Hybrid Digital-Analog Joint Source-Channel Codes for Broadcasting Correlated Gaussian Sources

Hamid Behroozi, Fady Alajaji and Tamás Linder

Department of Mathematics and Statistics
Queen’s University
Kingston, Ontario, Canada
behroozi@queensu.ca

Aug 13, 2009
Outline

1 Introduction
   - System Model
   - Problem Statement

2 Distortion Regions with Matched Bandwidth
   - Analog (Uncoded) Transmission
   - Joint Source-Channel Coding Schemes
   - Layering with Analog and Costa Coding
   - Layering with Analog, Superposition and Costa Coding

3 Conclusion
Broadcasting $k$ samples of a bivariate Gaussian source in $n = \lambda k$ uses of a power-limited broadcast channel to two users.

Bandwidth compression/expansion ratio: $\lambda = \frac{n}{k}$. We consider

- Matched bandwidth ($\lambda = 1$)
Problem Formulation

- Sources: $S_1(t)$ and $S_2(t)$ have zero mean and variance $\sigma_{S_1}^2$ and $\sigma_{S_2}^2$, respectively, and correlation coefficient $\rho \in (-1, 1)$.

- Joint Encoder: $X^n = \varphi (S_1^k, S_2^k)$, $\varphi : \mathbb{R}^k \times \mathbb{R}^k \to \mathbb{R}^n$

- Averaged power-limited transmitted sequence:
  \[
  \frac{1}{n} \sum_{t=1}^{n} E \left[ |X(t)|^2 \right] \leq P.
  \]

- Gaussian Broadcast Channel:
  - Receiver $i$ observes $Y_i(t) = X(t) + V_i(t)$, $i = 1, 2$
  - $V_i(t) \sim \mathcal{N}(0, \sigma_i^2)$ are independently distributed over $i$ and $t$, and are independent of the $X(t)$.
  - User 1: Weak User, User 2: Strong User
Based on its channel output \( Y_i^n \), user \( i \) provides an estimate \( \hat{S}_i^k = \psi_i (Y_i^n) \), where \( \psi_i : \mathbb{R}^n \rightarrow \mathbb{R}^k \) is a decoding function.

Fidelity Measure: Average squared error distortion

\[
\Delta_i = \frac{1}{k} E \left[ \sum_{t=1}^{k} |S_i(t) - \hat{S}_i(t)|^2 \right]
\]

For a particular coding scheme \((\phi, \psi_1, \psi_2)\), the performance is determined by the channel power constraint \( P \) and the incurred distortions \( \Delta_1 \) and \( \Delta_2 \) at the receivers.

For any given power constraint \( P > 0 \), the distortion region \( \mathcal{D} \) is defined as the convex closure of the set of simultaneously achievable distortion pairs at two users.
Our Goal

Goal

We aim to determine **achievable distortion regions** using **hybrid digital-analog (HDA) coding schemes** for two cases;
1) the source bandwidth equals the channel bandwidth,
2) broadcasting with bandwidth compression.

Note that the source-channel separation theorem does not hold in broadcasting correlated sources.
Distortion Regions with Matched Bandwidth

- Analog (Uncoded) Transmission
- Layering with Analog and Costa Coding
- Layering with Analog, Superposition and Costa Coding
1. Analog (Uncoded) Transmission

- Scaling the encoder input subject to the channel power constraint and transmitting it without explicit channel coding.

Why Uncoded?

- For a point-to-point transmission of a Gaussian source through an AWGN channel, uncoded is optimal (Goblick, 1965)
- For broadcasting a bivariate Gaussian source, below a certain SNR threshold, uncoded transmission is optimal (Bross, Lapidoth and Tinguely, 2008).
- In broadcasting a single Gaussian source to two users, uncoded achieves simultaneously the optimal distortion for both users (Chen and Wornell, 1998).
- For sending a bivariate Gaussian source over a Gaussian MAC, below an SNR threshold, uncoded is optimal.
Analog Transmission: Analysis

**Encoder:** \( X_a(t) = \tilde{\alpha} \sum_{i=1}^{2} a_i S_i(t), \)

\[
\tilde{\alpha} = \sqrt{\frac{P}{a_1^2 \sigma_{S_1}^2 + a_2^2 \sigma_{S_2}^2 + 2a_1 a_2 \rho \sigma_{S_1} \sigma_{S_2}}}, \quad a_i \geq 0.
\]

**Decoder:** MMSE estimator

**Distortion:** The set of simultaneously achievable distortion pairs at two users:

\[
D_i = \sigma_{S_i}^2 - \frac{\tilde{\alpha}^2 (a_i \sigma_{S_i}^2 + a_j \rho \sigma_{S_i} \sigma_{S_j})^2}{P + \sigma_i^2}, \quad i, j = 1, 2, \quad j \neq i
\]
Joint Source-Channel Coding Schemes

In order to exploit the advantages of both analog transmission and digital techniques, various HDA schemes have been introduced in the literature.

- Broadcasting a single memoryless Gaussian source under bandwidth mismatch, Mittal et al (02), Reznic et al (06)
- Broadcasting a Gaussian source with memory, Prabhakaran et al (05)
- Uncoded transmission for broadcasting correlated Gaussian sources, Bross et al (08)
- Inner and outer bounds for the distortion region in broadcasting a Gaussian mixture source, Reznic et al (06)
2. Layering with Analog and Costa Coding: Encoder

- Wyner-Ziv Encoder 1
- Wyner-Ziv Encoder 2
- Costa Encoder 1
- Costa Encoder 2

Bi-variate Source

$S^n_1, S^n_2$

$X^n_1, X^n_2, X^n_a$

$m_1, m_2$
Layering with Analog and Costa Coding: Decoder

\[ X^n \]

- **MMSE Estimator 1**
- **Costa Decoder 1**
- **Wyner–Ziv Decoder 1**

\[ Y^n_1 \]

\[ Y^n_2 \]

\[ V^n_1 \]

\[ V^n_2 \]

\[ U^n_1 \]

\[ U^n_2 \]

\[ S^n_1 \]

\[ S^n_2 \]
Layering with Analog and Costa Coding: Encoding

- **1st layer**: Uncoded transmission, $X_a$, of Power $P_a$ (meant for both strong and weak users)
- Now fix $P_1$ and $P_2$ to satisfy $P = P_a + P_1 + P_2$.
- **2nd layer**: $S_1$ is first Wyner-Ziv coded assuming an estimate of $S_1$ at the receiver as side information. The Wyner-Ziv index is then Costa encoded treating $X_a$ as an interference.
- $X_1^n = U_1^n - \alpha_1 X_a^n$ of power $P_1$ is then transmitted (meant to be decoded by the weak user).
- Costa scaling (inflation) factor $\alpha_1$ is set to be $\frac{P_1}{P_1 + P_2 + \sigma_1^2}$.
- **3rd layer**: $S_2$ is also Wyner-Ziv coded. The Wyner-Ziv index is then Costa encoded. $X_2^n = U_2^n - \alpha_2 (X_a^n + X_1^n)$ of power $P_2 = P - P_a - P_1$ is transmitted. Note: $\alpha_2 = \frac{P_2}{P_2 + \sigma_2^2}$.
- We merge all three layers and transmit $X^n = X_a^n + X_1^n + X_2^n$. 
Layering with Analog and Costa Coding: Decoding

An achievable distortion-region can be obtained by varying $P_a$, $P_1$ and $P_2$ subject to $P = P_a + P_1 + P_2$. For a given $P_a$, $P_1$ and $P_2$, the achievable pairs of distortion can be computed as follows.

- **MMSE estimation of $S_1^n$ from analog layer**
- **First, Wyner-Ziv bits are decoded from the 2nd layer by Costa decoding procedure.**
- **The side information is used in refining the estimate of $S_1^n$ for the weak user using the decoded Wyner-Ziv bits.**
- **From Wyner-Ziv distortion-rate function, $D_1 = D_1^* 2^{-2R_1'}$, where $D_1^*$ is the MMSE from the received $Y_1^n$.**

$$D_1 = D_1^* \left(1 + \frac{P_1}{P_2 + \sigma_1^2}\right)^{-1}, \quad D_1^* = \sigma_{S_1}^2 - \frac{\alpha^2(a_1\sigma_{S_1}^2 + a_2\rho\sigma_{S_1}\sigma_{S_2})^2}{P_a + P_1 + P_2 + \sigma_1^2}.$$
Strong user: First, an estimate of $S_n^2$ can be determined from the first and the second layers (as side information).

Based on $R'_2$ decoded Wyner-Ziv bits and side information, using the Wyner-Ziv distortion-rate function, $D_2 = D^*_2 2^{-2R'_2}$.

$D^*_2$ is the MMSE from the received $Y_n^2$ and the decoded $U_1^n$, i.e., $D^*_2 = \sigma^2_S - \Gamma_2^T \Upsilon_2^{-1} \Gamma_2,$

$$\Gamma_2 = \begin{bmatrix} \alpha (a_2 \sigma^2_S + a_1 \rho \sigma_S \sigma_1 \sigma_2) \\ \alpha_1 \alpha (a_2 \sigma^2_S + a_1 \rho \sigma_S \sigma_1 \sigma_2) \end{bmatrix},$$

$$\Upsilon_2 = \begin{bmatrix} P_a + P_1 + P_2 + \sigma^2_2 & P_1 + \alpha_1 P_a \\ P_1 + \alpha_1 P_a & P_1 + \alpha_1^2 P_a \end{bmatrix}.$$
3. Layering with Analog, Superposition and Costa Coding: Encoding

- This scheme also has three coding layers: analog, superposition, and Costa coding.
- In the second layer, we have two merged streams where $S^n_1$ is broadcasted to two users.
- The first source encoder is an optimal Wyner-Ziv encoder with rate $R''_1 = \frac{1}{2} \log(1 + \frac{(1-\lambda)P_1}{\lambda P_1 + P_a + P_2 + \sigma^2_1})$, and the second source encoder is an optimal Wyner-Ziv encoder for the residual error of the first encoder with rate $R''_2 - R''_1 = \frac{1}{2} \log(1 + \frac{\lambda P_1}{P_a + P_2 + \sigma^2_2})$.
- Then, we encode the Wyner-Ziv bits with capacity-achieving channel codes and transmit with powers $(1 - \lambda)P_1$ and $\lambda P_1$, respectively.
The final distortion in estimating $S^n_1$ at the weak user is

$$D_1 = D^*_1 2^{-2R_1''} = D^*_1 (1 + \frac{(1 - \lambda)P_1}{\lambda P_1 + P_a + P_2 + \sigma^2_1})^{-1}.$$ 

At the strong user, first an estimate of $S^n_1$ can be obtained

$$D^*_{12} = D^*_1 2^{-2R_2''} = D_1 (1 + \frac{\lambda P_1}{P_a + P_2 + \sigma^2_2})^{-1}.$$ 

Then we obtain an estimate of $S^n_2$ from the above estimate of $S^n_1$ with distortion: $D^*_2 = \sigma^2_{S_2} \left(1 - \rho^2 \left(1 - \frac{D^*_{12}}{\sigma^2_{S_1}}\right)\right).$

This estimate of $S^n_2$ acts as side information in refining the estimate of $S^n_2$ using the decoded Wyner-Ziv bits. Thus

$$D_2 = D^*_2 \left(1 + \frac{P_2}{\sigma^2_2}\right)^{-1}.$$
We transmit $n$ samples of a bivariate Gaussian source with the covariance matrix $\Lambda = \begin{bmatrix} 1 & 0.2 \\ 0.2 & 1 \end{bmatrix}$ in $n$ uses of a power-limited broadcast channel to two users with observation noise variances $\sigma_1^2 = -5 \text{ dB}$ and $\sigma_2^2 = 0 \text{ dB}$, respectively.

The two-user broadcast channel has the power constraint $P = 0 \text{ dB}$.

The boundaries of the distortion regions for the schemes presented are shown in this figure.

We observe that the layering with analog transmission and Costa coding outperforms all other schemes, including analog transmission.
Numerical Result (Matched Bandwidth)

Analog Transmission
Analog, Superposition and Costa Coding
Analog and Costa Coding

Distortion regions in broadcasting a bivariate source with the correlation coefficient $\rho = 0.2$. 
Conclusion

- We considered broadcasting a bivariate correlated Gaussian source to two users.
- Layered JSCC schemes for this problem were analyzed under matched bandwidth assumptions.
- We provided achievable distortion regions for different three-layered HDA coding schemes.
- Using numerical examples, we demonstrated that the layering with analog and Costa coding performs best.