Optimal Transmission for Dying Channels

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Application Scenarios

In a cognitive radio network or a sensor network, consider a point-to-point communication link subject to a fatal random attack.

Figure: Application scenarios of dying channel

How fast and reliable is the communication over a dying channel?
Outage Capacity Definition

- Model as a $K$-block fading channel with a random delay constraint.
- Adopt the outage capacity as the metric.

Definition

$$C_{\text{out}}(P, \eta) = \max_{K} \sup_{P_K: \sum_{i=1}^{K} P_k \leq KP} \left\{ R : \Pr\left\{ \frac{1}{K} \sum_{i=1}^{L} \log(1 + \alpha_i P_i) < R \right\} < \eta \right\},$$

where $L = \min([T], K)$. 
Uniform Power Allocation

- The only variable to optimize is the number of blocks $K$ when assuming uniform power allocation.

- We derive the lower and upper bounds of outage probability.

- When high SNR and Rayleigh fading are assumed, we obtain the closed-form solution for the optimal $K$. 
Lower and Upper Bounds

**Lower bound:**

\[
\Pr \left\{ \sum_{i=1}^{L} \log(1 + \alpha_i P) < KR \right\} \geq w_0 + \sum_{i=1}^{K-1} \left\{ F \left( \frac{e^{KR/i} - 1}{P} \right) \right\}^i w_i \\
+ \left\{ F \left( \frac{e^R - 1}{P} \right) \right\}^K w_K^*.
\]

**Upper bound:**

\[
\Pr \left\{ \sum_{i=1}^{L} \log(1 + \alpha_i P) < KR \right\} \leq w_0 + \sum_{i=1}^{K-1} \left\{ F \left( \frac{e^{KR} - 1}{P} \right) \right\}^i w_i \\
+ \left\{ F \left( \frac{e^{KR} - 1}{P} \right) \right\}^K w_K^*.
\]
For Rayleigh fading in high SNR regime, the outage probability is:

\[ p_{out}(K) \approx \xi e^{KR} + \frac{1}{PK e^{(\lambda-R)K}} + w_0, \]

where \( \xi = (1 - e^{-\lambda}) \frac{\beta/P}{1-\beta/P}, \quad \beta = e^{-\lambda}. \)

The optimal \( K \) is:

\[ K^* = \log \left[ \frac{\lambda + \log P - R}{\xi R} \right] \frac{1}{\lambda + \log P}. \]

Choose \( \lfloor K^* \rfloor \) or \( \lceil K^* \rceil \) that gives the smaller outage probability.
Simulation Results

- Rayleigh fading and exponentially distributed random attack time.

**Figure:** Outage probability vs. coding length, $R=1$ bps/Hz.
Optimal Power Vector

- General results:
  - The optimal power allocation has a non-increasing profile, i.e., $P_1 \geq P_2 \geq \cdots \geq P_K$.
  - When the fading gains are the same, the optimal coding length is $K = 1$ and $P_1 = P$.

- Conditioned on $K$, the optimization problem becomes

$$\min_{P_K} \Pr \left\{ \sum_{i=1}^{L} \log(1 + \alpha_i P_i) < K R \right\}$$

s.t. $\sum_{i=1}^{K} P_i \leq K P$.

- For some special cases, it can be cast into convex optimization problems.
High SNR Rayleigh Fading Case

- Rayleigh fading in high SNR regime:

$$\min_{P_K \in D_+} w_0 + \frac{c_1}{P_1} + \frac{c_2}{P_1 P_2} + \cdots + \frac{c_K}{\prod_{i=1}^K P_i}$$

s.t. $$\sum_{i=1}^K P_i \leq KP,$$

where $$D_+ = \{ \mathbf{P} \in \mathbb{R}_+^K : P_1 \geq P_2 \geq \cdots \geq P_K \geq 0 \}$$ is a convex cone.
Log-normal Fading Case

- Log-normal fading distribution

$$\min_{P_k} w_0 + \sum_{n=1}^{K} \frac{1}{2} \left\{ 1 + \text{erf} \left( \frac{KR - \sum_{i=1}^{n} \log P_i}{\sqrt{2n}} \right) \right\} w_n$$

s.t.

$$\sum_{i=1}^{K} P_i \leq KP$$

$$KR - \log P_1 \leq 0$$

$$KR - \log P_1 - \log P_2 \leq 0$$

$$\ldots$$

$$KR - \sum_{i=1}^{K} \log P_i \leq 0.$$
Simulation Results

- Exponentially distributed random attack time.
- Rayleigh fading and Log-normal fading.

![Simulation Results](image)

- Outage probability with non-uniform and uniform power allocation.
- Outage capacity v.s. number of blocks K.

![Graph 1](image)

- Numerical results provided as follows. Assume that the outage probability target is set as η.
- The simulation parameters are:
  - R = 1/λ = 5, average power P = 3 dB.
  - 1/λ = 4, average power P = 10 dB.

![Graph 2](image)

- The optimal power allocation leads to a significantly larger outage capacity over the uniform power allocation case.
- As K increases, the outage capacity with the optimal power allocation may even increase to a maximum value while the outage capacity with uniform power allocation monotonically decreases. This suggests that, with the potential of a random attack, we can still span the codeword over more than one block to exploit diversity.

*Fig. 4. Outage capacity v.s. number of blocks K, 1/λ = 4, average power P = 3.*

*Fig. 3. Outage probability with non-uniform and uniform power allocation.*

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Denoting \( c_i = w_i \left( e^{KR/i} - 1 \right) i \), we further simplify (20) as
\[
p_{out}(K) \approx w_0 + c_1 P_1 + c_2 P_1 P_2 + \cdots + c_K \prod_{i=1}^{K} P_i
\]
has a log-normal distribution, we can also approximate the problem as a convex one by minimizing the upper bound of the...
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Outage Probability Definition

**Definition**

The outage probability in the multiple parallel channels case is given as:

\[
p_{out}(R, P, N) = \Pr\left\{ \sum_{i=1}^{N} \frac{1}{K} \sum_{k=1}^{L_i} \log\left(1 + g_k^{(i)} P / N\right) < R \right\}
\]

\approx \Pr \left\{ \frac{1}{N} \sum_{i=1}^{N} Y_i < R / P \right\}, \tag{1}

where \( Y_i = \frac{1}{K} \sum_{k=1}^{L_i} g_k^{(i)} \).
Outage Probabilities

- The independent random attack case

\[ p_{\text{out}}(R, P, N) \approx \Phi\left( \frac{R/P - \mu_Y}{\sigma_Y / \sqrt{N}} \right). \quad (2) \]

- The \( m \)-dependent random attack case

\[ p_{\text{out}}(R, P, N) \approx \Phi\left( \frac{R/P - \mu_Y}{\sqrt{\nu_m / N}} \right). \quad (3) \]

where \( \nu_m = \frac{\mu_L \sigma_g^2}{K^2} + \frac{\mu_g^2 \sigma_L^2}{K^2} (1 + 2m \rho) \).
Outage Exponents

- The independent attack case

\[ \Lambda(s) := \log E \left[ \exp(sY_i) \right] = \log M_Y(s). \] (4)

- The \( m \)-dependent attack case

an approximate outage exponent can be quantified as:

\[ \mathcal{E}_{mdp}(R/P) \approx \frac{(\mu_Y - R/P)^2}{2 \upsilon_m}. \] (5)
Results

Number of sub-channels $N$ vs. Outage probability

- Analytical outage probability $R=0.5$ nats/s
- Simulated outage probability $R=0.5$ nats/s
- Analytical outage probability $R=2$ nats/s
- Simulated outage probability $R=2$ nats/s

Outage exponent vs. $\frac{r}{P}$

- $1/\lambda = 3$, independent attack
- $1/\lambda = 5$, independent attack
- $1/\lambda = 10$, independent attack
- $1/\lambda = 3$, $m$-dependent attack
- $1/\lambda = 5$, $m$-dependent attack
- $1/\lambda = 10$, $m$-dependent attack

Fig. 3. Outage probability convergence behavior of the independent case.

Fig. 4. Outage probability convergence behavior of the $m$-dependent case.

Fig. 5. Outage exponents for independent and $m$-dependent attack cases.

In Fig. 5, we compare the various outage exponent values for independent and $m$-dependent attack cases over the rate per unit cost $\frac{r}{P}$.

The average attack time $\frac{1}{\lambda}$ causes a larger average attack time in the $m$-dependent case compared to the independent case.

First, we see that the outage exponent for independent attack is $\rho = 0.571$, $m=1$, $Y=0.571$ and $P=2$. In this paper, we extended the previous single dying channel analysis to parallel dying channels, where each sub-channel is subject to a random attack during transmission.

In Fig. 5, we compare the various outage exponent values for independent and $m$-dependent attack cases over the rate per unit cost $\frac{r}{P}$.
Summary

- Considered a new type of channel existing in cognitive radio and sensor networks.
- Defined the outage capacity for dying channels.
- Investigated the power allocation and the optimal coding length.
  - There exists an optimal coding length.
  - The optimal power vector is non-increasing.
  - The power allocation problem is convex for some special cases.
- Studied the asymptotic outage probability and outage exponents for parallel dying channels.