

Can Iterative Decoding for Erasure Correlated Sources be Universal?

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Introduction & Motivation

We wish to transmit the outputs of two discrete memoryless correlated sources $(U_i^{(1)}, U_i^{(2)})$, for $i = 1, 2, \dots, k$ to a central receiver through two independent BMS channels with capacities C_1 and C_2 , respectively. The two sources are not allowed to collaborate and, hence, they use two independent encoding functions which map the k input symbols in to n_1 and n_2 output symbols respectively. The rates of the encoders are given by $R_1 = k/n_1$ and $R_2 = k/n_2$.

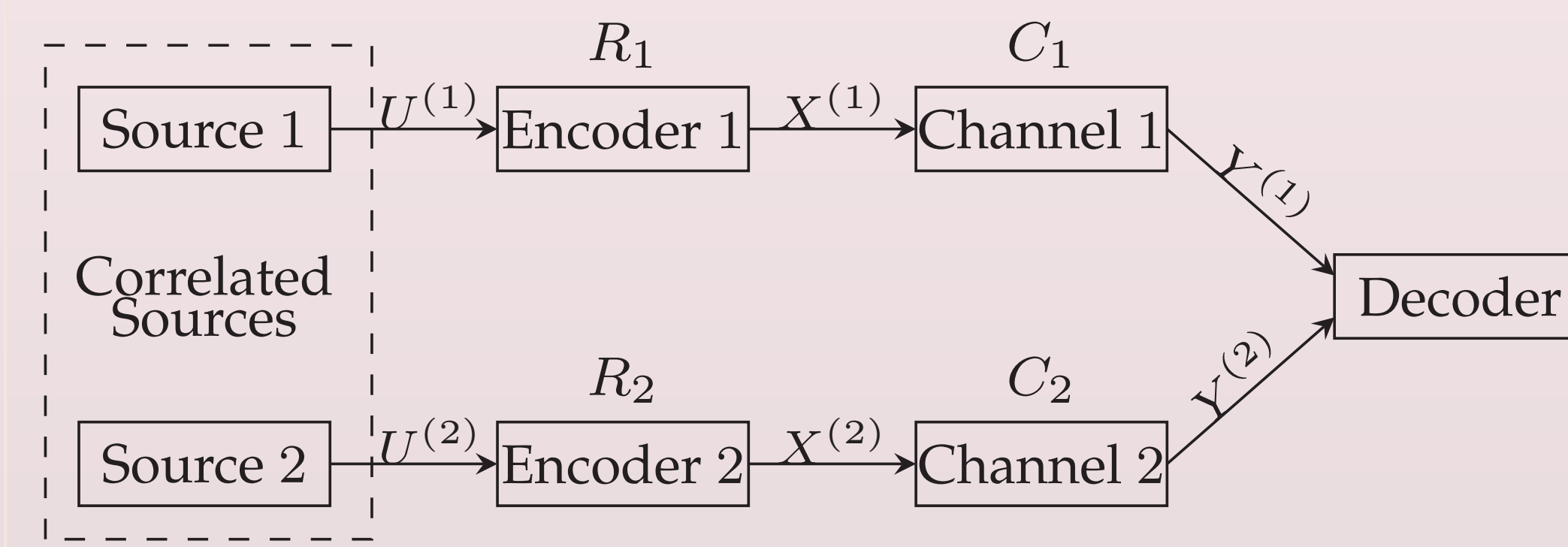
In such a problem, it is clear that one has to take advantage of the correlation between the sources to reduce the required bandwidth to transmit the information to the central receiver. Thus, this joint source-channel coding problem can be seen to be an instance of Slepian-Wolf coding in the presence of a noisy channel. From the Slepian-Wolf theorem, the sources can be reliably decoded at the receiver iff

$$\frac{C_i}{R_i} \geq H(U^{(1)}|U^{(2)}), \sum_i \frac{C_i}{R_i} \geq H(U^{(1)}, U^{(2)}),$$

for $i = 1, 2$. If ϵ_1 and ϵ_2 are known to the transmitters, the encoding functions can be chosen to have rates governed by (). However, it is unrealistic for the transmitters to have a priori knowledge of ϵ_1 and ϵ_2 . We then wish to find a universal source-channel coding scheme such that reliable transmission is possible over different channel parameters (ϵ_1, ϵ_2) . For a given pair of encoding functions and a joint decoding algorithm, the achievable rate region (ARR) is defined as the set of all channel parameters (ϵ_1, ϵ_2) for which the joint decoder achieves an arbitrarily low probability of error. The supremum of this region over all encoding and decoding functions is called the capacity region.

Our primary interest is to investigate whether there exist graph based codes and iterative decoding algorithms that are universal. Several code ensembles, including Luby Transform (LT) codes and LDPC codes, have been shown to achieve capacity with iterative decoding on a single user erasure channel. However, the universality of these ensembles for more complicated scenarios has not been studied well in the literature. Hence, the question of whether one can design a single graph based code and a decoding algorithm capable of universal performance is an open question.

System Model



Let Z_N be a sequence of i.i.d. Bernoulli p random variables. Define the correlation between the source sequences $U_N^{(1)}$ and $U_N^{(2)}$ by

$$(U_i^{(1)}, U_i^{(2)}) = \begin{cases} \text{i.i.d. Bernoulli } \frac{1}{2} \text{ r.v.s, if } Z_i = 0 \\ \text{same Bernoulli } \frac{1}{2} \text{ r.v. } U_i, \text{ if } Z_i = 1 \end{cases}$$

Consider transmission over erasure channels with erasure rates ϵ_1 and ϵ_2 . The Slepian-Wolf conditions are then given by

$$R_i \geq \frac{1-p}{1-\epsilon_i}, \sum_i R_i(1-\epsilon_i) \geq 2-p,$$

for $i = 1, 2$.

Density Evolution and Tanner Graphs

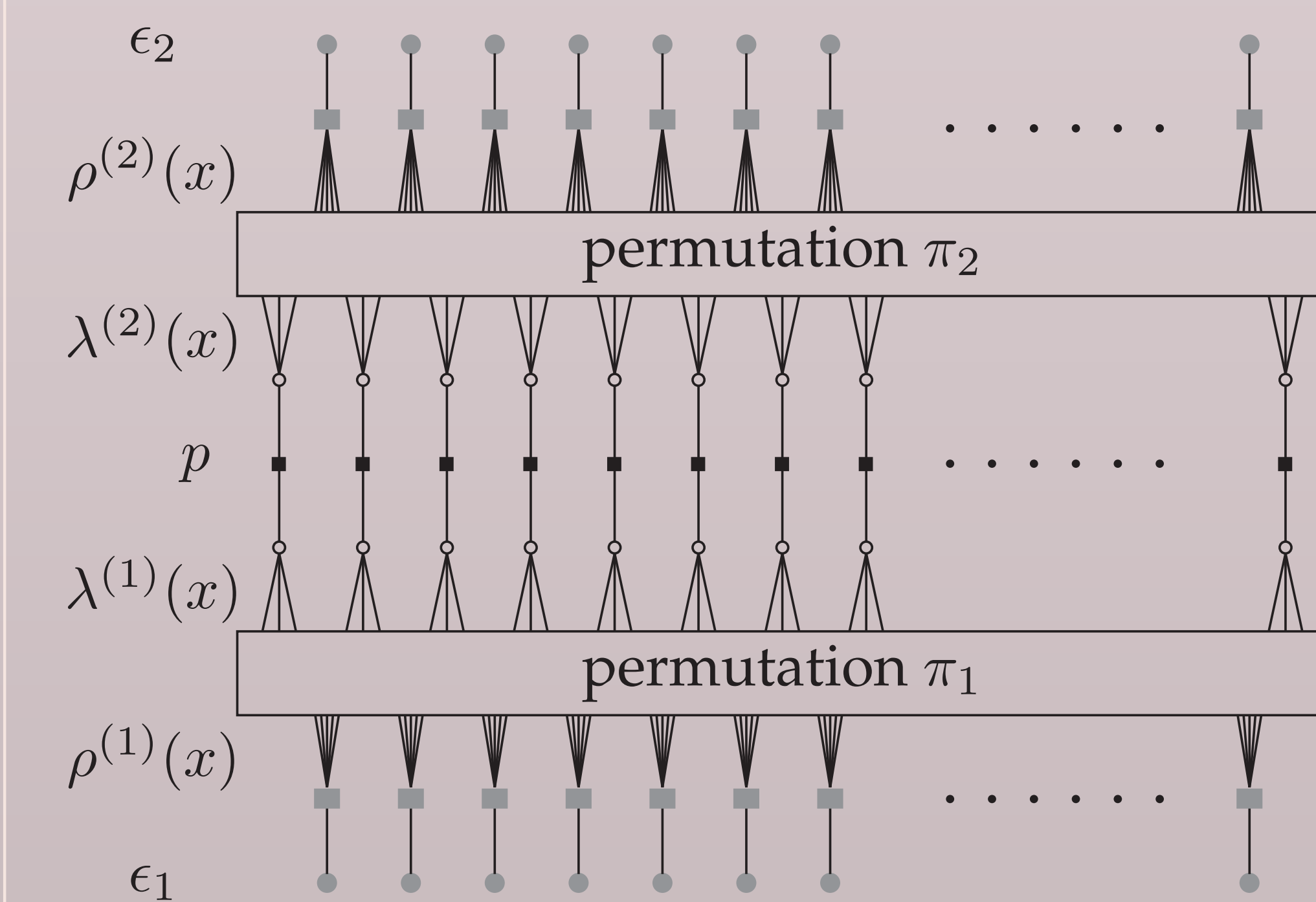


Fig. 1: Tanner Graph for an LP Code

The density evolution equations for the variable-to-check messages for the LT code are given below

$$\begin{aligned} x_{i+1} &= [(1-p) + p\bar{\lambda}(\epsilon_2, y_i)] \bar{\lambda}(\epsilon_1, x_i) \\ y_{i+1} &= [(1-p) + p\bar{\lambda}(\epsilon_1, x_i)] \bar{\lambda}(\epsilon_2, y_i), \end{aligned}$$

where $\bar{\lambda}(\epsilon, x) = \lambda(1 - (1-\epsilon)\rho(1-x))$.

Performance under Iterative Decoding

Using the density evolution equations, we can design codes using Linear Programming. The performance of the optimized LT code under iterative decoding is shown in Fig. 3. Also shown in Fig. 3 is the simulated density evolution threshold for LT code I. We show in [5] that LT codes cannot be universal for the two user Slepian-Wolf problem.

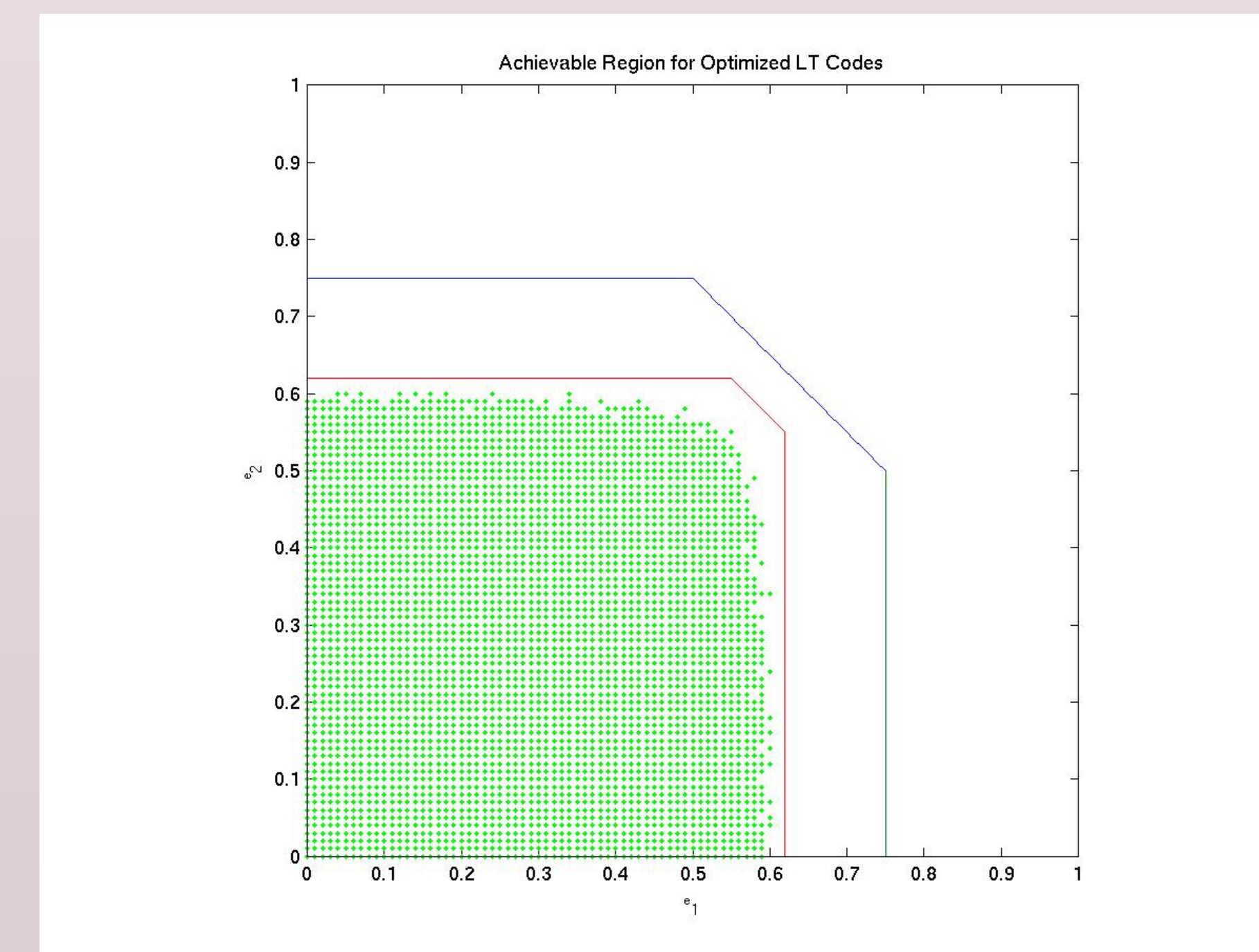


Fig. 3: ARR for the optimized LP Code

The performance of an optimized (for a single user channel) LDPC code under iterative decoding is shown in Fig. 4.

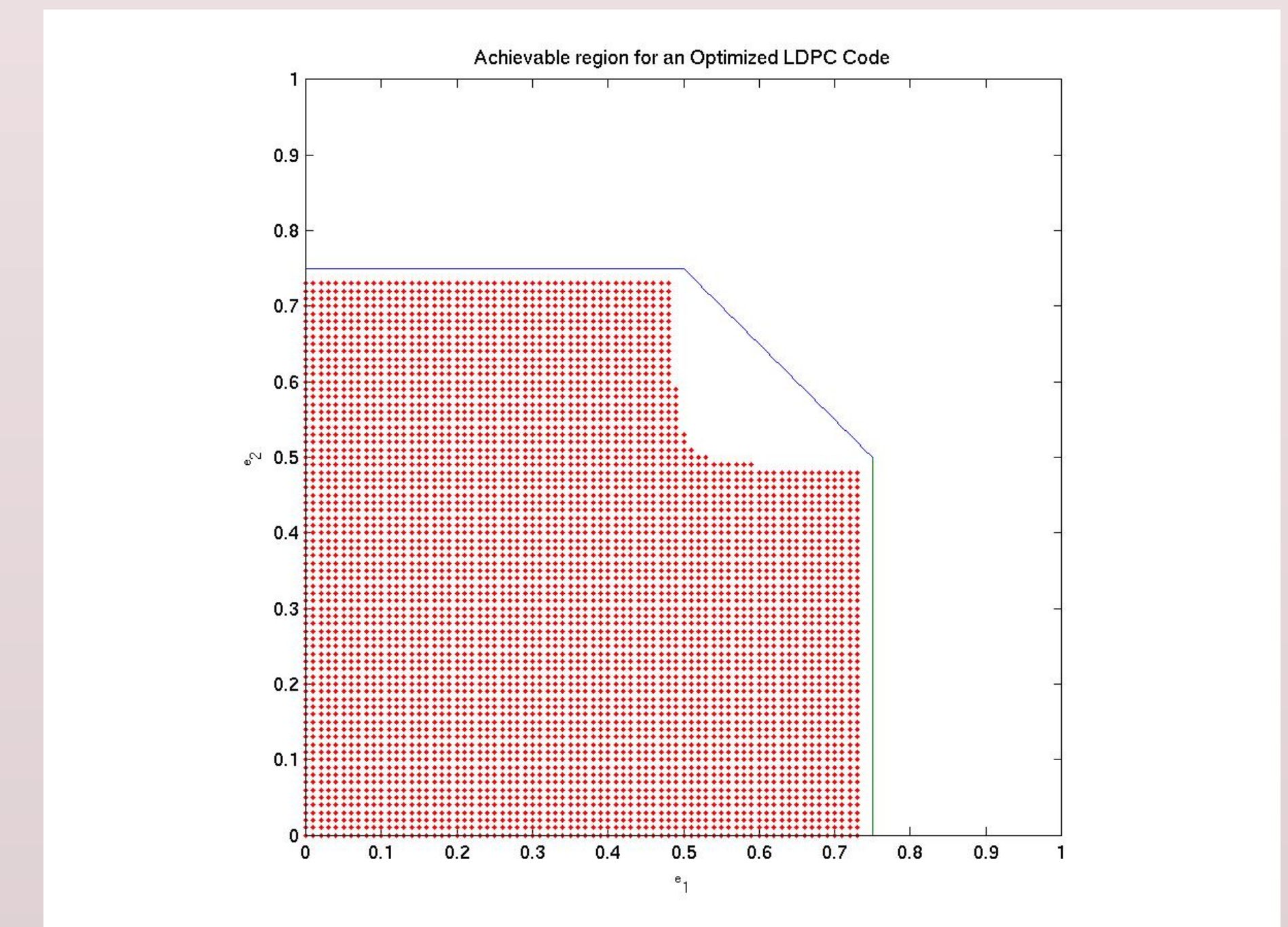


Fig. 4: ARR for an optimized LDPC Code

Conclusions and Future Work

These results essentially show that the problem in obtaining universality with the LT ensemble is essentially with the decoding algorithm rather than with code ensemble. This motivates us to find other decoding algorithms such as enhancements to message passing decoding that are nearly universal or to consider other code ensembles like protograph codes which may universally achieve capacity under iterative decoding.

References

- [1] D. S. Slepian, J. K. Wolf, Noiseless Coding of Correlated Information Sources, *IEEE Trans. Inform. Theory*, vol. 19, pp. 471-480, July 1973
- [2] M. Luby, LT Codes, *IEEE Trans. Foundations of Computer Science*, vol. 43, 2002
- [3] T. J. Richardson, A. Shokrollahi, R. Urbanke, Design of capacity-approaching low-density parity-check codes, *IEEE Trans. Inform. Theory*, vol. 47, Feb. 2001
- [4] T. J. Richardson, R. L. Urbanke, Modern Coding Theory, *Book Preprint*, Aug 2008
- [5] A. Yedla, H. D. Pfister, K. Narayanan, Can Iterative Decoding for Erasure Correlated Sources be Universal?, *submitted to Allerton '09*