NETWORK CODING OVERHEAD

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Introduction & Motivation

- Network coding **optimizes** multicast rates to max-flow min-cut bound (Ahlswede, et. al. ’00).

- Plain routing is **not** optimal & **Linear** network coding is **sufficient** (Li, et al. ’03, Koetter-Medard ’03).

- **Typical assumption**: Destinations know how packets are network-coded.

Protocol Information

Header ?	Data Symbols

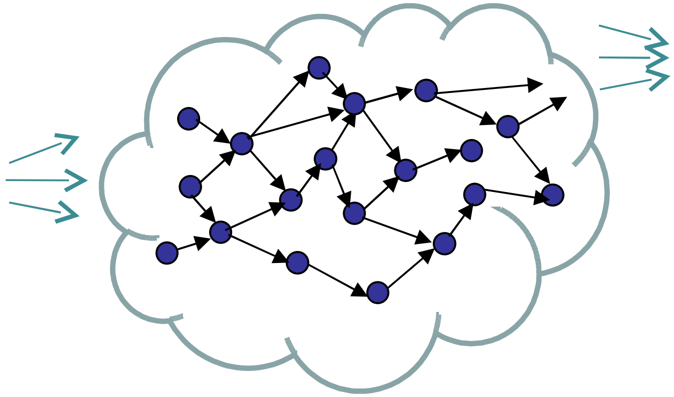
– Limiting effects of network coding **overhead**.

– Joint channel **coding/training** for robust network coding.

Network Coding Overhead

Network:

Source: $\mathbf{X}$



Any **Destination**:

$$\mathbf{Y} = \mathbf{G}\,\mathbf{X}$$

network coding matrix

(elements from finite field F_q)

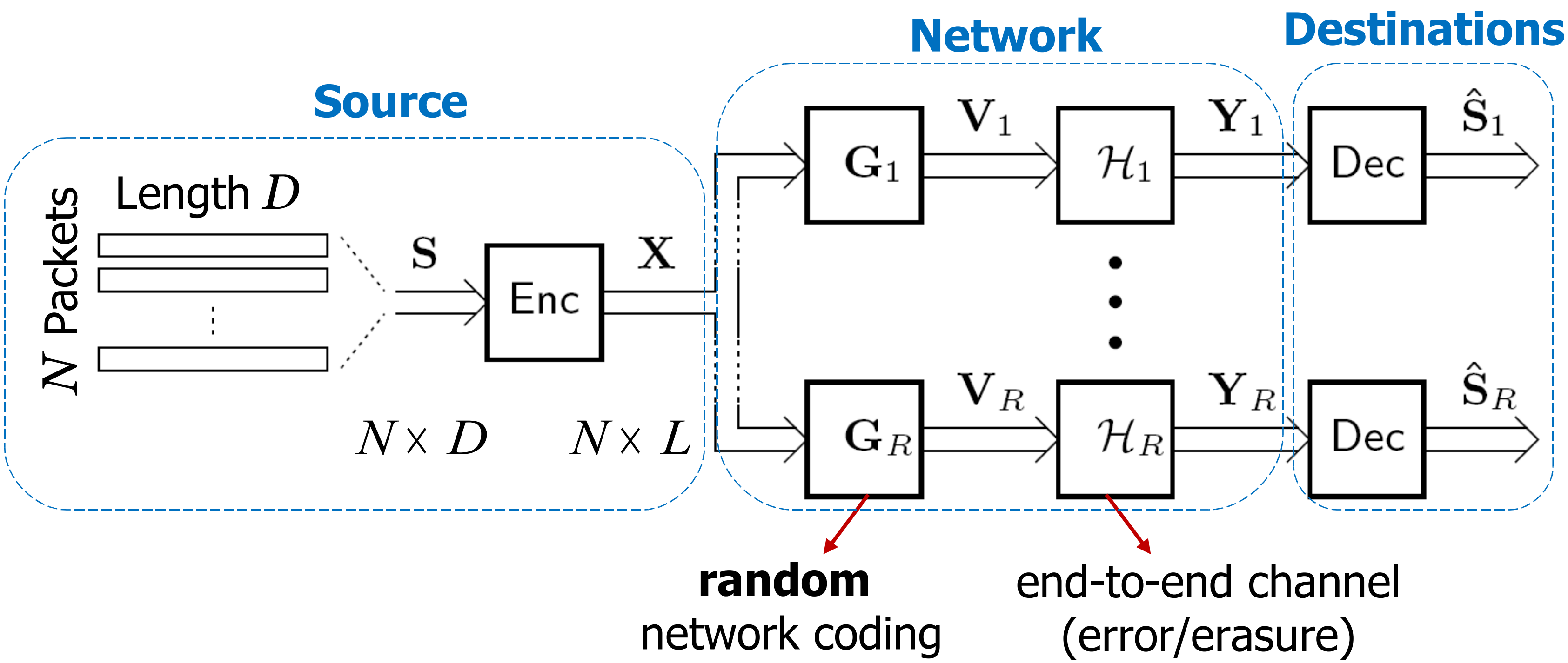
- Distributed solution: **Random network coding** (Ho, et. al '06) .
- Communicate "$\mathbf{G}$" to destinations in packet header:

G	Data

- **Challenge**: Decoding **impossible**, if error/erasure in "$\mathbf{G}$" .

System Model for Overhead Analysis

- **Question**: What is **overhead** for reliable communication of "$\mathbf{G}$" & data?



- **Unknown** topology & "$\mathbf{G}$" (*i.i.d* assumption).

Network Coding Meets MIMO: Training Approach

- Detection of network coding matrix $\mathbf{G}$ $\Leftrightarrow$ **MIMO** channel estimation.

- Source transmits **known training** sequence $\boldsymbol{X}_{\mathrm{T}}$.

- Destination $\boldsymbol{d}$ receives $\mathbf{Y}_d = \mathcal{H}_d(\underbrace{\mathbf{G}_d \boldsymbol{X}_{\mathsf{T}}})$.

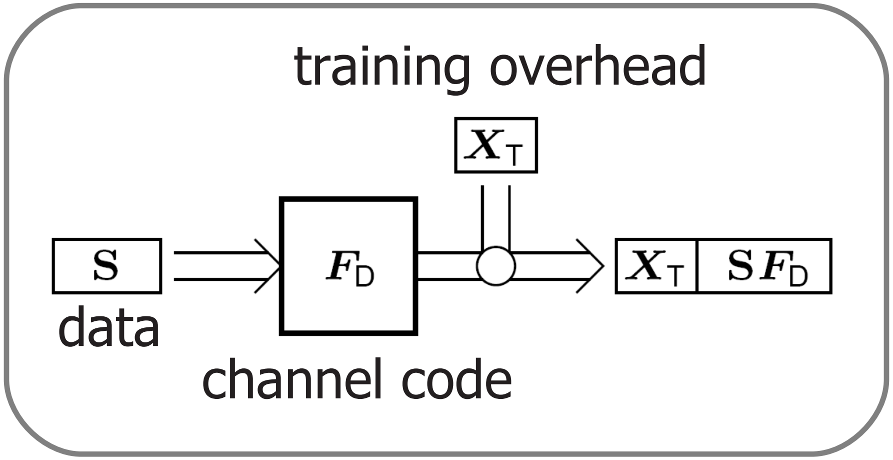
  Rows of $\mathbf{G}_d$ are encoded by $\boldsymbol{X}_{\mathrm{T}}$

- **Interpretation**: $\boldsymbol{X}_{\mathrm{T}}$ is a "code" generator matrix (e.g., MDS code)

  Rows of $\mathbf{G}_d$ are the transmitted "packets"

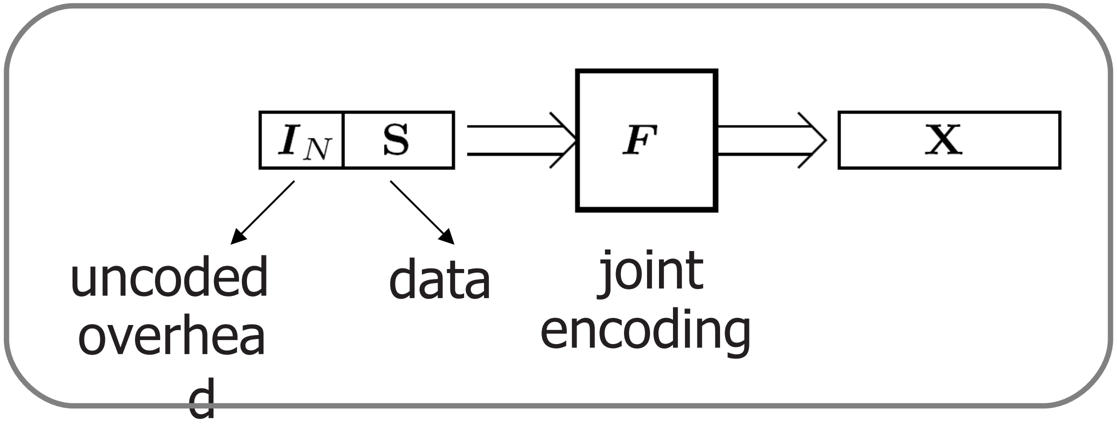
Encoding with Training

- **Separate** **training and coding** (STC).



$$\mathbf{X} = [\mathbf{X}_T \ \mathbf{S}\mathbf{F}_D] = [\mathbf{I}_N \ \mathbf{S}] \begin{bmatrix} \mathbf{X}_T & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_D \end{bmatrix}$$

- **Joint** **training and coding** (JTC).



$$\mathbf{X} = [\mathbf{I}_N \ \mathbf{S}]\mathbf{F}$$

Decoding Channel & Network Codes

(1) Decode **channel code**:

- $\hat{G}_d$ for random network coding matrix G_d
- $\hat{U}_d$ for network-coded data: $U_d = G_d\, S$

(S: source packets)

JTC: **Jointly** decode entire packet.

STC: **Separately** decode training and data parts.

} Maximum Likelihood (ML) estimates

(2) Decode **network code**:

– Solve $\hat{U}_d = \hat{G}_d\, \hat{S}_d$ for data $\hat{S}_d$ from ML estimates $\hat{G}_d$ and $\hat{U}_d$.

Throughput & Overhead

- Two types of decoding **error**: $\begin{cases} \text{Errors/erasures in packets} \\ \text{Rank-deficient network coding matrix } \hat{G}_d \end{cases}$

- **Decoding probability** P_d is derived via **random coding bounds**.

- End-to-end achievable **rate**:

$$\Lambda^* = \max_{N,D} \left[N \frac{D}{L} \min_d P_d \right]$$

- **Overhead**: $O = \dfrac{L - D}{L}$

N: independent messages per transmission

D: data symbols per packet

L: total packet length

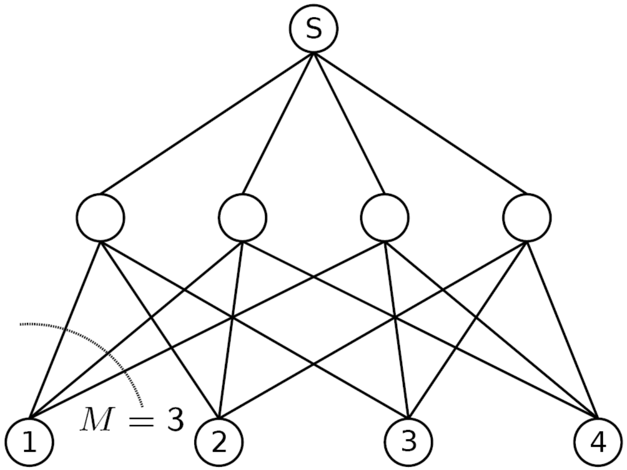
Limiting Effects of Overhead

Example 1: **Combination network**

1 source

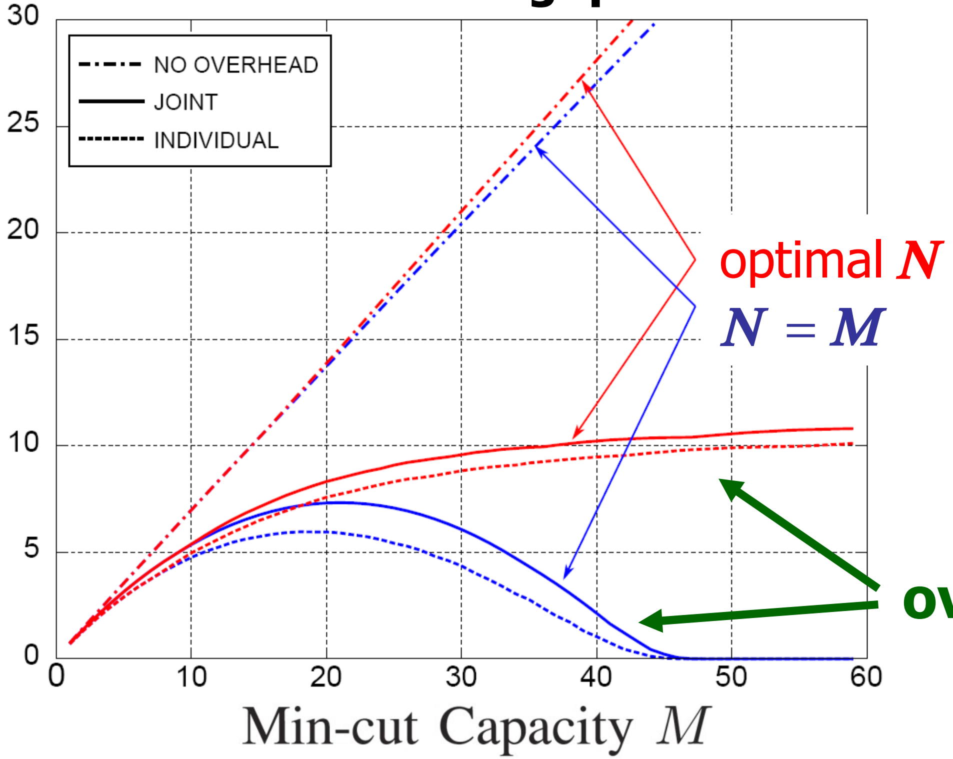
A layer-2 nodes

$\binom{A}{M}$ destinations



M: min-cut capacity

Network Throughput Λ



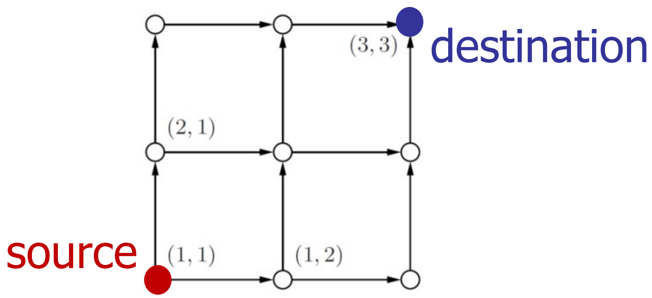
Field $q = 2^8$, 480 bits per packet,

$\binom{60}{M}$ combination network,

bit erasure probability 10^{-2}

Throughput-Overhead Performance (Cont'd)

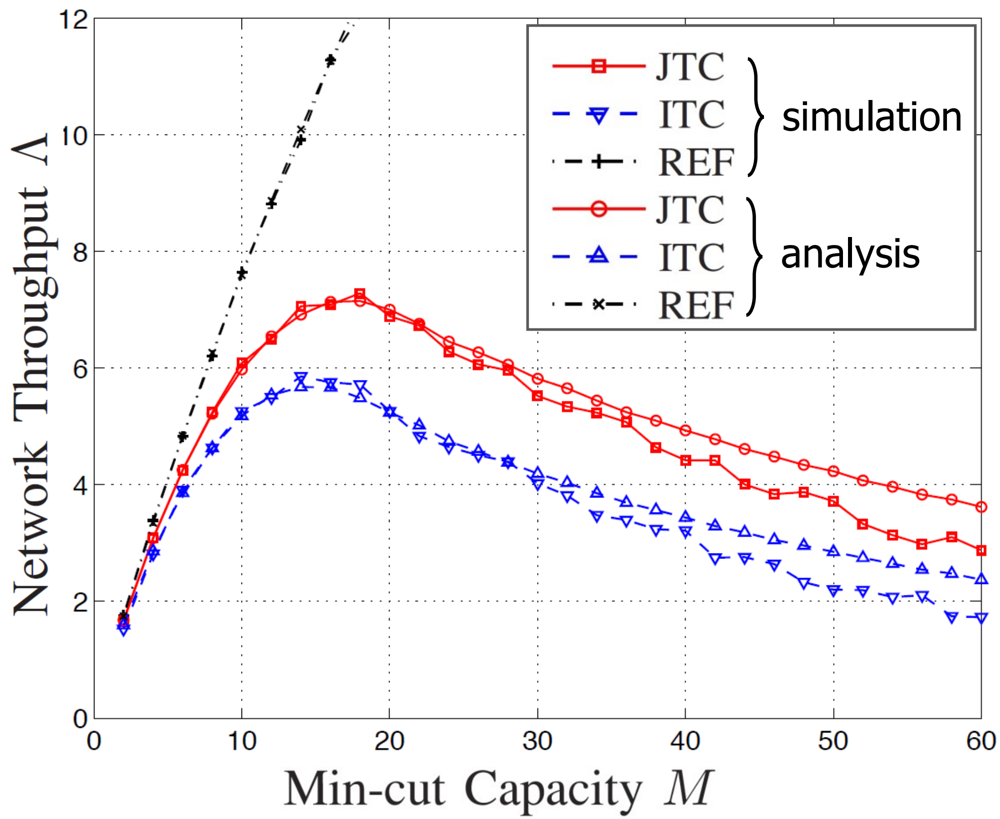
Example 2: **Grid network**



Each link carries $M/2$ packets per time slot

Field $q = 2^8$, 60 bits per packet

Bit error probability 10^{-3}



JTC: Joint training & coding

ITC: Individual training & coding

REF: Destination knows "**G**"

analysis: Random coding bound on probability of successful decoding

simulation: Average error rates over random coding matrices & error events with MDS channel codes

End-to-End Distributed Network Coding

- Oblivious to network topology (mobility effects, malicious behavior, …).

- **No** need to know network coding matrix "**G**" .

- Need to know min-cut capacity M (s.t. $N \leq M$)

  N : independent messages per transmission

- **Rate control**: Adapt N based on destination feedback (ACK/NACK).
  - Increase N , if channel & network codes are successfully decoded.
  - Decrease N , otherwise.

Conclusions

- **Training** can be combined with **channel coding**.
  - Enables **joint** decoding of data and network coding matrix.
  - Simplifies and balances protection of **overhead**.
- Network coding gain is **limited** by the necessary protocol overhead.
  - Overhead **grows** with the min-cut capacity.
- **Future Work**:
  - Extension to **general** error and erasure models.
  - Exploit **redundancy** in sequence of coding matrices.
  - **Adaptive** training and feedback schemes to learn error statistics.