ESIT 2018 Tutorials

Introduction to Random Matrix Theory and its Applications

Marco Chiani
University of Bologna

Abstract

Many mathematical models in information theory and physics are formulated by using matrices with random elements. In particular, the distribution of the eigenvalues of Gaussian, Wishart, and double Wishart random matrices plays a key role in multivariate analysis, including principal component analysis, analysis of large data sets, information theory, signal processing and mathematical physics. For example, in compressed sensing the probability that a randomly generated measurement matrix has a given restricted isometry constant is related to the distribution of the minimum and maximum eigenvalues of the matrix $\boldsymbol{M}=\boldsymbol{X}^{\dagger}\boldsymbol{X}$, and $\boldsymbol{X}$ is a submatrix of the measurement matrix. To mention another example, several stability problems in physics, in complex networks and in complex ecosystems are related to the probability that all eigenvalues of a random symmetric matrix (for instance with Gaussian entries) are negative. This probability is also important in mathematics, as it is related to the expected number of minima in random polynomials.

This tutorial will introduce the basic concepts in Random Matrix Theory, discussing in particular the distributions related to Wishart, double Wishart, and Gaussian symmetric/Hermitian random matrices, both real and complex, the eigenvalues limiting spectral support given by the Marcenko-Pastur and the semicircle laws, the Tracy-Widom laws. Applications will include Shannon capacity, compressive sensing, stability problems in complex ecosystems.