Efficient LLR Calculation for Iterative Decoding on Fading channels

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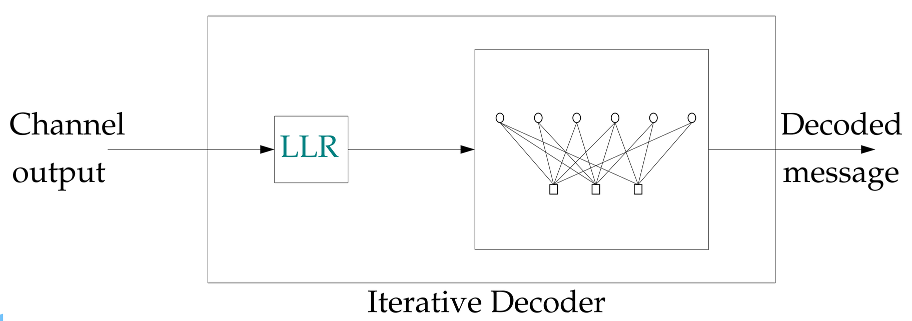
Introduction

Log-likelihood ratios (LLRs):

- very efficient metrics for soft decoding of many powerful codes, e.g., convolutional codes, turbo codes, low-density parity-check (LDPC) codes, etc.
- offer practical advantages: numerical stability, simplification of many decoding algorithms, etc.
- useful for message-passing algorithms in iterative decoding.



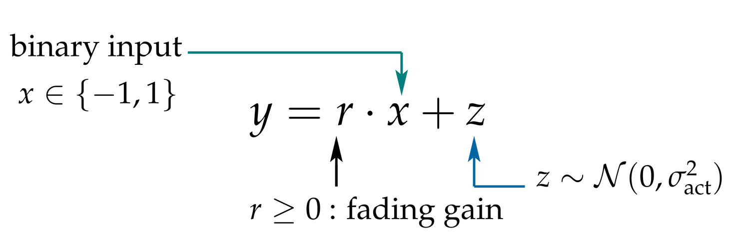
Challenges:

- Channel estimation techniques increase the complexity, cause overheads, and are imperfect.
- When no channel state information (CSI) is available at the receiver, LLRs are complicated functions of channel output [1].
- Decoder may not be able to handle extra complexity or delay.

To overcome these challenges, approximate LLRs should be calculated.

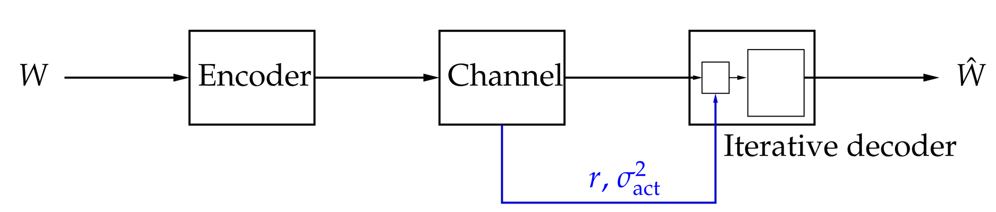
System Model

Uncorrelated fading channels:



- r changes independently for each channel use and its pdf is given.

1 - CSI at the receiver

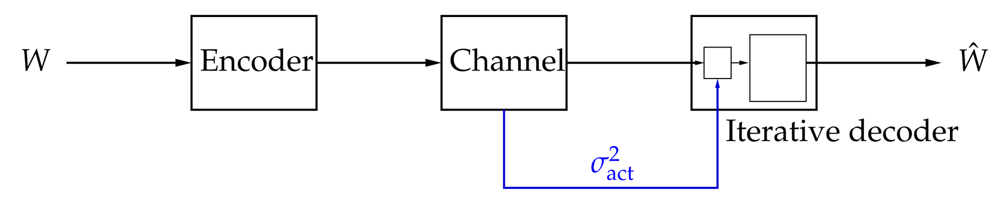


- BPSK:

$$l = \log \frac{p(x=+1|y,r)}{p(x=-1|y,r)} = \frac{2r}{\sigma_{\text{act}}^2} y \Longrightarrow \text{linear function of y}$$

- For non-binary modulations, LLR computation is complicated.

2 - No CSI at the receiver



- LLRs are complicated functions of y even for BPSK. On a Rayleigh channel [1]:

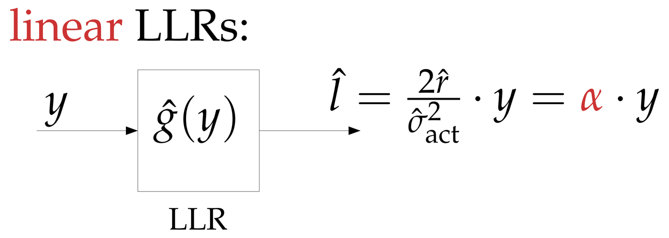
$$l = \log \frac{p(x=+1|y)}{p(x=-1|y)} = \log \frac{\Phi(y/\sqrt{2\sigma_{\text{act}}^2(1+2\sigma_{\text{act}}^2)})}{\Phi(-y/\sqrt{2\sigma_{\text{act}}^2(1+2\sigma_{\text{act}}^2)})},$$

where $\Phi(z) = 1 + \sqrt{\pi} z e^{z^2} \operatorname{erfc}(-z)$.

Symmetric LLR approximation:

$$\hat{l} = \hat{g}(y) \quad \text{where} \quad \hat{g}(-y) = -\hat{g}(y)$$

- To simplify use linear LLRs:



Questions:

- What linear LLR approximation gives the best performance?
- What if the noise power is also unknown?

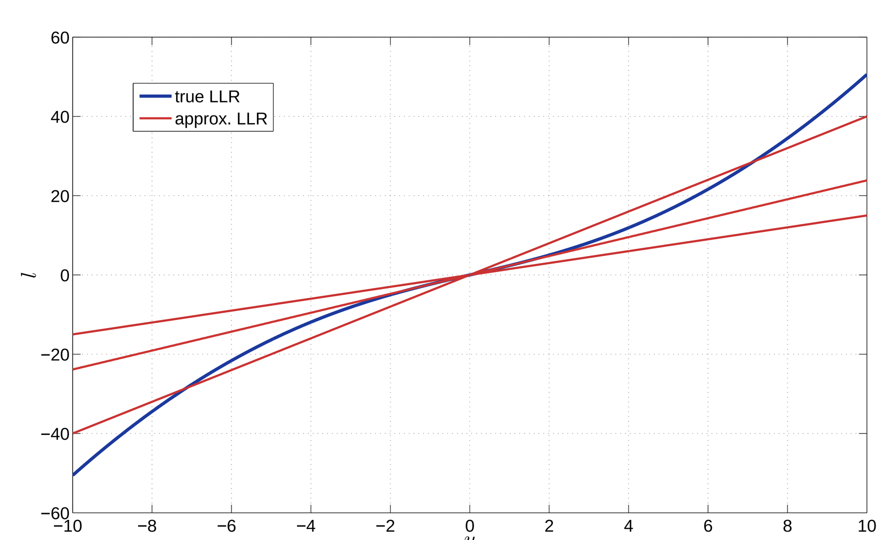


Fig. 1: True LLRs versus different approximate linear LLRs.

Optimum Linear LLR Approximation

Optimality Measures:

- minimum mean square error (MMSE):
  - min $E[|l - \hat{l}|^2]$
  - min $E[|r - \hat{r}|^2] \Longrightarrow \hat{r} = E[r]$ [2].
- based on LLR pdf:
  - minimizing the relative entropy between the real and approximate LLR pdfs
  - a new measure of LLR accuracy: maximize

$$\hat{C} = 1 - \int_{-\infty}^{\infty} \log_2(1 + e^{-\hat{l}}) f_{\hat{L}}(\hat{l}) d\hat{l}$$

where $f_{\hat{L}}(\hat{l})$ is the pdf of approximate LLRs. If true LLR pdf is used instead we get channel capacity.

Theorem 1: Between symmetric LLR calculations, $\hat{C}$ achieves its maximum only with true LLRs.

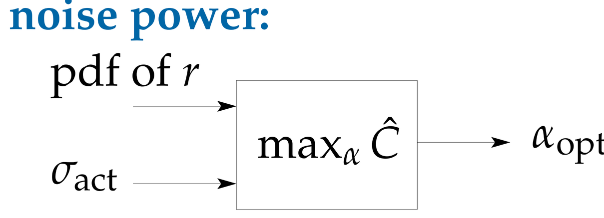
How to maximize $\hat{C}$?

$$\alpha_{\text{opt}} = \arg\max_{\alpha} \left\{ 1 - \int_{-\infty}^{\infty} \log_2(1 + e^{-\hat{l}}) f_{\hat{L}\{\alpha\}}(\hat{l}) d\hat{l} \right\}$$

Requires knowledge of noise power σ_{act}^2 and pdf of r.

Theorem 2: For a fixed σ_{act}, there exists a unique α which maximizes $\hat{C}$.

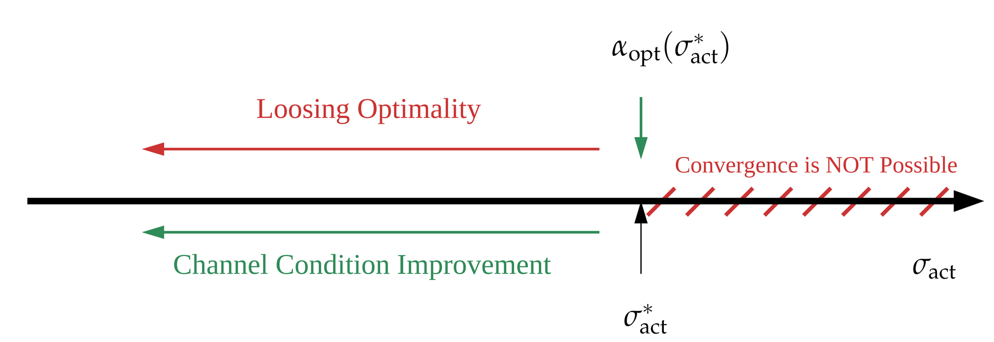
1- Known noise power:



one-variable convex optimization

2- Unknown noise power: $\sigma_{\text{act}} = ?$

- For any assumed σ_{act}, one can find α_{opt}.
- What to assume for σ_{act}?
  Idea: Maximize $\hat{C}$ at the highest noise standard deviation σ_{act}^* that a given code can tolerate under optimum linear LLRs ($\hat{l} = \alpha_{\text{opt}}(\sigma_{\text{act}}^*) \cdot y$).



- Justification: Channel condition improvement outweighs loss of optimality. Also, α_{opt} is not much sensitive to σ_{act}.

Simulation Results and Conclusion

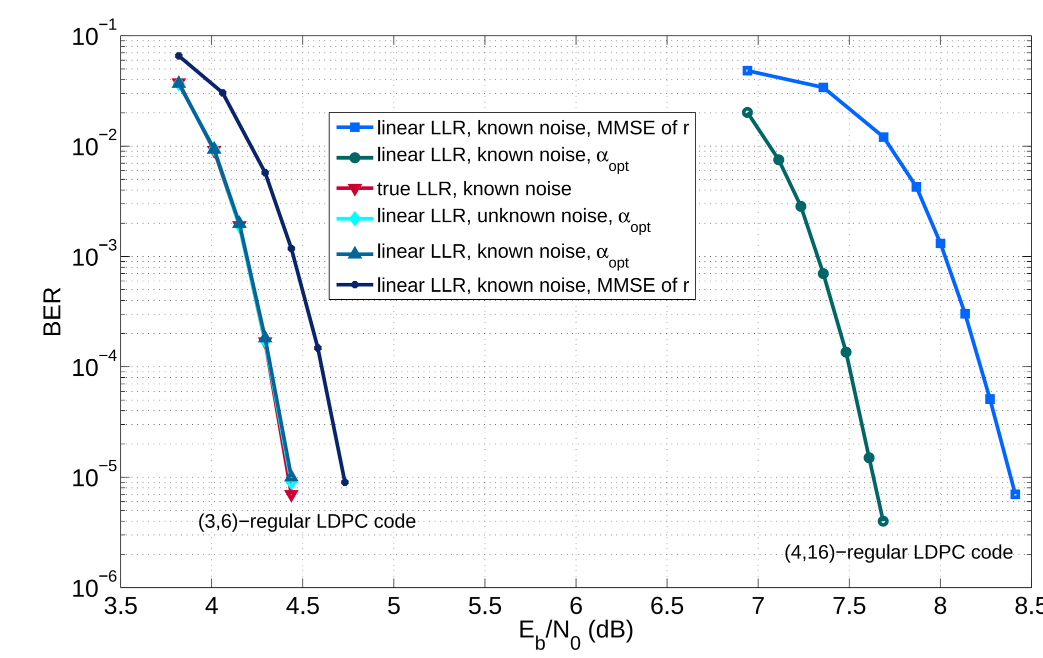


Fig. 2: BER comparison for LDPC codes of length 10^4 on normalized Rayleigh channel under different LLR calculations.

- A new measure for the accuracy of the approximate LLRs was introduced which outperforms other measures.
- Finding the optimum linear LLR functions is a convex optimization problem.
- With designed linear LLR functions, performance loss is extremely small (about 0.02 dB) compared to true LLR calculation even for the case that noise power is unknown at the receiver.
- The method can be extended to higher modulations with any LLR approximating function, e.g., piece-wise linear LLR functions.

References

[1] J. Hagenauer, "Viterbi decoding of convolutional codes for fading- and burst-channels," in *Zurich seminar on digital communications*, 1980.

[2] J. Hou, P. H. Siegel, and L. B. Milstein, "Performance analysis and code optimization of low-density parity-check codes on Rayleigh fading channels," *IEEE J. Sel. Areas Commun.*, vol. 19, no. 5, pp. 924–934, May 2001.