Joint State Sensing and Communication: Theory and Applications

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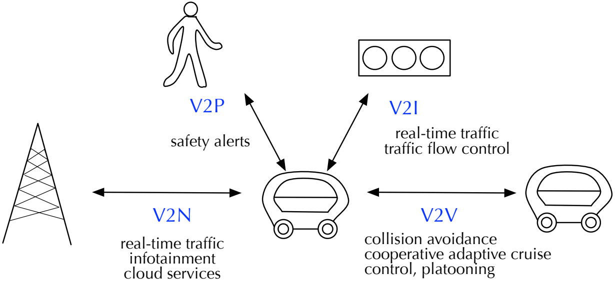
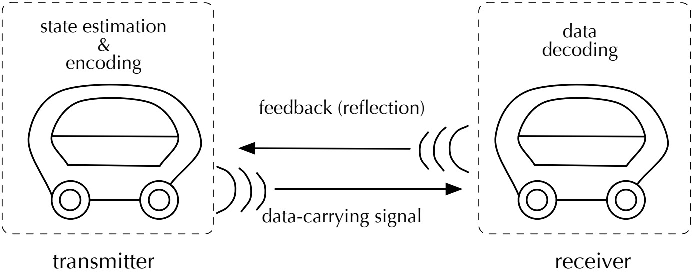
Introduction



- Future high-mobility networks must ensure both *connectivity* and *real-time adaptation*.
- A key-enabler is the ability to continuously track the dynamically changing environment, “state”, and react accordingly by exchanging information.

Example: Joint Radar and Vehicular Communication



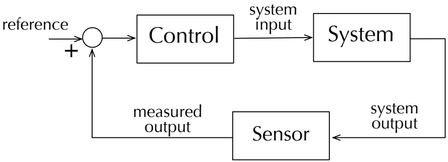
- The spectrum crunch encourages to use sensing and communication in the same frequency bands (e.g. IEEE S band shared between LTE and radar).
- One vehicle wishes to track the "state" (velocity, range) and simultaneously convey a message (safety/traffic-related).

Outline of my talk

- Part I: Preliminaries
  - Introduction
  - Channels with feedback
- Part II: Joint state sensing and communication
  - A single-user case
  - A two-user multiple access channel
- Part III: Vehicular applications
  - Joint radar and V2X communication
  - Performance analysis with multi-carrier modulation

Part I: Preliminaries

Feedback in our daily life

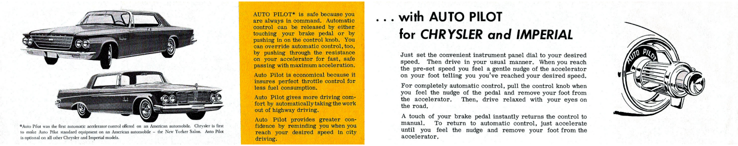


- Feedback enables a system to improve its capability by taking benefits from the response of actions and incorporating it into the design.
- Closed-loop control, rather than open-loop control without feedback.

Example 1: Thermostat

- Invented by Albert Butz in 1886, giving a birth to "Honeywell".
- Objective: keep the temperature constant in a room.
  - Reference: desired temperature
  - Control: switch on/off of boiler
  - Sensor: measures the temperature

Example 2: Cruise control of a car

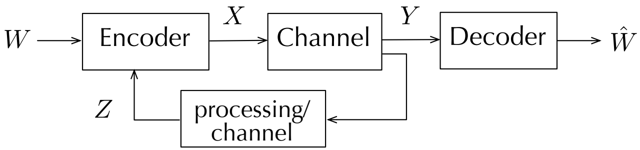


- Invented by Peerless and first commercialized for "Chrysler Imperial" in 1958.
- Objective: maintain speed whether up hill or down
  - Reference: desired speed
  - Control: accelerate or not
  - System: a channel with some disturbance (wind, hill).
  - Sensor: measures the speed.

Examples in communication standards

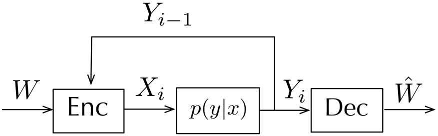
- Hybrid Automatic Request Control (HARQ)
  - included in High Speed Downlink/Uplink Packet Access (HSD/UPA) and LTE.
  - based on ACK/NACK feedback from users.
  - enables to improve error probability.
- Closed-loop MIMO
  - included in LTE
  - based on channel estimated at users.
  - A base station choose appropriate directions (precoder) to enhance data rate.

Feedback in communications



- Feedback enables a communication system to improve capacity, reliability or simplify encoding.
- Types of feedback.
  - Output feedback: $Z = Y$.
  - State feedback: estimated channel state given Y (processing).
  - Geralized feedback: Z is any causal function of Y (no processing).
- In information theory, feedback can be noise-free and even non-causal.

Feedback doesn't increase capacity of a memoryless channel [1]



The capacity of a memoryless channel with and without feedback is

$$C = \max_{P_X} I(X;Y)$$

achieved by

- Random encoding: to convey a message $w \in [1 : 2^{nR}]$, choose $x^n(w)$ from randomly and independently generated 2^{nR} sequences.
- Joint typicality decoding: choose $\hat{w}$ such that $(x^n(\hat{w}), y^n)$ are jointly typical.

[1] C. Shannon, "The zero error capacity of a noisy channel," IRE Trans. Information Theory, vol. 2, no. 3, 1956.

Converse: prove that we cannot transmit at $R > C$.

$$
\begin{aligned}
nR &= H(W) \\
&= I(W;Y^n) + H(W|Y^n) \\
&\leq I(W;Y^n) + n\epsilon_n \\
&= \sum_{i=1}^{n} I(W;Y_i|Y^{i-1}) + n\epsilon_n \\
&\leq \sum_{i=1}^{n} I(W,Y^{i-1}:Y_i) + n\epsilon_n \\
&= \sum_{i=1}^{n} I(W,Y^{i-1},X_i:Y_i) + n\epsilon_n \\
&= \sum_{i=1}^{n} I(X_i:Y_i) + n\epsilon_n
\end{aligned}
$$

Error probability for the channel w/o feedback

- A decoder makes an error if one of the following events occurs.

$$\mathcal{E}_1 = \{(X^n(1), Y^n) \notin \mathcal{T}\}, \quad \mathcal{E}_2 = \{(X^n(w), Y^n) \in \mathcal{T}, \ \forall w \neq 1\}$$

- Union bound

$$P(\mathcal{E}) = P(\mathcal{E}_1 \cup \mathcal{E}_1) \leq P(\mathcal{E}_1) + P(\mathcal{E}_2)$$

where by law of large number $\lim_{n\to\infty} P(\mathcal{E}_1) = 0$ and we have

$$\begin{aligned} P(\mathcal{E}_2) &\leq \sum_{w=2}^{2^{nR}} P((X^n(w), Y^n) \in \mathcal{T}) \\ &\leq \sum_{w=2}^{2^{nR}} 2^{-n(I(X;Y)-\epsilon)} \quad \text{joint typicality lemma} \\ &= 2^{-n(C-R-\epsilon)} \end{aligned}$$

Well-known results on output feedback [6]

- Feedback improves reliability of a memoryless channel
- The capacity of a two-user Gaussian multiple access channel (MAC) with feedback
- The capacity of a two-user erasure MAC
- An achievable rate region of a two-user Gaussian broadcast channel (BC)
- An achievable rate region of a Gaussian network with more than two users [2] [3]
- Tight bounds for a two-user Gaussian interference channel [4]
- Upper bounds of the K-user Gaussian MAC using dependence balance bounds [5]

[2] G. Kramer, "Feedback strategies for white Gaussian interference networks," IEEE Trans. Inf. Theory, vol. 48, no. 6, 2002.

[3] Ardestanizadeh et al., "Linear-feedback sum-capacity for Gaussian multiple access channels", IEEE Trans. Inf. Theory, vol. 58, no.1 2012

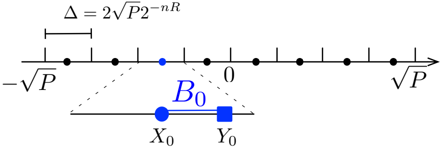
[4] C. Suh and D. Tse, "Feedback capacity of the Gaussian interference channel to within 2 bits", IEEE Trans. Inf. Theory, 2011

[5] E.Sula, "Sum-Rate Capacity for Symmetric Gaussian Multiple Access Channels with Feedback", ISIT'2018

[6] A. El Gamal and Y.-H. Kim, Network Information Theory, Cambridge University Press, 2011.

A Gaussian channel: Schalkwijik and Kailath

- A Gaussian channel $Y_i = X_i + B_i$ with $B_i \sim \mathcal{N}(0,1)$ and the input subject to $\frac{1}{n}\sum_{i=1}^{n} \mathbb{E}[|X_i|^2] \leq P$.
- Recursively send an estimation error seen by receiver.



$$
\begin{aligned}
X_0 &= \theta(w) & Y_0 &= \theta(w) + B_0 \\
X_1 &= \gamma_1 B_0 & Y_1 &= X_1 + B_1 \\
X_2 &= \gamma_2(B_0 - \mathbb{E}[B_0|Y_1]) & Y_2 &= X_2 + B_2 \\
&\vdots & &\vdots \\
X_n &= \gamma_n(B_0 - \mathbb{E}[B_0|Y^{n-1}]) & Y_n &= X_n + B_n
\end{aligned}
$$

- Receiver estimates

$$\hat{\theta}(w) = Y_0 - \mathbb{E}[B_0|Y^n] = \theta(w) + B_0 - \mathbb{E}[B_0|Y^n]$$

Error probability of Schalkwijik-Kailath 's scheme

- Orthogonality property implies that error $B_0 - \mathbb{E}[B_0|Y^i]$ is independent of Y^i for each i. The output sequence is i.i.d. Gaussian $Y_i \sim \mathcal{N}(0, 1+P)$.
- Write mutual information in two ways (exercise!):

$$\begin{aligned} I(B_0; Y^n) &= \sum_{i=1}^{n} I(B_0; Y_i|Y^{i-1}) = \dots \\ &= \frac{n}{2}\log(1+P) \triangleq C(P). \\ I(B_0; Y^n) &= h(B_0) - h(B_0|Y^n) = \frac{1}{2}\log\frac{1}{\mathsf{var}(B_0|Y^n)} \end{aligned}$$

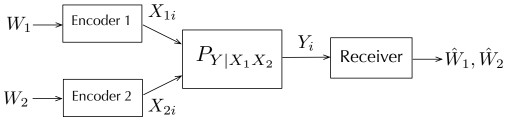
Error probability of Schalkwijik-Kailath 's scheme

- The estimate at receiver $\hat{\theta} \sim \mathcal{N}(\theta(w), 2^{-2nC(P)})$.
- The decoder makes an error if $|\theta - \hat{\theta}(w)| > \frac{\Delta}{2} = 2^{-nR}\sqrt{P}$ for any $w \in [1; 2^{nR}]$.
- The error probability is bounded by

$$\begin{aligned} P_e &= \Pr\left(|\theta - \hat{\theta}(1)| > 2^{-nR}\sqrt{P}\right) \\ &= 2Q(2^{n(C-R)}\sqrt{P}) \qquad \text{with } Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt \\ &\leq \sqrt{\frac{2}{\pi}} \exp\left(-\frac{2^{2n(C-R)}P}{2}\right) \qquad \text{with } Q(x) \leq \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \end{aligned}$$

- For $R < C(P)$, the error probability decays doubly exponentially !

Multiple Access Channel (MAC) without feedback

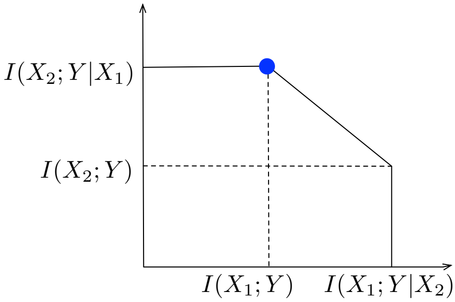


- Two transmitters wish to convey messages W_1, W_2 to the receiver, respectively.
- The capacity region of MAC w/o feedback is the convex hull of the union of [7]

$$\begin{aligned} R_1 &\leq I(X_1; Y|X_2) \\ R_2 &\leq I(X_2; Y|X_1) \\ R_1 + R_2 &\leq I(X_1, X_2; Y) \end{aligned}$$

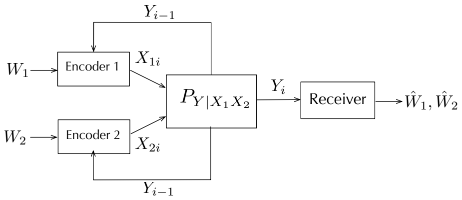
[7] An alternative expression is to use a time-sharing random variable Q.

Multiple Access Channel (MAC) without feedback



- Random encoding: to convey a message $w_k \in [1 : 2^{nR_k}]$, choose $x_k^n(w_k)$ from randomly and independently generated 2^{nR_k} sequences.
- Successive interference decoding
  - Find the unique message $\hat{w}_1$ such that $(x_1^n(\hat{w}_1), y^n) \in \mathcal{T}$.
  - Then, find the unique message $\hat{w}_2$ such that $(x_1^n(\hat{w}_1), x_2^n(\hat{w}_2), y^n) \in \mathcal{T}$.

MAC with output feedback



- Encoder 1 sends $X_{1i} = f_{1i}(W_1, Y_1^{i-1})$. Thanks to the feedback, two symbols (X_{1i}, X_{2i}) can be correlated.
- *Correlation* enables to reduce the multiuser interference and increase the sum rate.
  - Successive refinement of error seen by receivers.
  - A common message to be decoded by both encoders.

Gaussian MAC with feedback

- Consider the two-user Gaussian MAC

$$Y = X_1 + X_2 + B$$

with average power constraints $\frac{1}{n}\sum_{i=1}^{n}\mathbb{E}[|X_{ki}|^2] \leq P_k, \forall k = 1, 2$.

- The capacity region with feedback is given by

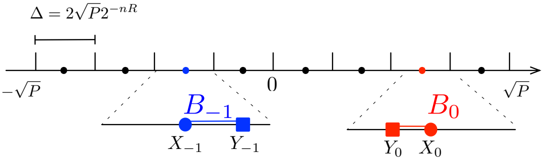
$$\begin{aligned} R_1 &\leq \frac{1}{2}\log(1 + P_1(1-\rho^2)) \\ R_2 &\leq \frac{1}{2}\log(1 + P_2(1-\rho^2)) \\ R_1 + R_2 &\leq \frac{1}{2}\log(1 + P_1 + P_2 + 2\rho\sqrt{P_1 P_2}) \end{aligned}$$

for some $\rho \in [0, 1]$.

- The sum capacity is given by ρ^*, solution of

$$\max_{\rho} \min\left\{\prod_{k=1}^{2}(1 + P_k(1-\rho^2), 1 + P_1 + P_2 + 2\rho\sqrt{P_1 P_2}\right\}$$

Ozarow's encoding $P_1 = P_2 = P$



$$
\begin{aligned}
\mathbf{X}_{-1} &= (\theta_1(w_1), 0) \\
\mathbf{X}_0 &= (0, \theta_2(w_2)) \\
\mathbf{X}_1 &= \gamma_1(B_{-1}, B_0) \\
&\vdots \\
\mathbf{X}_i &= \gamma_i(B_{-1} - \mathbb{E}[B_{-1}|Y^{i-1}], (-1)^{i-1}(B_0 - \mathbb{E}[B_{-1}|Y^{i-1}])) \\
&\vdots \\
\mathbf{X}_n &= \gamma_n(B_{-1} - \mathbb{E}[B_0|Y^{n-1}], (-1)^{n-1}(B_0 - \mathbb{E}[B_0|Y^{n-1}]))
\end{aligned}
$$

$$
\begin{aligned}
Y_{-1} &= \theta_1(w_1) + B_{-1} \\
Y_0 &= \theta_2(w_2) + B_0 \\
Y_1 &= X_{11} + X_{12} + B_1 \\
&\vdots \\
Y_i &= X_{1i} + X_{2i} + B_i \\
&\vdots \\
Y_n &= X_{1n} + X_{2n} + B_n
\end{aligned}
$$

- As for a single-user case, both encoders iteratively refine the receiver's error.

Ozarow: decoding and error analysis

- The decoder estimates

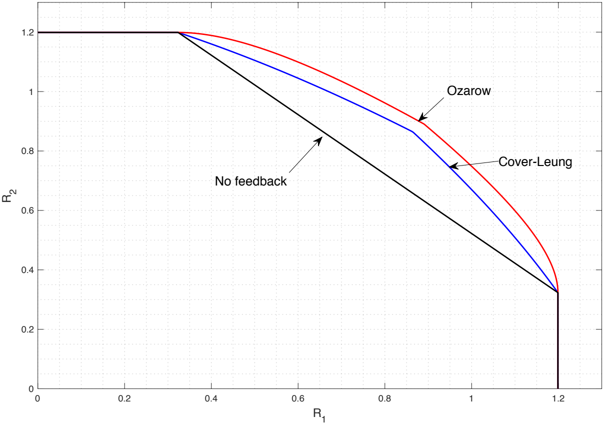
$$\hat{\theta}_1(w_1) = B_{-1} - \mathbb{E}[B_{-1}|Y^n] = \theta_1(w_1) + B_1 - \mathbb{E}[B_{-1}|Y^n]$$
$$\hat{\theta}_2(w_2) = B_0 - \mathbb{E}[B_0|Y^n] = \theta_2(w_2) + B_0 - \mathbb{E}[B_0|Y^n]$$

- It can be proved that the correlation $\mathbb{E}[X_{1i}X_{2i}] = \rho^*$ for any i.
- Following similar steps as a single user case, we can prove:

$$\hat{\theta}_k - \theta_k \sim \mathcal{N}(0, 2^{-2nC((1-\rho^*)P)}), \;\; k = 1, 2$$

- The error probability decays doubly exponentially as $n \to \infty$.

Gaussian MAC: two-user region



Binary erasure MAC

- Consider a binary erasure MAC

$$Y = X_1 + X_2$$

  where $X_1, X_2 \in \{0, 1\}$ and $Y \in \{0, 1, 2\}$.
- "Erasure" events occur when receiving $Y = 0 + 1 = 1 + 0 = 1$.
- The capacity of binary erasure MAC without feedback is (exercise)

$$R_1 \le 1, \quad R_2 \le 1, \;\; R_1 + R_2 \le \frac{3}{2}$$

- How much can we increase the sum rate via feedback ?

Two-phase schemes

1. $\frac{2}{3}$ bit/channel use
   - Phase 1: each user sends k uncoded bits
     $\rightarrow$ roughly $k/2$ bits are in "erasure".
   - Phase 2: only user 1 retransmits the erased bits.

$$R_{\text{user}} = \frac{k}{k + k/2} = \frac{2}{3}$$

2. 0.7602 bit/channel use [8]
   - Phase 1: each user sends k uncoded bits
   - Phase 2: two users "cooperatively" retransmit the erased bits by using 3 input-pairs $(0,0),(0,1),(1,1)$.

$$R_{\text{user}} = \frac{k}{k + \frac{k/2}{\log_2(3)}} = 0.7602$$

[8] Gaarder, Wolf, "The capacity region of a multiple-access discrete memoryless channel can increase with feedback", IEEE Trans. Inf. Theory, vol.21, no.1, 1975.

Optimal scheme: Cover-Leung

- An achievable region over a memoryless MAC:

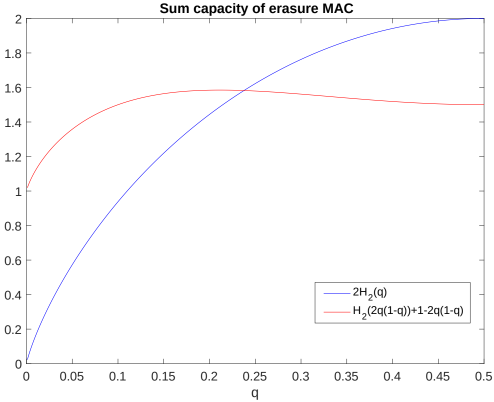
$$
\begin{aligned}
R_1 &\le I(X_1; Y | X_2, U) \\
R_2 &\le I(X_2; Y | X_1, U) \\
R_1 + R_2 &\le I(X_1, X_2; Y)
\end{aligned}
$$

  for some $P_U P_{X_1|U} P_{X_2|U}$.

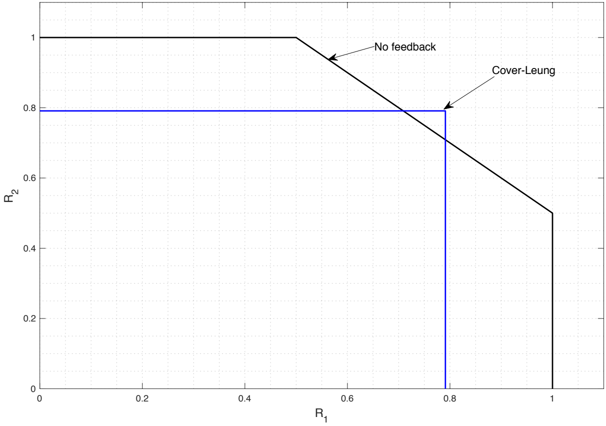
- The scheme yields the sum capacity over erasure MAC (exercise!)

$$
\begin{aligned}
C_{\text{sum}} &= \max_{P_{U,X_1,X_2}} \min\{H(X_1|U) + H(X_2|U), H(Y)\} \\
&= \max_q \min\{2H_2(q), H_2(2q\bar{q}) + 1 - 2q\bar{q}\} \\
&= 0.799
\end{aligned}
$$

Sum rate capacity of binary erasure MAC



Binary erasure MAC: two-user region



Cover-Leung: block Markov encoding/backward decoding

- Two encoders send (w_{1b}, w_{2b}) in block b of N channel uses for $b \in [1, B]$.
- At the end of block b, encoder 1 "estimates" $\tilde{w}_{2b}$ from a feedback Y_b^N.

$$(u^N(\tilde{w}_{2b-1}), x_1^N(w_{1b}|\tilde{w}_{2b-1}), x_2^N(\tilde{w}_{2b}|\tilde{w}_{2b-1}), y_b^N) \in \mathcal{T}$$

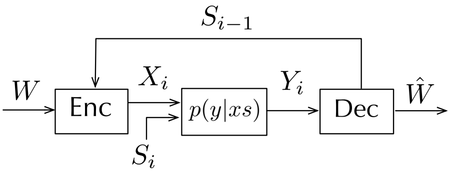
- In block $b+1$, both encoders send:
  - refinement information on w_{2b} by $u^N(w_{2b})$:
  - fresh messages w_{1b+1}, w_{2b+1} by $x_1^N(w_{1,b+1}|\tilde{w}_{2b}), x_2^N(w_{2,b+1}|w_{2b})$
- Backward decoding
  in block b, the decoder outputs w_{1b}, w_{2b-1} using the information from block $b+1$.

Block markov encoding and backward decoding

Block	1	2		$B-1$	B
X_2	$x_2^N(w_{21}\|1)$	$x_2^N(w_{22}\|w_{21})$	...	$x_2^N(w_{2,B-1}\|w_{2,B-2})$	$x_2^N(1\|w_{2,B-1})$
X_1	$x_1^N(w_{11}\|1)$	$x_1^N(w_{12}\|\tilde{w}_{21})$	...	$x_1^N(w_{1,B-1}\|\tilde{w}_{2,B-2})$	$x_1^N(w_{1B}\|\tilde{w}_{2,B-1})$
(X_1, Y)	$\tilde{w}_{21} \rightarrow$	$\tilde{w}_{22} \rightarrow$	...	$\tilde{w}_{2,B-1} \rightarrow$	0
Y	$\hat{w}_{11}$	$\leftarrow (\hat{w}_{12}, \hat{w}_{21})$	...	$\leftarrow (\hat{w}_{1,B-1}, \hat{w}_{2,B-2})$	$\leftarrow (\hat{w}_{1B}, \hat{w}_{2,B-1})$

- $\tilde{w}_{2b}$: user 2's message decoded by user 1 at the end of block b.
- w_{1b}: a private message of user 1 at block b.

Well-known results on state feedback



- The sum capacity scaling $M \log \log K$ in the MISO-BC with M transmit antennas and K users[9]
- The capacity region of an erasure BC (EBC) for $K \leq 3$ and for a symmetric EBC $K > 3$ [10][11]
- The DoF region of the MISO BC [12].
- many others...

[9] Sharif and Hassibi, "On the capacity of MIMO broadcast channels with partial side information", IEEE Trans. on info. Th., 2005

[10] C. C. Wang "The capacity region of two-receiver multiple-input broadcast packet erasure channels with channel output feedback", IEEE Trans. on Info. Th., 2014.

[11] M. Gatzianas et al., "Multiuser Broadcast Erasure Channel With Feedback-Capacity and Algorithms", IEEE Trans. on Inf. Th, 2013

[12] M. A. Maddah-Ali and D. N. C. Tse, "Completely Stale Transmitter Channel State Information is Still Very Useful," IEEE Trans. on Inf. Th, vol. 58, no. 7, 2012.

MISO and erasure BC

Erasure BC

The capacity region of K-user symmetrical erasure BC is given by [WangIT12, GatzianasIT13]

$$\sum_{k=1}^{K} \frac{1}{\underbrace{\boxed{1-\delta^k}}_{=\alpha_k}} R_{\pi_k} \leq 1, \quad \forall \pi$$

MISO BC

The DoF region of K-user MISO-BC with $M \geq K$ antennas is given by [MAT-IT12]

$$\sum_{k=1}^{K} \frac{1}{\underbrace{\boxed{k}}_{=\alpha_k}} \mathsf{DoF}_{\pi_k} \leq 1, \quad \forall \pi$$

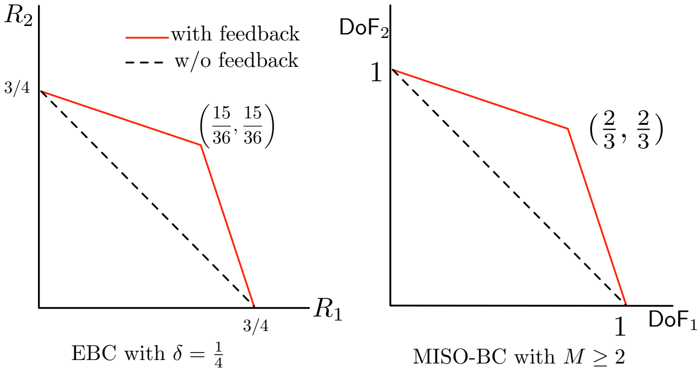
$$\mathsf{DoF} = \lim_{P\to\infty} \frac{R}{\log P}$$

- Both regions have a polyhedron structure characterized by

$$R_k = d_k = \begin{cases} \frac{1}{\sum_{k=1}^{|\mathcal{K}|} \frac{1}{\alpha_k}}, & k \in \mathcal{K} \\ 0, & k \notin \mathcal{K} \end{cases}$$

for $\mathcal{K} \subseteq \{1, \dots, K\}$.

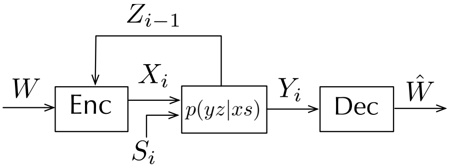
Two-user erasure/MISO-BC regions



Unified view on the schemes for EBC/MISO-BC

- Opportunistic multicasting can be repeated for a subset $\mathcal{J}$ of users for $\mathcal{J} \subseteq \{1, \ldots, K\}$.
- Algorithms for erasure BC and MISO-BC consist of K phases.
  - Phase 1: broadcast $V_1, \ldots, V_K$, each of dimension N.
  - Phases 2-K: generate $V_{\mathcal{J}}$ simultaneously useful for $\mathcal{J}$ and send sequentially for all $\mathcal{J}$.
- We can interpret phases 2 to K as *multicasting phase* of overheard symbols.
- Feedback enables to successively refine the multiuser interference (spatial/code dimension)

Well-known results on generalized feedback



- Generalized feedback refers to an additional causal channel output.
- An achievable rate region of a DM-MAC[13][14]
- An achievable rate region of a DM-BC [15] [16]
- An achievable region and outer bounds of DM interference channels [17]

[13] A. Carleial, "Multiple-access channels with different generalized feedback signals", IEEE Trans. on Inf. Th, 1982

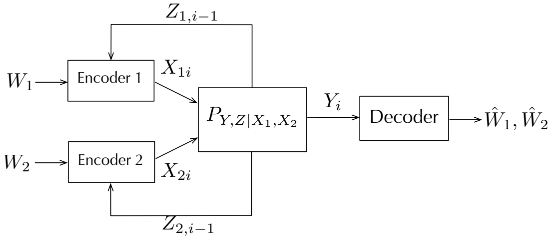
[14] F. Willems, "Information Theoretical Results for the Discrete Memoryless Multiple Access Channel", Ph. D. thesis, Katholieke Universiteit Leuven, Belgium, 1989.

[15] O. Shayevitz, and M. Wigger, "On the capacity of the discrete memoryless broadcast channel with feedback", IEEE Trans. on Inf. Th, 2013

[16] R.Venkataramanan and S. Pradhan, "An achievable rate region for the broadcast channel with feedback", IEEE Trans. on Inf. Th, 2013

[17] S. Yang and D. Tuninetti, "Interference channel with generalized feedback : Part I: Achievable region", IEEE Trans. on Inf. Th, 2011

Generalized feedback for MAC: Willems



- Encoder k observes an output $Z_{k,i-1}$ at time i.
- Feedback enables "transmitter cooperation".
- An achievable rate region is given by

$$
\begin{aligned}
R_1 &\le I(X_1;Y|X_2,V_1,U) + I(V_1;Z_2|X_2,U) \\
R_2 &\le I(X_2;Y|X_1,V_2,U) + I(V_2;Z_1|X_1,U) \\
R_1 + R_2 &\le \min\{I(X_1,X_2;Y), I(X_1,X_2;Y|V_1,V_2,U) \\
&\qquad + I(V_1;Z_2|X_2,U) + I(V_2;Z_1|X_1,U)\}
\end{aligned}
$$

Willems' scheme

In each block b, user 1

1. generates a private message $w_{11(b)}$ and another message $w_{12(b)}$ to be decoded by user 2.

2. sends
$$x_1^N(w_{12(b-1)}, \tilde{w}_{21(b-1)}, w_{12(b)}, w_{11(b)})$$
$\tilde{w}_{21(b-1)}$ was estimated from block $b-1$.

3. then estimates $\tilde{w}_{21(b)}$ from its feedback $z_1^N(b)$.

Willems' block Markov encoding and backward decoding

Block	1	2		$B-1$	B
X_1	$\underline{x}_1(1,1,w_{12(1)},w_{11(1)})$	$\underline{x}_1(w_{12(1)},\tilde{w}_{21(1)},w_{12(2)},w_{11(2)})$	...	$\underline{x}_1(w_{12(B-2)},\tilde{w}_{21(B-2)},w_{12(B-1)},w_{11(B-1)})$	$\underline{x}_1(w_{12(B-1)},\tilde{w}_{21(B-1)},1,1)$
(X_1,Z_1)	$\tilde{w}_{21(1)} \rightarrow$	$\tilde{w}_{21(2)} \rightarrow$	...	$\tilde{w}_{21(B-1)} \rightarrow$	0
X_2	$\underline{x}_2(1,1,w_{21(1)},w_{22(1)})$	$\underline{x}_2(\tilde{w}_{12(1)},w_{21(1)},w_{21(2)},w_{22(2)})$	...	$\underline{x}_2(\tilde{w}_{12(B-2)},w_{21(B-2)},w_{21(B-1)},w_{22(B-1)})$	$\underline{x}_2(\tilde{w}_{12(B-1)},w_{21(B-1)},1,1)$
(X_2,Z_2)	$\tilde{w}_{12(1)} \rightarrow$	$\tilde{w}_{12(2)} \rightarrow$	...	$\tilde{w}_{12(B-1)} \rightarrow$	0
Y	$\hat{w}_{11(1)},\hat{w}_{22(1)}$	$\leftarrow (\hat{w}_{12(1)},\hat{w}_{21(1)})\quad \hat{w}_{11(2)},\hat{w}_{22(2)}$	...	$\leftarrow (\hat{w}_{12(B-2)},\hat{w}_{21(B-2)})\quad \hat{w}_{11(B-1)},\hat{w}_{22(B-1)}$	$\leftarrow \hat{w}_{12(B-1)},\hat{w}_{21(B-1)}$

- $(w_{12(b-1)}, w_{21(b-1)})$: the common message from the previous block $b-1$, carried by U.
- $\tilde{w}_{21(b)}$: user 2's message decoded by user 1 at the end of block b, carried by V_2.
- $w_{11(b)}$: a private message of user 1 at block b.

Willems' scheme

- By letting R_{kj} denote the rate of $w_{kj(b)}$, we can prove that $P_e \to 0$ as $N \to \infty$.

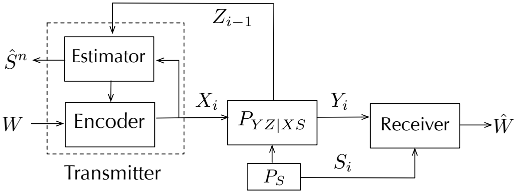
$$\begin{aligned}
R_{12} &\le I(V_1; Z_2 | X_2 U) \\
R_{21} &\le I(V_2; Z_1 | X_1 U) \\
R_{11} &\le I(X_1; Y | S X_2 V_1 U) \\
R_{22} &\le I(X_2; Y | S X_1 V_2 U) \\
R_{11} + R_{22} &\le I(X_1 X_2; Y | S V_1 V_2 U) \\
R_{12} + R_{21} + R_{11} + R_{22} &\le I(X_1 X_2; Y | S)
\end{aligned}$$

Summary of Part I

- Feedback enables a communication system to improve reliability, simplify encoding, or increase capacity.
- Achievable schemes build on *successive refinement*:
  - Linear: MMSE-based approaches, interference alignment
  - Non-linear: block-Markov encoding
- The capacity of many channels with feedback remains open.

Part II: Joint State Sensing and Communications

System Model



- Transmitter sends a message W and estimates a state sequence S^n via "generalized feedback": strictly causal channel output Z_{i-1}.
- Receiver decodes $\hat{W}$ from its observation Y^n and S^n (known perfectly).
- A memoryless state-dependent channel:

$$P_{WX^nS^nY^nZ^n}(w, \mathbf{x}, \mathbf{s}, \mathbf{y}, \mathbf{z}) = P(w) \prod_{i=1}^{n} P_S(s_i) \prod_{i=1}^{n} P(x_i|wz^{i-1}) P_{YZ|XS}(y_i z_i|x_i s_i).$$

Separation-based Approach

(a) IEEE 802.11p OFDM frame (b) IEEE 802.11ad frame

- Resources are divided into either sensing or data communications.
  - LTS: Physical Downlink Control Channel.
  - IEEE 802.11p combined with Direct Short Range Communication.
  - 3GPP-based Cellular Vehicle-to-Everything (C-V2X).
  - mmWave V2X based on IEEE 802.11ad.
- Limitations:
  - they performs poorly in high mobility scenarios or for a large state dimension.
  - the data rate degrades by dedicating more resources to state sensing.
- What is the optimal tradeoff between communication and sensing ?

Related Works

1. Capacity-distortion tradeoff with state only at transmitter
   - full or non-causal state [18]
   - strictly causal and causal state [19]
   - statistical state [20]
2. Channel with state available at transmitter or/and receiver [21]

[18] Sutivong et al., "Channel capacity and state estimation for state-dependent Gaussian channels", TIT 2005, Choudhuri et Mitra, "On Non-causal side information at the encoder", Allerton 2012

[19] Choudhuri et al., "Causal state communication", TIT 2013

[20] Zhang et al., "Joint transmission and state estimation: a constrained channel coding approach", TIT 2011

[21] El Gamal and Kim, Chapter 7 "Network Information Theory"

Some Definitions

- A $(2^{nR}, n)$ code consists of a message set, an encoder, a decoder, and a state estimator.
- The state estimate is measured by the expected distortion

$$\mathbb{E}[d(S^n, \hat{S}^n)] = \frac{1}{n}\sum_{i=1}^{n} \mathbb{E}[d(S_i, \hat{S}_i)]$$

- A rate distortion pair (R, D) is achievable if

$$\lim_{n\to\infty} P(\hat{W} \neq W) = 0$$

  and

$$\limsup_{n\to\infty} \mathbb{E}[d(S^n, \hat{S}^n)] \leq D.$$

- The capacity-distortion tradeoff $C(D)$ is the supremum of R such that (R, D) is achievable.

Main Result

Theorem

The capacity-distortion tradeoff of the state-dependent memoryless channel with the i.i.d. states is given by

$$C(D) = \max I(X;Y|S)$$

where the maximum is over all P_X satisfying $\mathbb{E}[d(S,\hat{S})] \leq D$ and the joint distribution of $SXYZ\hat{S}$ is given by

$$P_X(x) P_S(s) P_{YZ|XS}(yz|xs) P_{\hat{S}|XZ}(\hat{s}|xz)$$

.

Converse

Use Fano's inequality and usual steps:

$$
\begin{aligned}
R &\le \frac{1}{n} I(W; Y^n | S^n) + \epsilon_n \\
&\le \frac{1}{n} \sum_{i=1}^{n} [H(Y_i|S_i) - H(Y_i|X_i, Y^{i-1}, W, S^n) + \epsilon_n \\
&= \frac{1}{n} \sum_{i=1}^{n} I(X_i; Y_i | S_i) + \epsilon_n \quad \text{Markov chain } (W, Y^{i-1}, \{S_l\}_{l \neq i}) - (S_i, X_i) - Y_i \\
&\le \frac{1}{n} \sum_{i=1}^{n} C\left(\mathbb{E}[d(S_i, \hat{S}_i)]\right) + \epsilon_n \qquad \text{definition of } C(\cdot) \\
&\le C\left(\frac{1}{n} \sum_{i=1}^{n} \mathbb{E}[d(S_i, \hat{S}_i)]\right) + \epsilon_n \qquad \text{concavity of } C(\cdot) \\
&\le C(D)
\end{aligned}
$$

Achievability

- Encoder: random coding for fixed P_X and reconstruction function $\hat{s}(x,z)$ that achieve $C(\frac{D}{1+\epsilon})$ given a target distortion D.
- Decoder: jointly typicality decoding.
- Expected distortion: by defining $d_{\max} = \max_{(s,\hat{s})} d(s,\hat{s}) < \infty$,

$$\limsup_{n\to\infty} \mathbb{E}[d(S^n,\hat{S}^n)] \le \limsup_{n\to\infty} P_e d_{\max} + (1-P_e) \underbrace{(1+\epsilon)\mathbb{E}[d(S,\hat{S})]}_{\text{typical average lemma}}$$

$$\le \limsup_{n\to\infty} P_e d_{\max} + (1-P_e)(1+\epsilon)D$$

$$= (1+\epsilon)D \qquad P_e \to 0 \text{ if } R < I(X;Y|S)$$

This proves the achievability of $(C(\frac{D}{1+\epsilon}), D)$.

- From the continuity of $C(x)$ in x, the desired result follows as $\epsilon \to 0$. .

Numerical Method for Optimization

- Suppose that the input X^n has a cost constraint B.
- Consider a cost function $b(X^n) = \frac{1}{n}\sum_{i=1}^{n} b(X_i)$ such that $\limsup_{n\to\infty} \mathbb{E}[b(X^n)] \le B$.
- The optimization problem can be stated as

$$\begin{aligned} \text{maximize} \quad & I(X;Y|S) \\ \text{subject to} \quad & \mathbb{E}[d(S,\hat{S})] \le D. \\ & \mathbb{E}[b(X)] \le B \end{aligned}$$

- For the joint distribution $P_X P_S P_{YZ|XS} P_{\hat{S}|XZ}$, the estimator $\hat{s}(x,z)$ can be computed a priori.

Numerical Method for Optimization

The problem can be rewritten in terms of P_X [22]:

$$\begin{aligned} \text{maximize} \quad & \mathcal{I}(P_X, P_{Y|XS}|P_S) \\ \text{subject to} \quad & \sum_x b(x)P_X(x) \le B \\ & \sum_x c(x)P_X(x) \le D \end{aligned}$$

where we define the mutual information functional

$$\mathcal{I}(P_X, P_{Y|XS}|P_S) = \sum_s P_S(s) \sum_x \sum_y P_X(x)P_{Y|XS}(y|xs) \log \frac{P_{Y|XS}(y|xs)}{P_{Y|S}(y|s)}.$$

and

$$c(x) = \sum_{z\in\mathcal{Z}} P_{Z|X}(z|x) \sum_{s\in\mathcal{S}} P_{S|XZ}(s|xz)d(s, \hat{s}(x, z)).$$

[22] P_X denotes a feasible input distribution s.t. $P_X(x) \ge 0, \forall x \in \mathcal{X}$ and $\sum_x P_X(x) = 1$

A New Problem

- Assume that a feasible set of P_X satisfying cost and distortion constraints is non-empty.
- The solution does not necessarily satisfy both constraints with equality.
- Consider a parametric form of the optimization problem.

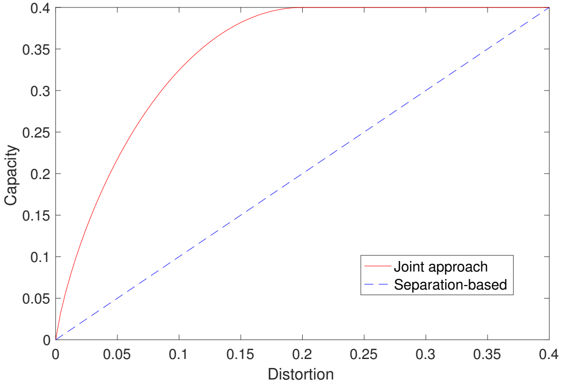
$$\max_{P_X : \sum_x b(x) P_X(x) \leq B} \mathcal{I}(P_X, P_{Y|XS} | P_S) - \mu \sum_x c(x) P_X(x)$$

  where $\mu \geq 0$ is a fixed parameter.
- We propose a modified Blahut-Arimoto.

Example 1: Binary channel with multiplicative states

- A binary channel $Y = SX$ with a Bernoulli distributed state s.t. $P_S(1) \stackrel{\Delta}{=} p_s \in [0, 1/2]$.
- Based on the Hamming distortion function $d(s, \hat{s}) = s \oplus \hat{s}$, characterize the input distribution $P_X(0) \stackrel{\Delta}{=} p \in [0, 1/2]$ that maximizes $C(D)$.
- Two extreme points:
  - If $p = 0$ (by sending always $X = 1$), $D_{\min} = 0$ but $C(D) = 0$.
  - If $p = 1/2$, then $C(D_{\max}) = p_s$ and $D_{\max} = p_s/2$.

$C(D)$ of Binary Channel with $p_s = 0.4$



- For a given p_s, we have

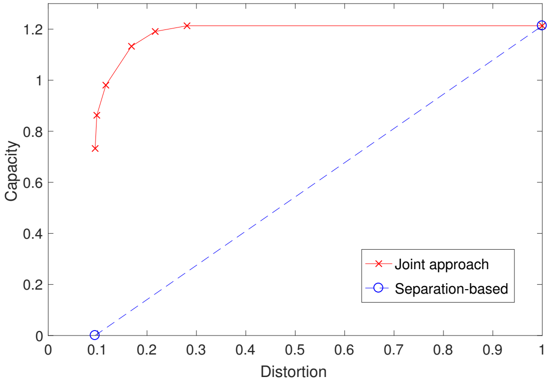
$$C(p) = p_s H_2(p), \quad D(p) = p_s p$$

- A separation-based approach achieves a time-sharing between $(D, C) = (0, 0)$ and (p_s, p_s).

Example 2: a real Gaussian channel with Rayleigh fading

- A real fading channel $Y_i = S_i X_i + N_i$ where
  - S_i, N_i are i.i.d. Gaussian distributed with zero mean and unit variance
  - $\{X_i\}$ satisfies the average power constraint $\frac{1}{n}\sum_i \mathbb{E}[|X_i|^2] \leq P$.
- Quadratic distortion function: the expected distortion is $\mathbb{E}\left[\frac{1}{1+|X|^2}\right]$.
- Two extreme points:
  - $D_{\min}$ achieved by 2-ary pulse amplitude modulation (PAM).
  - $C_{\max} = \mathbb{E}[\log(1 + |S|^2 P)]$ achieved by Gaussian input.

$C(D)$ of Gaussian channel with $P = 10$ dB



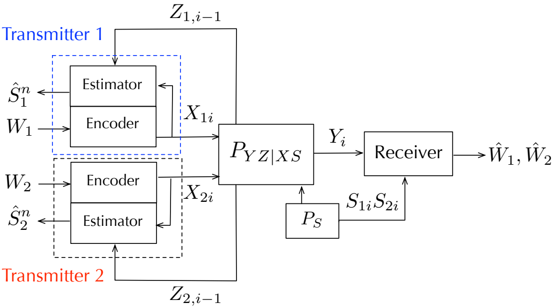
Separation-based approach: a time-sharing between

- $(D_{\min}, 0)$ by dedicating full resources to state estimation
- $(D_{\max}, C_{\max})$ with $D_{\max} \triangleq \mathrm{var}[S] = 1$, by ignoring feedback and sending data with Gaussian distribution.

Remarks

- Joint sensing and communication potentially yields a large gain with respect to a separation-based approach.
- Even restricting to a memoryless case, this preliminary result presents a first step towards a unified framework.
- Yet, feedback is only useful for state estimation for the single-user memoryless channel.
- Can feedback enhance joint sensing and communication over a multiuser channel ?

Multiple Access Channel Model



- Encoder k wishes to convey a message W_k and simultaneously estimate the state S_k.
- A memoryless MAC:

$$\prod_{i=1}^{n} P_S(s_i) P_{YZ_1Z_2|X_1X_2S}(y_i, z_{1i}, z_{2i}|x_{1i}, x_{2i}, s_i)$$
$$P(x_{1i}|x_1^{i-1}, z_1^{i-1}) P(x_{2i}|x_2^{i-1}, z_2^{i-1}).$$

Some Definitions

- (R_1, R_2, D_1, D_2) is achievable if

$$P(\hat{W}_k \neq W_k) \leq \epsilon, \quad k = 1, 2$$

and

$$\mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} d_k(S_{ki}, \hat{S}_{ki})\right] \leq D_k + \epsilon \;\; k = 1, 2.$$

- $\mathcal{C}(D_1, D_2)$ is the closure of achievable (R_1, R_2) for specified D_1, D_2.
- Idealized estimator $\psi_1^*(x_1, x_2, z_1, z_2)$ that knows also (x_2, z_2).

$$c_1(x_1, x_2) = \mathbb{E}[d_1(s_1, \psi_1^*(x_1, x_2, z_1, z_2))|X_1 = x_1, X_2 = x_2].$$

- Achievable estimator $\underline{\psi}_1^*(x_1, v_2, z_1)$ that uses knowledge of v_2

$$\underline{c}_1(x_1, v_2) = \mathbb{E}[d_1(s_1, \underline{\psi}_1^*(x_1, v_2, z_1))|X_1 = x_1, V_2 = v_2]$$

Contributions

- Outer bound: extension of the Tandon-Ulukus bounds[23] to the state-dependent MAC with distortion constraints.
- Achievability: extension of Willems' scheme [24] to the same context.

[23] R. Tandon and S. Ulukus, "Dependence balance based outer bounds for Gaussian networks with cooperation and feedback", IEEE Trans. Info. Theory, vol. 57, no. 7, 2011.

[24] F. Willems, "Information Theoretical Results for the Discrete Memoryless Multiple Access Channel", Ph. D. thesis, Katholieke Universiteit Leuven, Belgium, 1989.

Achievability

Theorem

An achievable rate region of the state-dependent memoryless MAC with i.i.d. states is given by

$$\begin{aligned} R_1 &\le I(X_1; Y|X_2V_1US) + I(V_1; Z_2|X_2U) \\ R_2 &\le I(X_2; Y|X_1V_2US) + I(V_2; Z_1|X_1U) \\ R_1 + R_2 &\le \min\{I(X_1X_2; Y|S), I(X_1X_2; Y|SV_1V_2U) \\ &\quad + I(V_1; Z_2|X_2U) + I(V_2; Z_1|X_1U)\} \end{aligned}$$

where $V_1X_1 - U - V_2X_2$ *and* $UV_1V_2 - X_1X_2 - YZ_1Z_2$ *form Markov chains, and where*

$$\begin{aligned} \mathbb{E}[\underline{c}_1(X_1, V_2)] &\le D_1 \\ \mathbb{E}[\underline{c}_2(V_1, X_2)] &\le D_2. \end{aligned}$$

Our proposed scheme: encoding

Block	1	2		$B-1$	B
X_1	$\underline{x}_1(1,1,w_{12}^1,w_{11}^1)$	$\underline{x}_1(w_{12(1)},\tilde{w}_{21(1)},w_{12(2)},w_{11(2)})$	...	$\underline{x}_1(w_{12(B-2)},\tilde{w}_{21(B-2)},w_{12(B-1)},w_{11(B-1)})$	$\underline{x}_1(w_{12(B-1)},\tilde{w}_{21(B-1)},1,1)$
(X_1,Z_1)	$\tilde{w}_{21(1)} \rightarrow$	$\tilde{w}_{21(2)} \rightarrow$	...	$\tilde{w}_{21(B-1)} \rightarrow$	0
X_2	$\underline{x}_2(1,1,w_{21(1)},w_{22(1)})$	$\underline{x}_2(\tilde{w}_{12(1)},w_{21(1)},w_{21(2)},w_{22(2)})$	...	$\underline{x}_2(\tilde{w}_{12(B-2)},w_{21(B-2)},w_{21(B-1)},w_{22(B-1)})$	$\underline{x}_2(\tilde{w}_{12(B-1)},w_{21(B-1)},1,1)$
(X_2,Z_2)	$\tilde{w}_{12(1)} \rightarrow$	$\tilde{w}_{12(2)} \rightarrow$	...	$\tilde{w}_{12(B-1)} \rightarrow$	0
(Y,S)	$\hat{w}_{11(1)},\hat{w}_{22(1)}$	$\leftarrow (\hat{w}_{12(1)},\hat{w}_{21(1)})$ $\hat{w}_{11(2)},\hat{w}_{22(2)}$	...	$\leftarrow (\hat{w}_{12(B-2)},\hat{w}_{21(B-2)})$ $\hat{w}_{11(B-1)},\hat{w}_{22(B-1)}$	$\leftarrow \hat{w}_{12(B-1)},\hat{w}_{21(B-1)}$

- Same block-markov encoding/backward decoding as Willems except that the receiver now observes the states S

State estimation and distortion analysis

- User 1 estimates the state sequence $s_1^N(b)$ as

$$\hat{s}_1^N(b) = \underline{\psi}_1^*(x_1^N(b), v_2^N(b), z_1^N(b))$$

- The distortion for a fixed message pair (w_1, w_2) with $w_k = \{w_{k1}^b, w_{k2}^b\}$.

$$\begin{aligned}
& d_1^{(n)}(w_1, w_2) \\
& \overset{(a)}{\leq} P_e^{(n)}(w_1, w_2) d_{\max} + (1 - P_e^{(n)}(w_1, w_2))(1 + \epsilon) \\
& \qquad \frac{1}{n} \sum_i \mathbb{E}\left[\mathbb{E}[d_1(S_{1i}, \underline{\psi}_1^*(x_1, v_2, Z_{1i}))|X_{1i} = x_1, V_{2i} = v_2]\right] \\
& \overset{(b)}{=} P_e^{(n)}(w_1, w_2) d_{\max} + (1 - P_e^{(n)}(w_1, w_2))(1 + \epsilon) \mathbb{E}\left[\underline{c}_1(X_1, V_2)\right].
\end{aligned}$$

- Averaging over all message pairs, we obtain the desired result.

$$\limsup_{n \to \infty} d_1^{(n)} \leq \mathbb{E}[\underline{c}_1(X_1, V_2)] \leq D_1.$$

State-dependent erasure MAC

- An erasure MAC with binary states

$$Y = S_1 X_1 + S_2 X_2$$

S_1, S_2 are i.i.d. Bernoulli with p_s.

- The sum capacity without feedback is

$$\begin{aligned} R_{\text{sum}-\text{no}-\text{fb}}(\infty) &= \max_{P_Q P_{X_1|Q} P_{X_2|Q}} I(X_1, X_2; Y|SQ) \\ &= \max_a 2p_s \overline{p}_s H_2(a) + p_s^2 H_3(a^2, 2a\bar{a}, \bar{a}^2) \\ &= 2p_s \overline{p}_s + \frac{3p_s^2}{2} \end{aligned}$$

An achievable sum rate with output feedback

- Focus on the symmetric rate $R_1 = R_2$ and let

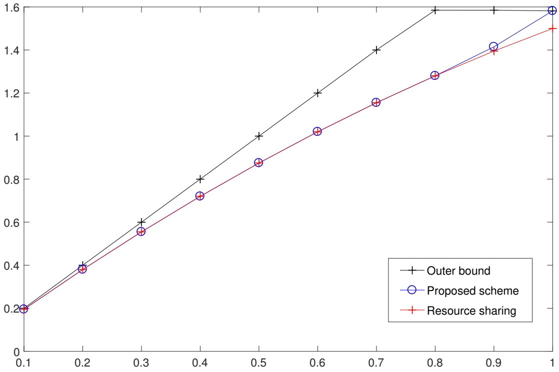
$$X_k = V_k \oplus \Theta_k = U \oplus \Sigma_k \oplus \Theta_k, \quad k = 1, 2$$

where U, Σ_k, Θ_k are Bernoulli distributed with p, q, r.

- An achievable sum rate can be computed.

$$\begin{aligned} R_{\mathrm{sum-fb}}(\infty) = \max_{p,q,r} \min\{ & I(X_1X_2;Y|S), I(X_1X_2;Y|SV_1V_2U) \\ & + I(V_1;Y|X_2U) + I(V_2;Y|X_1U)\} \end{aligned}$$

Unconstrained sum rate



- Feedback is useful only when p_s gets closed to 1.

Minimum distortion

- Distortion is smaller if $X_k = V_k$ (no private message).
- Two simple estimators
  1. the state is perfectly known:

$$\psi_1^*(1,0,0) = 0, \quad \psi_1^*(1,0,1) = 1$$
$$\psi_1^*(1,1,0) = 0, \quad \psi_1^*(1,1,2) = 1$$

  2. the state is erased.

$$\psi_1^*(x_1, x_2, y) = \mathbf{1}\{p_s > 1/2\}$$

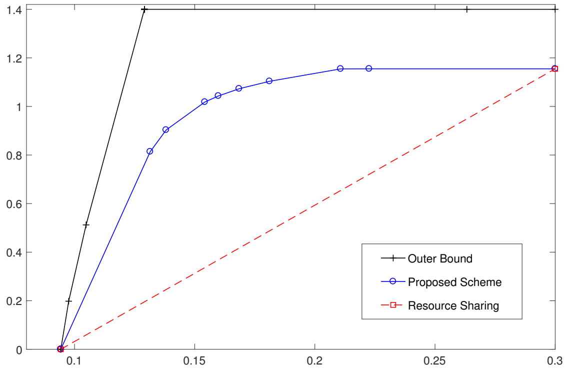
  for $(x_1, x_2, y) \in \{(0,0,0), (0,1,0), (0,1,1), (1,1,1)\}$.

- The minimum distortion is a solution of

$$\min_{p,q} \sum_{(x_1,x_2)} P_{X_1,X_2}(x_1,x_2) c_1(x_1,x_2)$$

achieved by letting $X_1 = X_2 = U$ (zero rate).

Tradeoff between sum rate and distortion



- Our proposed scheme provides a significant gain

Summary of Part II

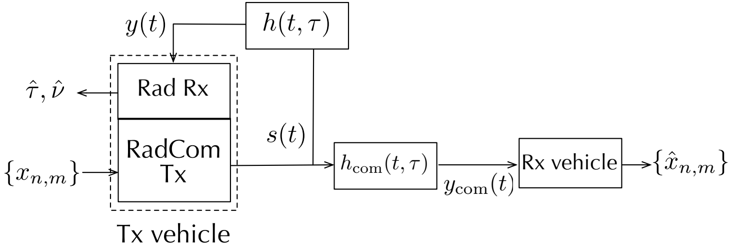
- Feedback enables a joint sensing and communication system to improve capacity-distortion tradeoff compared to a resource-sharing scheme.
- Open problems include:
  - A single user channel with memory
  - MAC with spatially correlated states
  - MAC with asymmetric states
  - Broadcast channel
- More details are available on arXiv [25][26].

[25] M. Kobayashi, G. Caire, and G. Kramer, "Joint State Sensing and Communication: Optimal Tradeoff for a Memoryless Case", in *2018 IEEE Int. Symp. Inf. Theory, Vail, CO*, June, 2018. arXiv:1805.05713

[26] M. Kobayashi, H. Hamad, G. Kramer, G. Caire, "Joint State Sensing and Communication over Memoryless Multiple Access Channels", to be presented at *2019 IEEE Int. Symp. Inf. Theory, Paris, France*, July, 2019. arXiv:1902.03775

Part III: Joint Radar and Vehicular Communications

System Model



- A total bandwidth of B Hz operating at the carrier frequency f_c [Hz].
- A transmit vehicle is equipped with a full-duplex monostatic radar.
- The transmitter sends data $\{x_{n,m}\}$ while estimating range r and velocity v.

$$\nu = \frac{2vf_c}{c}, \quad \tau = \frac{2r}{c}$$

Time-Frequency Selective Channel

- Consider P targets, each represented by a single LOS pathwith (τ_p, ν_p).
- Radar channel

$$h(t, \tau) = \sum_{p=0}^{P-1} h_p \delta(\tau - \tau_p) e^{j2\pi\nu_p t}$$

- Radar received signal without noise

$$y(t) = \int h(t, \tau) s(t - \tau) d\tau = \sum_{p=0}^{P-1} h_p s(t - \tau_p) e^{j2\pi\nu_p t}$$

- Communication channel

$$h_{\mathrm{com}}(t, \tau) = g_0 e^{j\pi\nu_0 t} \delta\left(\tau - \frac{\tau_0}{2}\right)$$

Related Works

- Range and velocity estimation using OFDM [27], [28]
- Range and velocity estimation using OTFS [29]
- Joint radar and communication based on resource sharing [30][31]

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[27] Strum et al., "Waveform design and Signal processing aspects for fusion of wireless communications and radar sensing", Proc. IEEE 2011

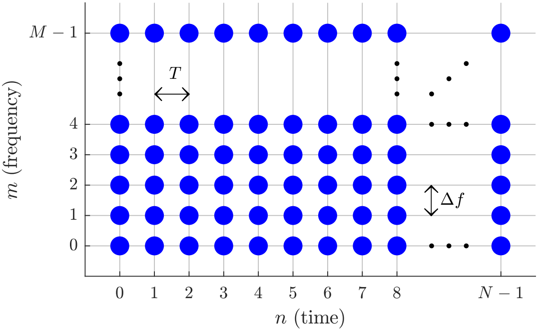
[28] D. H. Nguyen and R. W. Heath, "Delay and Doppler processing for multi-target detection with IEEE 802.11 OFDM signaling", ICASSP 2017

[29] P. Raviteja et al., "Orthogonal Time Frequency Space (OTFS) Modulation Based Radar System", preprint on arxiv.1901.09300

[30] P. Kumari et al., "IEEE 802.11ad-Based Radar: An Approach to Joint Vehicular Communication-Radar System", IEEE Trans. Vehicular Technology, vol. 67, no. 4, 2018

[31] P. Kumari et al., "Performance Trade-Off in an Adaptive IEEE 802.11 Waveform Design for a Joint Automotive Radar and Communication System", ICASSP'2017.

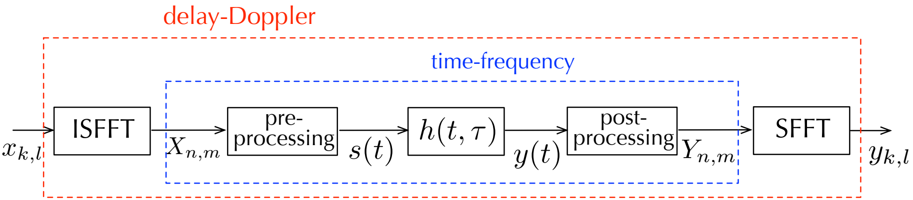
Transmission using M subcarriers and N time slots



- Total bandwidth is divided in M subcarriers, i.e. $B = M\Delta f$.
- $T = \frac{1}{\Delta f}$ is one symbol duration, $T_{\text{frame}} = NT$.
- $\{x_{n,m}\}$ satisfies average power constraint $\mathbb{E}[|x_{n,m}|^2] \leq P$.
- The parameters are chosen such that

$$\nu_{\max} < \Delta f \,, \quad \tau_{\max} < T$$

OFDM and OTFS



- Cyclic prefix OFDM uses Inverse DFT/DFT in time-frequency domain.
- OFTS is a modulation patented by Cohere [32] using the Zak transform[33].
- Mapping from delay-Doppler to time-frequency domains:

$$X[n,m] = \frac{1}{\sqrt{NM}} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} x_{k,l} e^{j2\pi\left(\frac{nk}{N} - \frac{ml}{M}\right)}$$

[32] R. Hadani et al., "Orthogonal time frequency space modulation," in Wireless Commun. and Networking Conf. (WCNC), 2017

[33] H. Bolcskei and F. Hlawatsch, "Discrete zak transforms, polyphase transforms, and applications," IEEE Trans. Signal Proc., vol. 45, no. 4, 1997.

Cyclic Prefix OFDM

- Consider a cyclic prefix (CP) of length $T_{\rm cp} > \tau_{\max}$ to avoid inter-symbol-interference (ISI).
- An OFDM symbol is of duration $T_o = T_{\rm cp} + T$.
- Pre-processing: CP and Inverse DFT

$$s(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} x_{n,m} \mathrm{rect}(t - nT_o) e^{j2\pi m \Delta f (t - T_{\rm cp} - nT_o)}$$

- Post-processing: sampling every $\frac{T}{M}$, CP-removal, and DFT

$$y_{n,m} \approx \sum_{p=0}^{P-1} h_p e^{j2\pi n T_o \nu_p} e^{-j2\pi m \Delta f \tau_p} x_{n,m}$$

where we used the approximation $\nu_{\max} \ll \Delta f$.

Doppler signal estimation and CRLB[34]

- Problem: estimate $\boldsymbol{\theta} = (A, f, \phi)$ from a finite noisy samples:

$$y_n = \underbrace{Ae^{j(2\pi f n + \phi)}}_{s_n} + w_n, \quad n = 1, \dots, N$$

where $w_n \sim \mathcal{N}_{\mathbb{C}}(0, N_0)$.

- Cramér Rao Lower Bound (CRLB) presents the lower bound of the error variance for any unbiased estimator.

$$\sigma^2_{\theta_i} \geq [\mathbf{I}(\boldsymbol{\theta})^{-1}]_{i,i}, \quad i = 1, 2, 3$$

where 3×3 Fisher information matrix $\mathbf{I}(\boldsymbol{\theta})$

$$[\mathbf{I}(\boldsymbol{\theta})]_{i,j} = \frac{2}{N_0} \mathrm{Re} \left\{ \sum_n \left[\frac{\partial s_n}{\partial \theta_i} \right]^* \left[\frac{\partial s_n}{\partial \theta_j} \right] \right\}, \quad i, j \in [1, 3]$$

[34] M. A. Richards, Fundamentals of radar signal processing. Tata McGraw-Hill Education, 2005.

Doppler signal estimation and CRLB

- Fischer information matrix

$$\mathbf{I}(\boldsymbol{\theta}) = \frac{2}{N_0}\begin{bmatrix} N & 0 & 0 \\ 0 & 4\pi^2 A^2 \frac{N(N-1)(2N-1)}{6} & \pi A^2 N(N-1) \\ 0 & \pi A^2 N(N-1) & NA^2 \end{bmatrix}$$

- The error variance of A, f, θ denoted is lower bounded respectively by

$$\sigma_A^2 \geq \frac{N_0}{2N}$$
$$\sigma_f^2 \geq \frac{6}{4\pi^2 \mathsf{snr} N(N^2-1)}$$
$$\sigma_\phi^2 \geq \frac{2N-1}{\mathsf{snr} N(N+1)}$$

where we let $\mathsf{snr} = \frac{A^2}{N_0}$ denote the SNR.

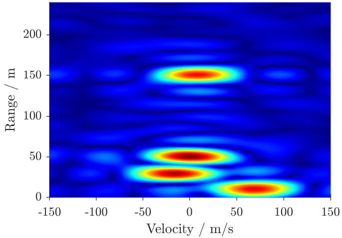
Maximum Likelihood estimator with OFDM ($P = 1$)

1. Compute the DFT/IDFT output $Z(\nu, \tau)$.

$$Z(\nu,\tau) \triangleq \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} z_{n,m} A_{n,m} e^{-j2\pi\nu n T_o} e^{j2\pi m \Delta f \tau}$$

   where $z_{n,m}$ is noisy output, $A_{n,m} = |x_{n,m}|$ is known.

2. Choose $(\hat{\nu}, \hat{\tau})$ maximizing $|Z(\nu,\tau)|^2$ over Γ.
3. Let the channel gain be $\hat{h} = Z(\hat{\nu},\hat{\tau}) / \left(\sum_{n,m} A_{n,m}^2\right)$.



CRLB for OFDM

Lemma

In the regime of large M and N, the CRLB of $f = T_o \nu$ and $\mathrm{t} = \Delta f \tau$ are given by

$$\sigma_{\hat{\mathrm{f}}}^2 \geq \frac{6}{|h|^2 P (2\pi)^2 M N (N^2 - 1)},$$
$$\sigma_{\hat{\mathrm{t}}}^2 \geq \frac{6}{|h|^2 P (2\pi)^2 M N (M^2 - 1)}.$$

- The result covers a special case of constant-amplitude modulation[35].

[35] K. M. Braun,"OFDM Radar Algorithms in Mobile Communication Networks", Ph.D. dissertation, Karlsruhe Institute of Technology, 2014

OTFS

- Pre-processing:

$$s(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} X[n, m]\, g_{\mathrm{tx}}(t - nT)\, e^{j2\pi m\Delta f(t-nT)}$$

- Post-processing: matched filter and sampling at $t = nT, f = m\Delta f$.

$$Y(t, f) = C_{r,g_{\mathrm{rx}}}(t, f) = \int y(t')\, g_{\mathrm{rx}}^{*}(t' - t)\, e^{-j2\pi f t'} dt'$$

- After SFFT, the output of dimension NM in delay-Doppler domain :

$$\mathbf{y} = \sum_{p=0}^{P-1} h_p' \boldsymbol{\Psi}^p(\tau_p, \nu_p)\mathbf{x} + \boldsymbol{w}$$

This holds for any transmit/receive pulse pair.

ML estimator and CRLB for OTFS ($P = 1$)

- Focus on a practical case of a rectangular pulse[36].

$$C_{g_{\mathrm{rx}}, g_{\mathrm{tx}}}(\tau, \nu) = \frac{e^{-j\pi\nu(T+\tau)}}{\pi\nu T} \sin\left(\pi\nu\left(T - |\tau|\right)\right)$$

limiting ISI between n and $n-1$.

- Following the similar steps as OFDM, the ML estimator is given by

$$(\hat{\tau}, \hat{\nu}) = \text{arg} \max_{(\tau,\nu)\in\Gamma} \frac{\left|\boldsymbol{x}^H \boldsymbol{\Psi}(\tau, \nu)^H \boldsymbol{y}\right|^2}{\boldsymbol{x}^H \boldsymbol{\Psi}(\tau, \nu)^H \boldsymbol{\Psi}(\tau, \nu)\, \boldsymbol{x}}$$

- CRLB can be derived by computing the Fischer information matrix.

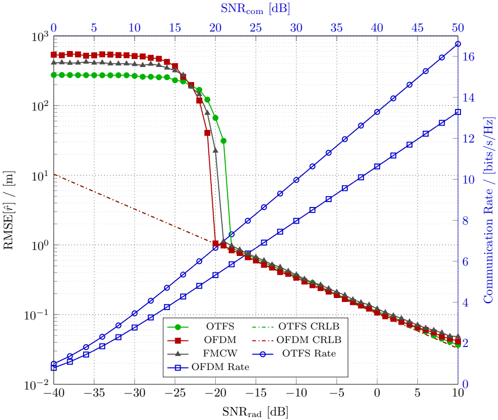
[36] P. Raviteja et al., "Interference cancellation and iterative detection for orthogonal time frequency space modulation", IEEE Transactions on Wireless Communications, vol. 17, no. 10, 2018.

Numerical example: setup

IEEE 802.11p[37]	
$f_c = 5.89$ GHz	$M = 64$
$B = 10$ MHz	$N = 50$
$\Delta f = B/M = 156.25$ kHz	$T_{\rm cp} = \frac{1}{4}T = 1.6\,\mu$s
$T = 1/\Delta f = 6.4\,\mu s$	$T_{\rm o} = T_{\rm cp} + T = 8\,\mu$s
$r_{\max}^{\rm otfs} < Tc/2 \simeq 960$ m	$r_{\max}^{\rm ofdm} < T_{\rm cp}c/2 \simeq 240$ m
$\sigma_{\rm rcs} = 1$ m^2	$G = 100$
$r = 20$ m	$v = 80$ km/h

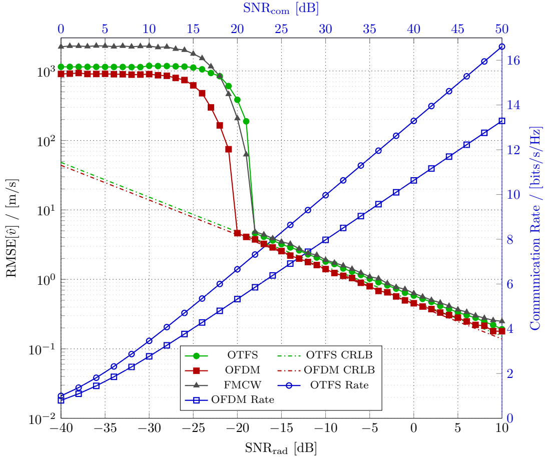
[37] D. H. Nguyen and R. W. Heath, "Delay and Doppler processing for multi-target detection with IEEE 802.11 OFDM signaling", ICASSP 2017

Numerical example: range estimation



- Under a simplified scenario, OFDM/OTFS can yield a significant data rate without compromising radar estimation.

Numerical example: velocity estimation



Summary of Part III

- OFDM and OTFS achieve as accurate radar estimation as FMCW by sending data "for free".
- The range estimation is limited by the CP length, i.e. $r_{\max} < cT_{\rm cp}/2$ yielding poor performance at mmWave bands.
- OTFS performs better at a significant higher complexity.
- Future works include
  - Joint beam alignment and target tracking
  - Waveform design robust to ICI and ISI.
- More details are found on arXiv [38]

[38] L. Gaudio, M. Kobayashi, B. Bissinger, G. Caire, "Performance Analysis of Joint Radar and Communication using OFDM and OTFS", to be presented at ICC Workshop 2019, arXiv:1902.01184