Impact of Network Topology Knowledge on Fairness: A Geometric Approach

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Motivation

- Throughput-fairness is a commonly desired objective in network resource allocation.
- Fair allocations often depend on the topology (state) of the network.
- Wireless topology is difficult to know precisely/accurately.

Question: How much fairness can be lost when there are errors in network topology knowledge?

Outline of Approach

1. Model topology-dependent resource allocation problem as a knowledge-dependent mapping from 'topology' space to throughput space.
2. Determine geometric properties of fairness measure.
3. Specify a geometric error model.
4. Analyze.

1. Network Model: Scheduling in Multiuser Uplink

- Scheduled multiple-access/single-clique
- Fixed code rate vector \( \mathbf{r} \)
- Infinite backlog

Topology fully described by service rate vector: \( \mathbf{q} \)
Topology knowledge (service rate vector estimate): \( \hat{\mathbf{q}} \)

Fair Allocation: Find \( \mathbf{t}^* \) such that \( \mathbf{1}^T \mathbf{t}^* = 1 \) and \( \mathbf{t}^*_i / r_i = \lambda \) for all \( i \) and some \( \lambda > 0 \), i.e.

\[
\mathbf{t}^*_i(q_i, \lambda) = \left( \frac{\mathbf{1}^T}{\lambda} \sum_{j=1}^{N} \frac{1}{q_j} \right)^{-1}.
\]

Estimated Throughput Vector: \( \hat{\mathbf{t}} = \mathbf{1} \Lambda \hat{\mathbf{q}} \)
Realized Throughput Vector: \( \mathbf{x} = \left[ \frac{\mathbf{1}^T \hat{\mathbf{q}}}{\mathbf{1}^T \mathbf{q}} \right] \left[ \mathbf{1}^T \hat{\mathbf{q}} / N \right] \)

2. Geometry of Fairness Measure

Jain’s Index:

\[
f = \left( \sum_{i=1}^{N} \frac{x_i}{\hat{x}_i} \right)^2 \left( \sum_{i=1}^{N} \frac{\hat{x}_i}{x_i} \right)^2 = \cos(b)^2.
\]

3. Error Model

Error vector: \( \Delta \mathbf{q} = \mathbf{q} - \hat{\mathbf{q}} \)
Bound on error magnitude: \( \| \Delta \mathbf{q} \| \leq \delta \)
Fairness loss:

\[
e(\delta, \hat{\mathbf{q}}) = 1 - \min_{\| \Delta \mathbf{q} \| \leq \delta} J(\hat{\mathbf{q}}, \mathbf{q}).
\]

Diagram of error model and search space in two domains

\[\begin{align*}
\hat{q} &= (\hat{q}_1, \hat{q}_2) \\
q &= (q_1, q_2) \\
\delta &= 1 - \min_{\| \Delta \mathbf{q} \| \leq \delta} J(\hat{\mathbf{q}}, \mathbf{q}).
\end{align*}\]

4. Main Results

Theorem 1 [Two-Users] Let the topology error be bounded by \( \delta \), and let a topology estimate \( \hat{\mathbf{q}} = (\hat{q}_1, \hat{q}_2) \), with \( \hat{q}_1 > \hat{q}_2 \) be given. The worst case error, \( \Delta \mathbf{q}(\hat{\mathbf{q}}) \), in a two-user network is

\[
\Delta \mathbf{q}(\hat{\mathbf{q}}) = \left( \frac{\delta}{\mathbf{1}^T \hat{\mathbf{q}}} \right) \hat{\mathbf{q}} - \left( \frac{\delta}{\mathbf{1}^T \mathbf{q}} \right) \mathbf{q}.
\]

and the fairness loss is

\[
e(\delta, \hat{\mathbf{q}}) = \left( \frac{\delta}{\mathbf{1}^T \hat{\mathbf{q}}} \right)^2 \left( \frac{\mathbf{1}^T \mathbf{q}}{\mathbf{1}^T \hat{\mathbf{q}}} \right)^2
\]

where

\[
\gamma = \left( \frac{\mathbf{1}^T \mathbf{q}}{\mathbf{1}^T \hat{\mathbf{q}}} \right) - 1.
\]

Theorem 2 [N-Users] The loss in fairness in an \( N \)-user scheduled network is given by the optimization problem

\[
e(\delta, \hat{\mathbf{q}}) = \max_{\mathbf{w}} \sum_{i,j} \left( \frac{w_i - w_j}{\hat{q}_i} \right)^2 \left( \frac{\hat{q}_j}{\hat{q}_i} \right)^2
\]

where \( \mathbf{w} = \left[ w_1, \ldots, w_N \right] \) is such that \( \sum_{i=1}^{N} w_i = |\mathbf{q}| \) and \( (\mathbf{w}, \mathbf{q}) = 0 \).

Table 1: Topology estimates for two-user simulation

<table>
<thead>
<tr>
<th>δ</th>
<th>( q_1 )</th>
<th>( q_2 )</th>
<th>( | \mathbf{q} | )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASYM-LO</td>
<td>0.2000</td>
<td>0.6284</td>
<td>0.8485</td>
</tr>
<tr>
<td>SYMM-LO</td>
<td>0.3000</td>
<td>0.4243</td>
<td>0.5000</td>
</tr>
<tr>
<td>SYMM-HI</td>
<td>0.6000</td>
<td>0.3000</td>
<td>0.8485</td>
</tr>
</tbody>
</table>

Figure 1: Fairness loss versus normalized error in a two-user network with topology estimates as listed in Table 1.

Remarks

Remark 1: Fixed coding rates have no impact on fairness loss — they are not an unknown and can be accounted for perfectly.

Remark 2: Worst-case error places more emphasis on the users with weaker channels, therefore it is more important to have precise measurements for weaker users.

Remark 3: If the channel is more asymmetric, the loss in fairness is exaggerated.

Remark 4: Fairness loss only dependent on normalized error; thus, in order to achieve the same level of fairness, weaker channels require more precise measurement.

Extensions

- Other allocation problems
- Other fairness measures [2]
- Other error models (e.g., stochastic)

Related Work
