Multicasting in Large Random Networks: Bounds on Minimum Energy per Bit

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Abstract

We consider scaling laws for maximal energy efficiency of communicating a message to all the nodes in a random wireless network, as the number of nodes in the network becomes large. Two cases of large wireless networks are studied —

- Dense random networks, and
- Extended random networks

We also establish an information-theoretic lower bound on the minimum energy per bit for multicasting that holds for arbitrary wireless networks when the channel state information is not available at the transmitters.

Upper bounds are obtained by constructing a simple flooding scheme that requires no information at the receivers about the channel states or the locations and identities of the nodes.

Channel Model

A few key assumptions:

- AWGN with circularly symmetric fading:

$$y_j = \sum_{i=1}^{k} h_{ij} x_i + z_j \qquad j = 1, 2, ..., k$$

  where, $h_{ij} \in \mathbb{C}$ is i.i.d. over time and independent at different nodes.
- No channel state information at transmitters. CSIR does not matter.
- Cost of transmitting $x \in \mathbb{C}$ is $|x|^2$.
- Total energy consumption is the sum of costs at all the nodes.

Converse

Define:

$$G(\mathcal{R}) \triangleq \frac{1}{|\mathcal{R}|} \left(\max_{i \in \{1,...,k\}} \sum_{j \in \mathcal{R} \setminus \{i\}} \mathbb{E}[|h_{ij}|^2] \right) \tag{1}$$

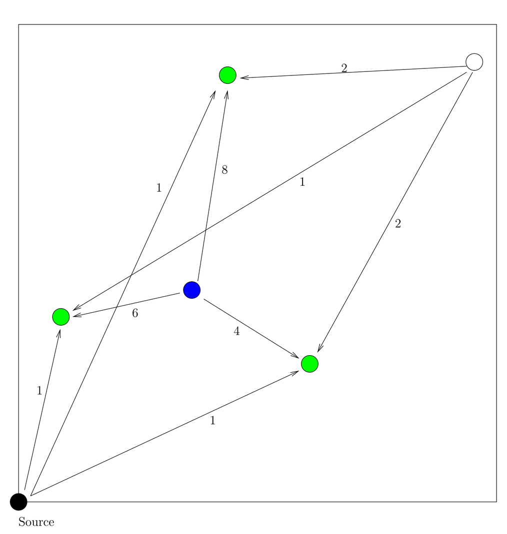

Figure 1: Effective network radius: an example

Theorem 1. *In a network with k nodes, where node 1 is the source node and the destination set is $\mathcal{R} \subset \{2, ..., k\}$, the required minimum energy per bit satisfies*

$$\frac{E_b}{N_0}_{min}\left(\mathcal{R}\right) \geq \frac{\log_e 2}{G(\mathcal{R})}$$

where G is the effective network radius.

Remarks:

- Tighter bounds can be obtained by the following definition

$$G(\mathcal{R}) \triangleq \min_{\substack{\mathcal{R}' \subseteq \mathcal{R}: \\ \mathcal{R}' \neq \phi}} \frac{1}{|\mathcal{R}'|} \left(\max_{i \in \{1,...,k\}} \sum_{j \in \mathcal{R}' \setminus \{i\}} \mathbb{E}[|h_{ij}|^2] \right) \tag{2}$$

- Result is tight for point-to-point channel, [Verdú, 2002].

Achievability

We propose an achievable scheme which operates in wideband, and is described over T time slots. One of the rules for decoding is that the received energy per bit at the decoder should exceed $N_0 \log_e 2$, which can be achieved using, e.g., *pulse position modulation* which identical codebooks at all the nodes.

$\underline{\texttt{FLOOD}(E_{b1},\, E_{b2})}$:

At the beginning of time slot t (for $t > 1$), each node (except the source node) follows the following protocol

1. If the node is able to decode a message successfully for the first time in the previous time slot, then it transmits the same message in the current slot with any value of energy per bit greater than E_{b2}.
2. Else, keep quiet.

The source node, on the other hand, only transmits in the 1^{st} time slot and with any value of energy per bit greater than E_{b1}.

The energy consumption of $\texttt{FLOOD}(E_{b1},\, E_{b2})$ is:

$$E_{b\text{flood}} \leq E_{b1} + (k-1) E_{b2} \tag{3}$$

Large Random Networks

- Large random networks : k nodes are placed randomly, independently and uniformly over a square of area A_k. The source node is always placed at the origin.
- Path loss function:
  –
$$g(r) = r^{-\alpha} \quad \text{for all } r \geq r_0, \quad \text{and } \alpha > 2$$
  –
$$g(0) \leq \bar{g}$$

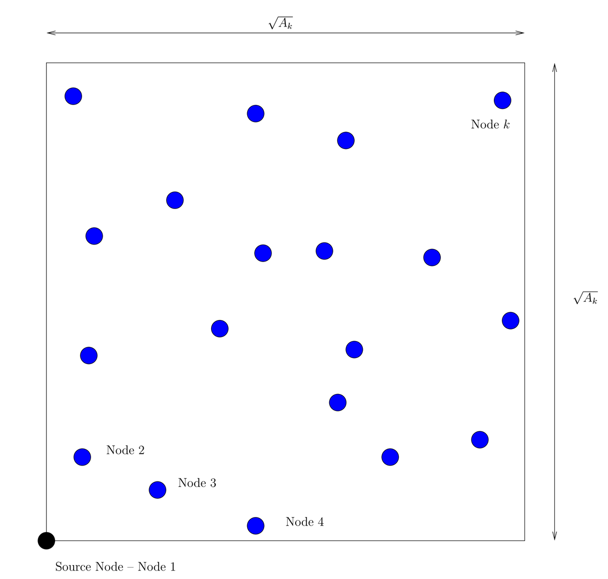

Figure 2: Large random networks

Dense Random Networks

For dense networks: $A_k = o\left(\frac{k}{\log k}\right)$.

Theorem 2. *With probability 1, the node placement is such that the following hold*

$$c_1 \leq \frac{1}{A_k} \frac{E_b}{N_0}_{\min}$$

$$\frac{1}{A_k} \frac{E_b}{N_0}_{flood} \leq c_2$$

for all but a finite number of k, where $c_1, c_2 > 0$ are constants depending only on the parameters of the path loss model.

Achieved using:

$$\texttt{FLOOD}\left(\Theta(1), \Theta\left(\frac{A_k}{k}\right)\right) \tag{4}$$

Extended Random Networks

Node density $\lambda = \frac{k}{A_k}$ is a constant.

Theorem 3. *With probability 1, the node placement is such that the following hold*

$$c_1 \leq \frac{1}{k} \frac{E_b}{N_0}_{\min}$$

$$\frac{1}{k (\log k)^{\alpha/2}} \frac{E_b}{N_0}_{flood} \leq c_2$$

for all but a finite number of k, where $c_1, c_2 > 0$ are constants depending only on the path loss model and λ.

Achieved using the point-to-point multihopping scheme:

$$\texttt{FLOOD}\left(\Theta((\log k)^{\alpha/2}), \Theta((\log k)^{\alpha/2})\right) \tag{5}$$