2\}$	15.6	15.5

Algebraic Construction over $\mathbb{Z}$

In order to encode K messages:

- Choose K relatively prime integers: $M_1, \ldots, M_K \in \mathbb{Z}$.
- The message alphabets are $\mathcal{W}_1 = \mathbb{Z}/M_1\mathbb{Z}, \ldots, \mathcal{W}_K = \mathbb{Z}/M_K\mathbb{Z}$.
- Encode messages to M-PAM, where $M = \prod_{k=1}^{K} M_k$:

$$\rho(w_1, \ldots, w_K) = \frac{M}{M_1} w_1 + \cdots + \frac{M}{M_K} w_K \mod M.$$

- This map corresponds to the Chinese remainder theorem:

$$\mathbb{Z}/M_1\mathbb{Z} \times \cdots \times \mathbb{Z}/M_K\mathbb{Z} \to \mathbb{Z}/M\mathbb{Z}.$$

- Rate $R_k = \log_2 M_k$ b/dim.

Algebraic Construction over $\mathbb{Z}$

Distance with no side information

- $d_0 =$ min distance of M-PAM $\mathbb{Z}/M\mathbb{Z} = 1$

Distance with side information configuration S

- Receiver knows $w_k = a_k$, $k \in S$
- Expurgated codebook consists of the points

$$x = \sum_{k \in S} \frac{M}{M_k} a_k + \sum_{k \notin S} \frac{M}{M_k} w_k \mod M, \text{ where } w_k \text{ are unknown integers}$$

- Difference between codewords $\Delta x = \sum_{k \notin S} \frac{M}{M_k} \Delta w_k \mod M$
- Minimum distance d_S

$$\min |\Delta x| = \min \left| \sum_{k \notin S} \frac{M}{M_k} \Delta w_k \right| = \gcd \left(\frac{M}{M_k}, k \notin S \right) = \prod_{k \in S} M_k$$

Algebraic Construction over $\mathbb{Z}$

Side Information Gain

- Distance with side information

$$d_S = \prod_{k \in S} M_k = \prod_{k \in S} 2^{R_k} = 2^{\sum_{k \in S} R_k} = 2^{R_S}$$

- Side information gain

$$\Gamma = \min_S \frac{10 \log_{10} \left(\frac{d_S^2}{d_0^2} \right)}{R_S} \approx 6 \text{ dB/b/dim}$$

Construction over $\mathbb{Z}[i]$ and $\mathbb{Z}[\omega]$

- Choose K relatively-prime numbers $M_1, \ldots, M_K \in \mathbb{D}$
- Message alphabets: $\mathcal{W}_1 = \mathbb{D}/M_1\mathbb{D}, \ldots, \mathcal{W}_K = \mathbb{D}/M_K\mathbb{D}$
- Rate: $R_k = \log_2 |M_k|$ b/dim
- Constellation: $\mathcal{X} = \mathbb{D}/M\mathbb{D}$, where $M = M_1\, M_2 \cdots M_K$
- Encoding: Using Chinese remainder theorem

$$\rho(w_1, \ldots, w_K) = \frac{M}{M_1} w_1 + \cdots + \frac{M}{M_K} w_k \bmod M\mathbb{D}$$

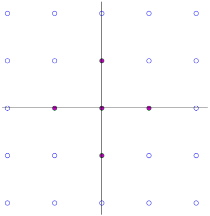
- Minimum distance:

$$d_0 = \mathrm{d}_{\min}(\mathbb{D}) = 1 \qquad d_S = \left| \gcd\left(\frac{M}{M_k}, k \notin S \right) \right| = \left| \prod_{k \in S} M_k \right| = 2^{R_S}$$

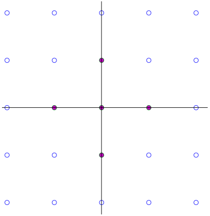
- Side information gain: $\Gamma = 20 \log_{10} 2 \approx 6$ dB/b/dim

$\mathbb{D} = \mathbb{Z}[i]$, $K = 2$, $(M_1, M_2) = (1+2i, 1-2i)$, $M = 5$

$w_1 \in \mathbb{Z}[i]/(1+2i)\mathbb{Z}[i] \cong \mathbb{F}_5$



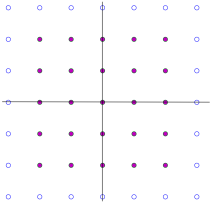
$w_2 \in \mathbb{Z}[i]/(1-2i)\mathbb{Z}[i] \cong \mathbb{F}_5$



$$x = w_1 \frac{M}{M_1} + w_2 \frac{M}{M_2} \bmod M\mathbb{D}$$

$$x = w_1(1-2i) + w_2(1+2i) \bmod 5\mathbb{Z}[i]$$

$5 = (1+2i)(1-2i)$
$\gcd(1+2i, 1-2i) = 1$



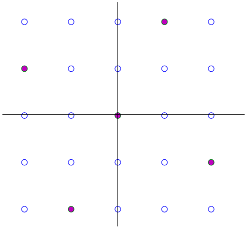
$\mathcal{X} = \mathbb{Z}[i]/5\mathbb{Z}[i]$

$\boxed{d_0 = \mathrm{d}_{\min}(\mathbb{Z}[i]) = 1}$

$$x = w_1(1-2i) + w_2(1+2i) \bmod 5\mathbb{Z}[i]$$

$$S = \{1\}$$

$$x = \Big(w_2(1+2i) + \text{constant}\Big) \bmod 5\mathbb{Z}[i]$$

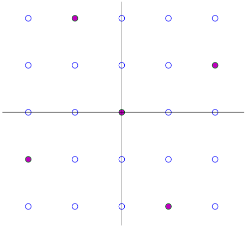


$$d_S = |1+2i| = \sqrt{5}$$

$$R_S = \tfrac{1}{2}\log_2 5$$

$$S = \{2\}$$

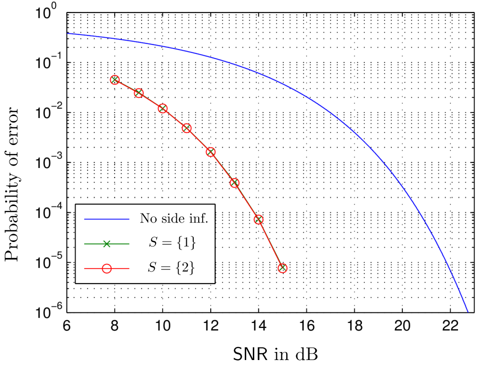
$$x = \Big(w_1(1-2i) + \text{constant}\Big) \bmod 5\mathbb{Z}[i]$$



$$d_S = |1-2i| = \sqrt{5}$$

$$R_S = \tfrac{1}{2}\log_2 5$$

$$\Gamma = \min_{S=\{1\},\{2\}} \frac{10\log_{10}(d_S^2/d_0^2)}{R_S} = \frac{10\log_{10} 5}{\frac{1}{2}\log_2 5} \approx 6 \text{ dB/b/dim}$$

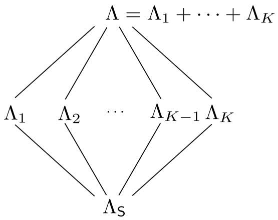


SNR gain over $S = \varnothing$ at $P_e = 10^{-5}$

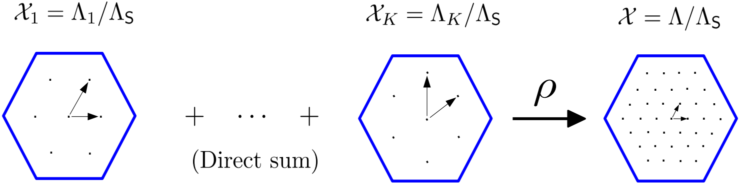
S	Actual gain	Predicted $\Gamma \times R_S$ dB
$\{1\}$	6.9	6.9
$\{2\}$	6.9	6.9

How to construct codes in higher dimensions?

Use a family of lattices



Lattice index code:



Lattice index codes

Definition

A *lattice index code* consists of nested lattice codes $\Lambda_1/\Lambda_\mathsf{s}, \ldots, \Lambda_K/\Lambda_\mathsf{s}$ such that the encoding map $\rho : \Lambda_1/\Lambda_\mathsf{s} \times \cdots \times \Lambda_K/\Lambda_\mathsf{s} \to \Lambda/\Lambda_\mathsf{s}$,

$$\rho(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_K) = (\boldsymbol{x}_1 + \cdots + \boldsymbol{x}_K) \mod \Lambda_\mathsf{s},$$

is one-to-one.

- $\Lambda_1/\Lambda_\mathsf{s}, \ldots, \Lambda_K/\Lambda_\mathsf{s}$ are subgroups of $\Lambda/\Lambda_\mathsf{s}$ (under addition mod Λ_s)
- One-to-one map ensures unique decodability and implies

$$\Lambda_1/\Lambda_\mathsf{s} \times \cdots \times \Lambda_K/\Lambda_\mathsf{s} \cong \Lambda/\Lambda_\mathsf{s} \text{ (as groups)}$$

- Rate $R_k = \dfrac{1}{n} \log_2 \dfrac{\mathrm{Vol}(\Lambda_\mathsf{s})}{\mathrm{Vol}(\Lambda_k)}$
- Minimum distance

$$d_0 = \mathrm{d}_{\min}(\Lambda) \qquad\qquad d_S = \mathrm{d}_{\min}\left(\sum_{k \notin S} \Lambda_k\right)$$

Properties

Effective codebook with side information configuration $S \subset \{1, \ldots, K\}$

- ..is a translate of the lattice code $\left(\sum_{k \notin S} \Lambda_k\right) / \Lambda_\mathsf{s}$
- Coding gain is the center density of the coding lattice $\sum_{k \notin S} \Lambda_k$

$$\delta\left(\sum_{k \notin S} \Lambda_k\right) = \frac{\left(r_{\text{pack}}\left(\sum_{k \notin S} \Lambda_k\right)\right)^n}{\mathrm{Vol}\left(\sum_{k \notin S} \Lambda_k\right)}$$

- When $S = \varnothing$, i.e., no side information, coding gain $= \delta(\Lambda)$

Distance gain due to side information

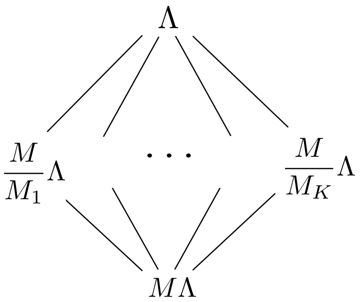
$$\frac{d_S}{d_0} = 2^{R_S} \times \left[\frac{\delta\left(\sum_{k \notin S} \Lambda_k\right)}{\delta(\Lambda)}\right]^{\frac{1}{n}}$$

Lemma

If Λ is a densest lattice in $\mathbb{R}^n$ then $\Gamma(\Lambda/\Lambda_\mathsf{s}) \leq 20 \log_{10} 2 \approx 6$ dB/b/dim

Construction using Chinese Remainder Theorem

- Let $\mathbb{D} = \mathbb{Z}$, $\mathbb{Z}[i]$ or $\mathbb{Z}[\omega]$ and Λ be any $\mathbb{D}$-lattice
  - Λ can be a known lattice with large coding gain
- Let $M_1, \ldots, M_K \in \mathbb{D}$ be relatively prime, $M = \prod_{k=1}^{K} M_k$
- Scale Λ by $\dfrac{M}{M_1}, \ldots, \dfrac{M}{M_K}$ to generate a family of lattices.



- Rate $R_k = \log_2 |M_k|$ b/dim.
- $\delta\left(\sum_{k \notin S} \Lambda_k\right) = \delta(\Lambda)$ for any S
- $d_0 = \mathrm{d_{min}}(\Lambda)$
- $\dfrac{d_S}{d_0} = 2^{R_S}$ for any S
- $\Gamma \approx 6$ dB/b/dim

Further Algebraic Constructions

Over general algebraic number fields (Huang, `ISIT'15`)

- Use the ring of integers $\mathcal{O}_{\mathbb{K}}$ of an algebraic number field $\mathbb{K}$
- The elements of $\mathcal{O}_{\mathbb{K}}$ can be embedded into a lattice
- Construct lattice codes using Chinese remainder theorem in $\mathcal{O}_{\mathbb{K}}$
- All the K messages can be allowed to take values from the same finite field
- If $\mathbb{K}$ is totally real: diversity gain in Rayleigh fading channel
- Both minimum Euclidean distance and minimum product distance improve with side information

Over Hurwitz quaternionic integers

- Non-commutative Euclidean domain that is geometrically equivalent to D_4^* lattice
- Corresponding lattice code has a larger coding gain than $\mathbb{Z}[i]$ and $\mathbb{Z}[\omega]$ constructions and provides more options for encoding rates

Summary of algebraic construction

- Using nested lattice codes for physical-layer index coding
- Algebraic labelling of codewords using Chinese remainder theorem
- Side information gain of at least 6 dB/b/dim
- Effective codebook at the receivers are also nested lattice codes
  - Can employ lattice decoding at the receivers

However..

- Decoding complexity is high for large dimensions
- Message sizes are not powers of 2

In the next section..

- We design a concatenated scheme that can be decoded with low-complexity iterative detection
- The modulation scheme (inner code) will encode integer number of bits while ensuring side information gain

Index Coding using Multidimensional PAM

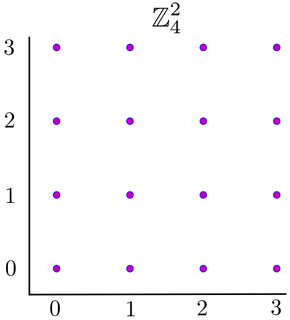
- **Message alphabet**:

$$\mathcal{W}_1 = \cdots = \mathcal{W}_K = \mathbb{Z}/M\mathbb{Z} \triangleq \mathbb{Z}_M$$

- **Codebook**: K-tuples over $\mathbb{Z}_M$

$$\mathcal{X} = \mathbb{Z}_M \times \cdots \times \mathbb{Z}_M = \mathbb{Z}_M^K$$

- Embed $\mathcal{X}$ into $\mathbb{R}^K$ using natural map



- Use a linear encoder to map messages to codeword

$$\begin{aligned} \mathcal{W}_1 \times \cdots \times \mathcal{W}_K &\to \mathcal{X} \\ \mathbb{Z}_M^K &\to \mathbb{Z}_M^K \end{aligned}$$

- Generate codeword $\boldsymbol{x}$ as linear combination of $\boldsymbol{c}_1, \ldots, \boldsymbol{c}_K \in \mathbb{Z}_M^K$:

$$\boldsymbol{x} = \rho(w_1, \ldots, w_K) = \sum_{k=1}^{K} w_k \boldsymbol{c}_k \bmod M.$$

Properties

- Code is fully characterized by the $K \times K$ generating matrix over $\mathbb{Z}_M$

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{c}_1 \ \boldsymbol{c}_2 \ \cdots \ \boldsymbol{c}_K \end{bmatrix}$$

- Code length $n =$ number of messages K.
- All messages are encoded at the same rate

$$R_1 = \cdots = R_K = \frac{1}{K} \log_2 M \text{ b/dim.}$$

- ρ is bijective $\Leftrightarrow \boldsymbol{C}$ is invertible, i.e., $\det(\boldsymbol{C})$ is a unit in $\mathbb{Z}_M$
- Minimum distance with no side information $d_0 = d_{\min}(\mathbb{Z}_M^K) = 1$

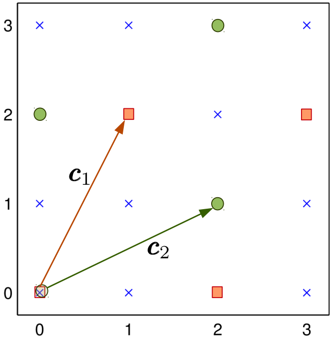
Properties

- Using linearity property, $d_S = d_{\min}\left(\mathscr{C}_S\right)$, where

$$\mathscr{C}_S = \left\{ \sum_{k \notin S} w_k \boldsymbol{c}_k \,\middle|\, w_k \in \mathbb{Z}_M, k \notin S \right\} \text{ is a } \mathbb{Z}_M\text{–linear code}$$

- d_S can be computed using shortest-vector algorithm for lattices
- Therefore $\Gamma = \min\limits_S \dfrac{10\log_{10}(d_S^2/d_0^2)}{R_S}$ can be efficiently computed using numerical techniques

Example: 16-QAM index code using $\mathbb{Z}_4$



- $\boldsymbol{C} = (\boldsymbol{c}_1 \ \boldsymbol{c}_2) = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$.
- $\det(\boldsymbol{C}) = -3 \bmod 4 = 1 \in U(\mathbb{Z}_4)$.
- $S = \{1\}$:

$$\mathscr{C}_S = \{w_2\boldsymbol{c}_2 \,|\, w_2 \in \mathbb{Z}_4\},$$
$$d_S = 2.$$

- $S = \{2\}$:

$$\mathscr{C}_S = \{w_1\boldsymbol{c}_1 \,|\, w_1 \in \mathbb{Z}_4\},$$
$$d_S = 2.$$

$\boxed{\Gamma \approx 6 \text{ dB/b/dim}}$

Computer Search for Good Codes

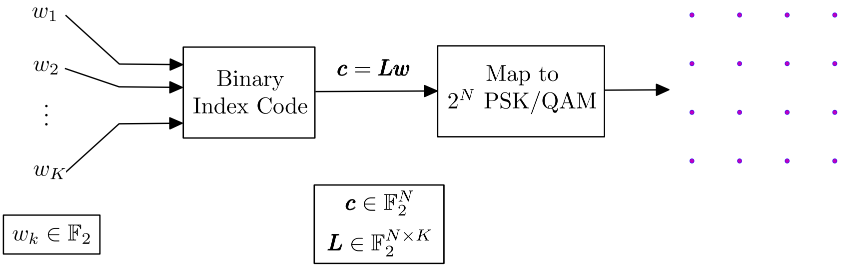
- We restrict search space to codes with circulant $\boldsymbol{C}$ matrix.
- Table gives the first column of $\boldsymbol{C}$ and Γ for the best codes.

M	$K = n$			
	2	3	4	5
4	$(1,2)^\mathsf{T}$ 6.02	$(1,2,2)^\mathsf{T}$ 4.52	$(1,1,3,0)^\mathsf{T}$ 3.01	$(1,2,1,3,0)^\mathsf{T}$ 3.76
8	$(1,2)^\mathsf{T}$ 4.65	$(1,2,0)^\mathsf{T}$ 3.49	$(1,0,3,3)^\mathsf{T}$ 4.01	$(1,7,2,2,5)^\mathsf{T}$ 4.70
16	$(1,12)^\mathsf{T}$ 6.02	$(1,2,10)^\mathsf{T}$ 5.24	$(1,4,10,8)^\mathsf{T}$ 5.57	$(1,14,11,12,5)^\mathsf{T}$ 5.28
32	$(1,6)^\mathsf{T}$ 5.85	$(1,22,14)^\mathsf{T}$ 5.73	$(1,10,14,2)^\mathsf{T}$ 5.80	$(1,24,27,15,26)^\mathsf{T}$ 5.77
64	$(1,36)^\mathsf{T}$ 6.04	$(1,38,60)^\mathsf{T}$ 5.73	$(1,38,20,30)^\mathsf{T}$ 5.85	$(1,16,18,55,21)^\mathsf{T}$ 5.82

Modulation scheme ensures large side information gain

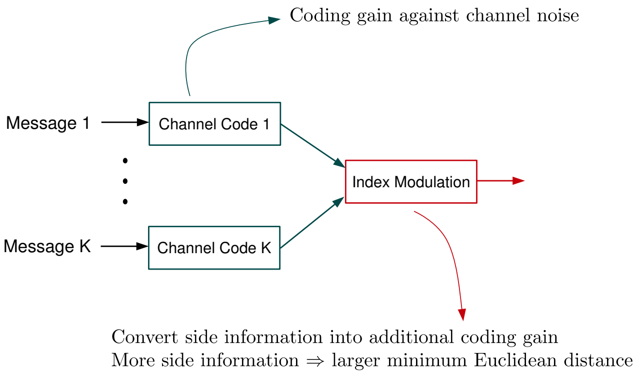
Further Modulation Schemes for Index Coding
(Mahesh & Rajan, arXiv:1603.03152)

Multiple receivers with general message demands



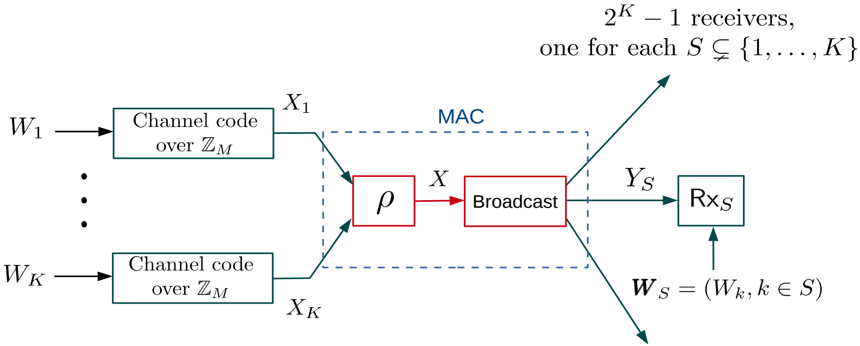
- The binary index code reduces the codeword length N (hence, the constellation size) while meeting the demands of the receivers
- The bit labelling of QAM/PSK allows receivers with sufficient side information to achieve gains in minimum Euclidean distance

Coded Index Modulation



- concatenated scheme: coding gain + side information gain.
- converts the channel into a MAC with many receivers.
- can perform close to channel capacity.

Coded Index Modulation



- If minimum Hamming distance of channel codes is d_H, then minimum squared Euclidean distance at Rx_S is at least $d_H \times d_S^2$.
- Capacity of MAC with many receivers: Ulrey `Inf.& Cont.'75`

Achievable rate region

Notation:

- $2^K - 1$ receivers indexed by S.
- $\mathsf{Rx}_S = (\mathsf{SNR}_S, S)$ observes Y_S.
- $\boldsymbol{X}_S = (X_k, k \in S)$, $\boldsymbol{X}_{S^c} = (X_k, k \notin S)$.
- $X_1, \ldots, X_K$ have distributions $p(x_1), \ldots, p(x_K)$.
- Inner code/modulation $(\rho, \mathcal{X})$ has dimension n.

Assumption:

- If $S \subset S'$, then $\mathsf{SNR}_S \geq \mathsf{SNR}_{S'}$

Theorem

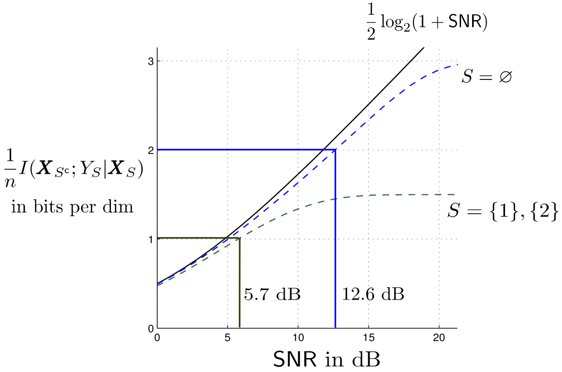
$(R_1, \ldots, R_K)$ achievable if and only if for every Rx_S:

$$\frac{1}{n} I(\boldsymbol{X}_{S^c}; Y_S \,|\, \boldsymbol{X}_S) \geq R - R_S.$$

$K = 2$, 64-QAM, $\Gamma = 4.66$ dB/b/dim

- Want $R_1 = R_2 = 1$ b/dim
- $2^K - 1 = 3$ receivers: $S = \varnothing, \{1\}, \{2\}$

$$\frac{1}{n} I(\boldsymbol{X}_{S^c}; Y_S | \boldsymbol{X}_S) \geq \begin{cases} R_1 + R_2 = 2 & \text{if } S = \varnothing \\ R_2 = 1 & \text{if } S = \{1\} \\ R_1 = 1 & \text{if } S = \{2\} \end{cases}$$



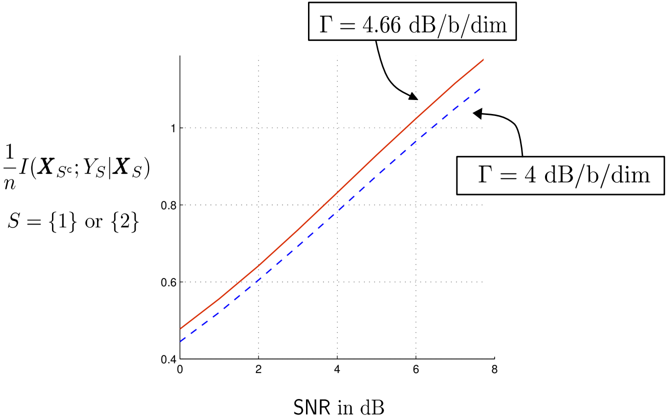
Dependence on Γ

- Larger $I(\boldsymbol{X}_{S^c}; Y_S | \boldsymbol{X}_S)$, for all $S \Rightarrow$ larger rates.
- Fano's inequality:

$$I(\boldsymbol{X}_{S^c}; Y_S | \boldsymbol{X}_S) \geq (1 - \mathsf{P}_e(\boldsymbol{X}_{S^c} | Y_S, \boldsymbol{X}_S)) \log_2 \prod_{k \in S^c} |\mathcal{X}_k| - 1$$

- Larger $\Gamma \Rightarrow$ larger d_S for all $S \Rightarrow$ smaller P_e simultaneously for all S
- We expect a modulation with larger Γ to perform better.

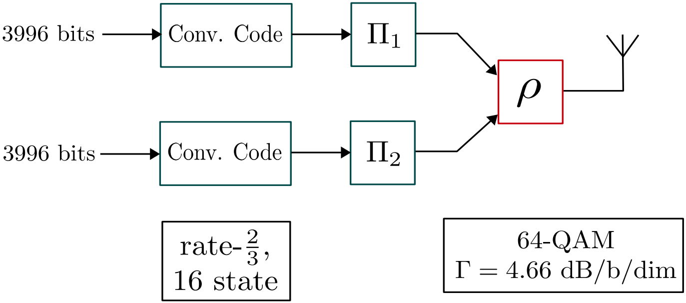
Example: 64-QAM, $K = 2$ messages



SNR gain of 0.7 dB at $\frac{1}{n} I(\boldsymbol{X}_{S^c}; Y_S | \boldsymbol{X}_S) = 1$.

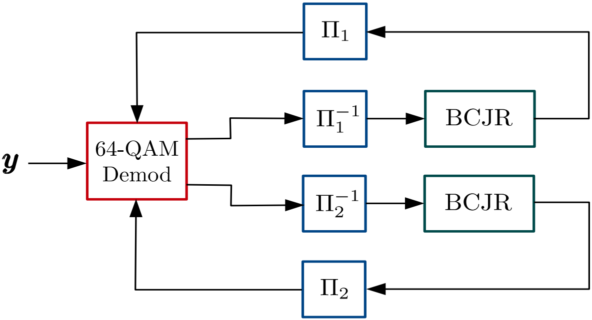
Encoder

- $K = 2$ messages, $R_1 = R_2 = 1$ b/dim.

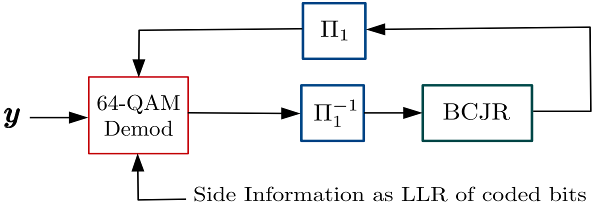


Decoder

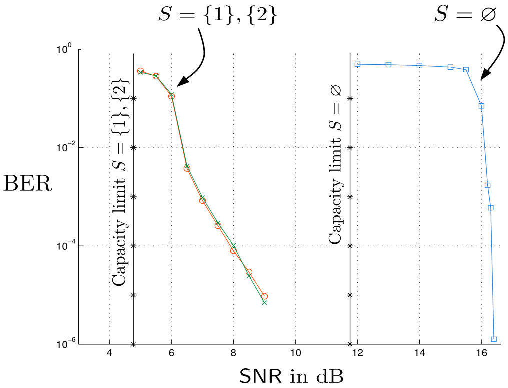
$\underline{S = \emptyset}$



$\underline{S = \{2\}}$



Simulation Result

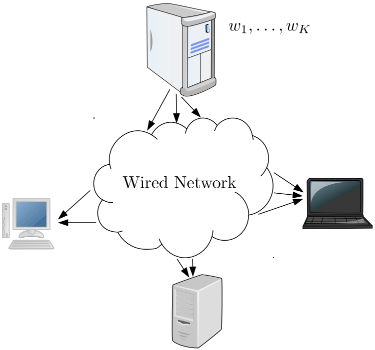


Gap to capacity (with Gaussian input alphabet) at BER 10^{-4}:

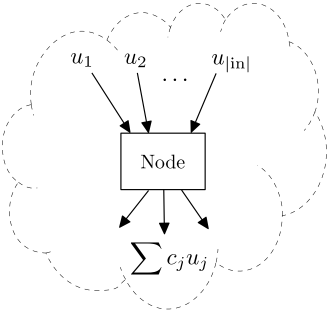
- 3.3 dB for $S = \{1\}, \{2\}$.
- 4.2 dB for $S = \varnothing$.

Wireline Multicasting
(Koetter & Médard `T-IT Mar'03`)

Multicast network



Linear network coding over $\mathbb{F}_q$

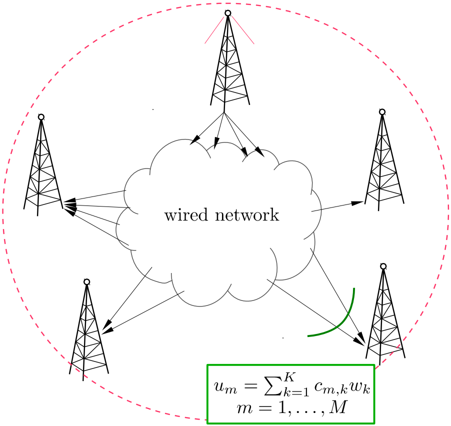


A network coding solution exists iff $\text{max-flow} \geq K$.

$\text{max-flow} =$ maximum number of edge-disjoint paths from the source to each of the receivers

Broadcasting with Coded Side Information at the Receivers

(Natarajan, Hong, Viterbo arXiv:1509.01332)



- Suppose $\text{max-flow} < K$.
- A wireless signal can supplement the wireline network.
- Symbols from wireline network serve as side information to decode wireless signal: linear combinations of source messages
- $\Rightarrow$ Broadcasting with coded side information at the receivers

Theorem: Lattice codes achieve the capacity of a wireless broadcast channel with coded side information at the receivers

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Wireline Multicasting

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