Distortion Exponent for Multiple Description Coding

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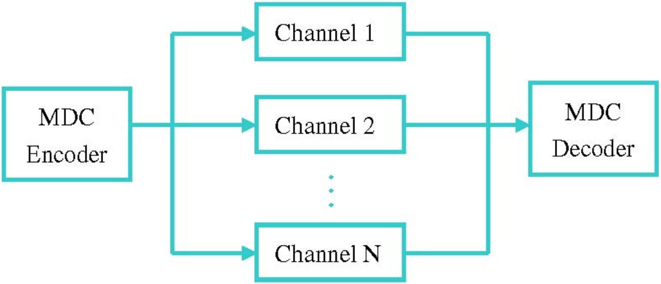
Motivation

1. Distortion exponent has been proposed as performance measure to characterize how fast the average distortion decays to zero when the SNR increases to infinity.
2. All related work considered the symmetric SNR. If they considered MDC case, that is only for two descriptions. (Layered source coding (*Gunduz et al*), MDC (*Laneman et al*), hybrid analog-digital coding (*Caire et al*))
3. We consider asymmetric SNR and general number of descriptions.
4. For MDC, we use the most recently available achievable rate distortion region and some bounds. (*Kramer, Tian, et al*)

Model



Model cont.

1. The distortion exponent is defined as

$$\Delta = -\lim_{SNR\to\infty} \frac{\log D_{\mathrm{ave}}}{\log SNR}.$$

2. Channel model

$$Y_c = \sqrt{\mathrm{SNR}_c}\, H_c X_c + Z_c, \quad c \in \{1, \cdots, N\},$$

where X_c is Gaussian source, Z_c is Gaussian noise, H_c is Rayleigh fading gain.

3. Outage probability

$$P_{\mathrm{out}}(R) = \Pr[I \leq R_c],$$

where $I = \log(1 + |H_c|^2 SNR)$.

4. Bandwidth expansion/compression factor

$$b = R_s / R_c,$$

where R_s is source coding rate.

Problem Formulation

The optimization problem is

$$D_{\mathrm{DMC}} = \min_{R_1,\ldots,R_N} \sum_{\mathcal{A}\subseteq\{1,..,N\}} p_{\mathcal{A}} D_{\mathcal{A}}$$

$$\textit{subject to} : \textit{Rate Distortion region},$$

where $D_{\emptyset} = 1$, and $p_{\mathcal{A}} = \Pr[I_i > R_i \;\; \forall i \in \mathcal{A}, \;\; I_j \leq R_j \;\; \forall j \in \mathcal{A}^c]$ and rate distortion region is

$$D_{\mathcal{A}} \geq \frac{|\mathbf{Q}_{\mathcal{A}}|}{|\mathbf{1}_{\mathcal{A}}\mathbf{1}_{\mathcal{A}}^T + \mathbf{Q}_{\mathcal{A}}|} = \frac{1}{1 + \mathbf{1}_{\mathcal{A}}^T \mathbf{Q}_{\mathcal{A}}^{-1} \mathbf{1}_{\mathcal{A}}},$$

$$\frac{|\mathbf{Q}_{\mathcal{A}}|}{\prod_{k\in\mathcal{A}}(1 + \mathbf{Q}_{\{kk\}})} \geq \mathrm{e}^{-b\sum_{k\in\mathcal{A}} R_k}.$$

Noise Cancellation Approximation

For example, for $N = 3$, the descriptions are

$$U_1 = X + W_1 = X - \sigma_1 Z_1$$

$$U_2 = X + W_2 = X + \sigma_2(\rho_{12} Z_1 - \sqrt{1-\rho_{12}^2} Z_2)$$

$$U_3 = X + W_3 = X + \sigma_3\big(\rho_{13} Z_1 + \sqrt{1-\rho_{13}^2}(\rho_{23|1} Z_2 + \sqrt{1-\rho_{23|1}^2} Z_3)\big).$$

And we have

$$U_{12} = X - \frac{\sqrt{1-\rho_{12}^2}}{\frac{\rho_{12}}{\sigma_1} + \frac{1}{\sigma_2}} Z_2, \quad \sigma_{12} \triangleq \frac{\sqrt{1-\rho_{12}^2}}{\frac{\rho_{12}}{\sigma_1} + \frac{1}{\sigma_2}}.$$

Similarly, we get

$$\sigma_{13} \triangleq \frac{\sqrt{1-\rho_{13}^2}}{\frac{\rho_{13}}{\sigma_1} + \frac{1}{\sigma_3}}, \quad \sigma_{23} \triangleq \frac{\sqrt{1-\rho_{23}^2}}{\frac{\rho_{23}}{\sigma_2} + \frac{1}{\sigma_3}}, \quad \sigma_{123} \triangleq \frac{\sqrt{1-\rho_{23|1}^2}}{\frac{\rho_{23|1}}{\sigma_{12}} + \frac{1}{\sigma_{13}}}.$$

Parametrization

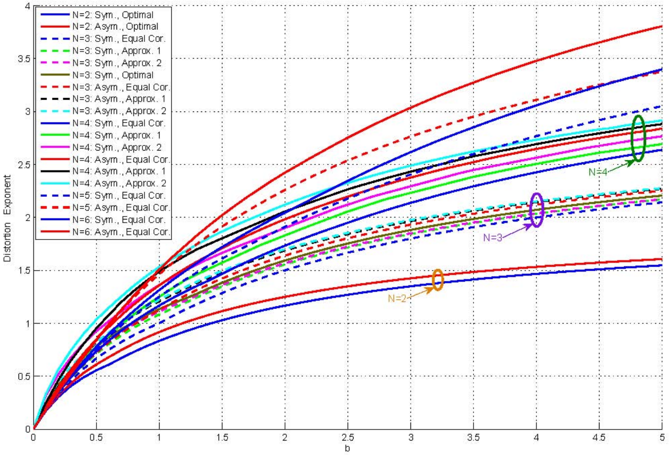
Inspired by *DMT, Zheng*, for some $x > 1$, we propose the following parametrization.

$$\begin{aligned}
&\mathrm{SNR}_c = x^{\beta_c}, \quad \beta_c \in \mathbb{R}, \\
&R_c = \log(1 + x^{r_c}), \quad r \in \mathbb{R}, \\
&D_{\mathcal{A}} = \frac{1}{1 + x^{d_{\mathcal{A}}}}, \quad d_{\mathcal{A}} \in \mathbb{R},\ d_{\emptyset} = -\infty, \\
&1 - \rho_{12}^2 = x^{-\theta_{12}}, \quad 1 - \rho_{13}^2 = x^{-\theta_{13}}, \quad 1 - \rho_{23|1}^2 = x^{-\theta_{23|1}}.
\end{aligned}$$

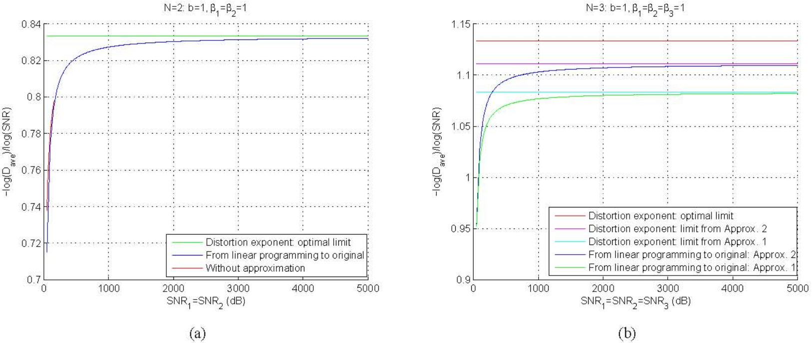
Linear conditions can be obtained

$$\begin{aligned}
&d_{12} = \theta_{12} + \max\{d_1, d_2\}, \quad d_{13} = \theta_{13} + \max\{d_1, d_3\}, \\
&d_{23} = \min\{\theta_{12}, \theta_{13}\} + \max\{d_2, d_3\}, \\
&d_{123} = \theta_{23|1} + \max\{d_{12}, d_{13}\}.
\end{aligned}$$

Numerical result



Numerical result cont.



Conclusion and Future Work

1. Achievable distortion exponent for MDC with any number of descriptions is proposed
2. Consider other channel diversity schemes such as relay channels
3. Consider the power allocation schemes at the transmitter