NETWORK CODING OVERHEAD

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Introduction & Motivation


- Plain routing is **not** optimal & **Linear** network coding is **sufficient** (Li, et al. ’03, Koetter-Medard ’03).

- **Typical assumption**: Destinations know how packets are network-coded.

Protocol Information

- Limiting effects of network coding **overhead**.

- Joint channel **coding/training** for robust network coding.
Network Coding Overhead

Source: $X$  

Any Destination:

$$Y = G X$$

network coding matrix
(elements from finite field $F_q$)

- Distributed solution: **Random network coding** (Ho, et. al ’06).
- Communicate “$G$” to destinations in packet header: $G$ | Data
- **Challenge:** Decoding **impossible**, if error/erasure in “$G$”.
System Model for Overhead Analysis

- **Question:** What is **overhead** for reliable communication of “G” & data?

- **Unknown** topology & “G” (i.i.d assumption).

```
N Packets
\[ \text{Length } D \]

Source \[ \rightarrow \text{Enc} \rightarrow X \]

\[ N \times D \]

\[ N \times L \]

Network

| \[ G_1 \] | \[ \mathcal{H}_1 \] | \[ Y_1 \] | Dec
| --- | --- | --- | ---

| \[ \cdots \] | \[ \cdots \] | \[ \cdots \] | \[ \cdots \]

Destinations

\[ \hat{S}_1 \]

\[ \hat{S}_R \]

random network coding

end-to-end channel (error/erasure)
Network Coding Meets MIMO: Training Approach

- Detection of network coding matrix $G \iff$ MIMO channel estimation.

- Source transmits **known training** sequence $X_T$.

- Destination $d$ receives $Y_d = \mathcal{H}_d(G_dX_T)$.

  Rows of $G_d$ are encoded by $X_T$

- **Interpretation:** $X_T$ is a “code” generator matrix (e.g., MDS code)

  Rows of $G_d$ are the transmitted “packets”
Encoding with Training

- **Separate** training and coding (STC).

\[
X = [X_T \ SF_D] = [I_N \ S] \begin{bmatrix} X_T & 0 \\ 0 & F_D \end{bmatrix}
\]

- **Joint** training and coding (JTC).

\[
X = [I_N \ S]F
\]
Decoding Channel & Network Codes

(1) Decode channel code: \[
\begin{cases}
\hat{G}_d & \text{for random network coding matrix } G_d \\
\hat{U}_d & \text{for network-coded data: } U_d = G_d S
\end{cases}
\]

JTC: Jointly decode entire packet.
STC: Separately decode training and data parts.

(2) Decode network code:

- Solve \[
\hat{U}_d = \hat{G}_d \hat{S}_d
\]
  for data \(\hat{S}_d\) from ML estimates \(\hat{G}_d\) and \(\hat{U}_d\).
Throughput & Overhead

- Two types of decoding error:
  - Errors/erasures in packets
  - Rank-deficient network coding matrix \( \hat{G}_d \)

- Decoding probability \( P_d \) is derived via random coding bounds.

- End-to-end achievable rate:
  \[
  \Lambda^* = \max_{N,D} \left[ N \frac{D}{L} \min_d P_d \right]
  \]

- Overhead:
  \[
  O = \frac{L - D}{L}
  \]

- \( N \): independent messages per transmission
- \( D \): data symbols per packet
- \( L \): total packet length
Limiting Effects of Overhead

Example 1: **Combination network**

1 source

\( A \) layer-2 nodes

\( \binom{A}{M} \) destinations

**Network Throughput** \( \Lambda \)

\( M \): min-cut capacity

Field \( q = 2^8 \), 480 bits per packet,

bit erasure probability \( 10^{-2} \)

**overhead kicks in**
Example 2: **Grid network**

Each link carries $M/2$ packets per time slot

Field $q = 2^8$, 60 bits per packet

Bit error probability $10^{-3}$

**JTC**: Joint training & coding

**ITC**: Individual training & coding

**REF**: Destination knows “G”

**analysis**: Random coding bound on probability of successful decoding

**simulation**: Average error rates over random coding matrices & error events with MDS channel codes
End-to-End Distributed Network Coding

• Oblivious to network topology (mobility effects, malicious behavior, ...).

• **No** need to know network coding matrix “\(G\).”

• Need to know min-cut capacity \(M\) (s.t. \(N \leq M\)) \(N\) : independent messages per transmission

• **Rate control:** Adapt \(N\) based on destination feedback (ACK/NACK).
  
  – Increase \(N\), if channel & network codes are successfully decoded.
  
  – Decrease \(N\), otherwise.
Conclusions

• **Training** can be combined with **channel coding**.
  – Enables **joint** decoding of data and network coding matrix.
  – Simplifies and balances protection of **overhead**.

• Network coding gain is **limited** by the necessary protocol overhead.
  – Overhead **grows** with the min-cut capacity.

• **Future Work:**
  – Extension to **general** error and erasure models.
  – Exploit **redundancy** in sequence of coding matrices.
  – **Adaptive** training and feedback schemes to learn error statistics.