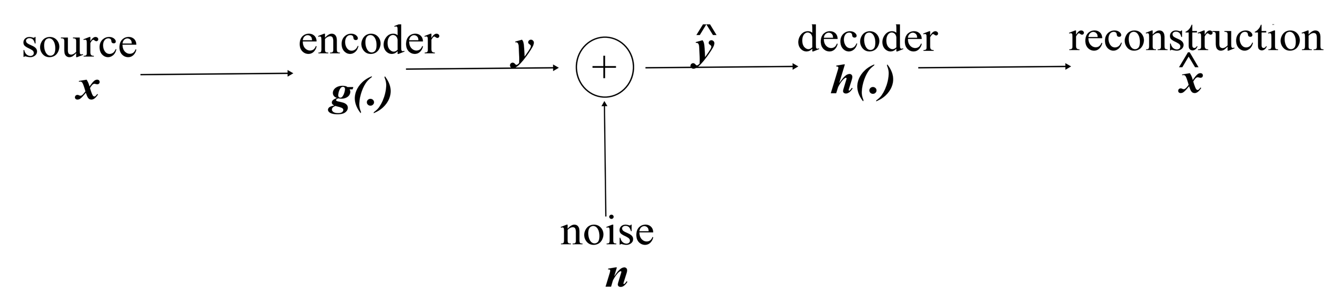
Optimal Mappings for Joint Source Channel Coding

Emrah Akyol, Kenneth Rose, Tor Ramstad

Electrical and Computer Engineering, UCSB
Electrical Engineering, Norwegian University of Science and Technology, Norway
(eakyol, rose)@ece.ucsb.edu, ramstad@iet.ntnu.no

Setup



$\mathbf{x}$: $\boldsymbol{m}$-dimensional source
$\mathbf{y}$: $\boldsymbol{k}$-dimensional channel symbol
$\mathbf{n}$: $\boldsymbol{k}$-dimensional noise, independent of $\mathbf{x}$
$g(\cdot) : \mathbb{R}^m \rightarrow \mathbb{R}^k$, $h(\cdot) : \mathbb{R}^k \rightarrow \mathbb{R}^m$.
Aim: Minimize MSE ($\mathbb{E}(|\mathbf{x} - \hat{\mathbf{x}}|^2)$) subject to average power constraint $P(\mathbf{g}) \leq P$ over $\mathbf{g}$ and $\mathbf{h}$.
$k > m \rightarrow$ bandwidth expansion
$k < m \rightarrow$ bandwidth compression

Suboptimal Example Mappings

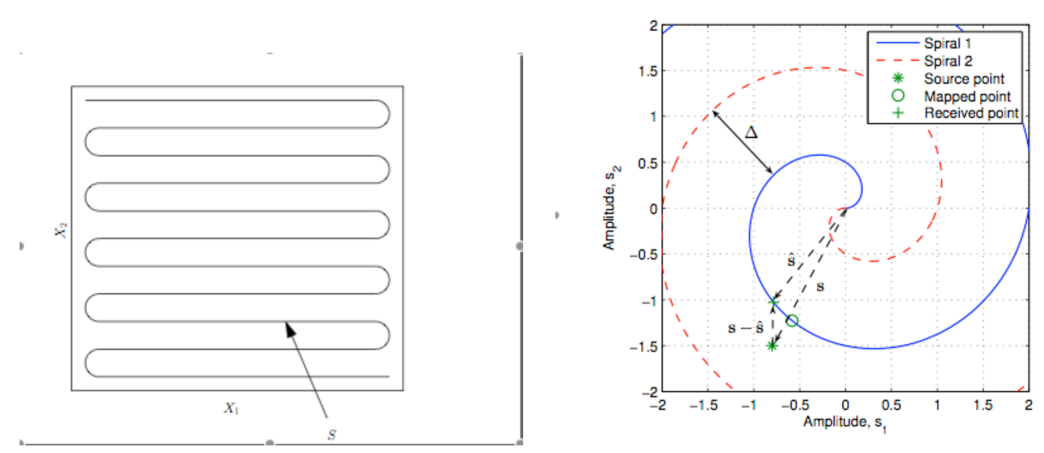


Figure: Example mappings for Gaussian source-channel

Shannon 1949, Kotelnikov 1959, Chung 2000, Hekland 2005, Floor 2009

Solution

For a fixed encoder $\mathbf{g}(\cdot)$, optimal decoder in terms of known quantities, is

$$\mathbf{h}(\hat{\mathbf{y}}) = \frac{\int \mathbf{x}\, f_X(\mathbf{x})\, f_N[\hat{\mathbf{y}} - \mathbf{g}(\mathbf{x})]\, d\mathbf{x}}{\int f_X(\mathbf{x}) f_N[\hat{\mathbf{y}} - \mathbf{g}(\mathbf{x})]\, d\mathbf{x}}$$

For a fixed decoder, the optimal encoder minimizes

$$J(\mathbf{g}) = D(\mathbf{g}) + \lambda\{\mathbf{P}(\mathbf{g}) - \mathbf{P}\}$$

over the mapping $\mathbf{g}(\cdot)$. Necessary condition: $\nabla J(g) = 0$

Main Result

For any given communication scheme to be optimal, the following conditions should be satisfied by encoder and decoder

$$\mathbf{g}(\mathbf{x}) = \frac{1}{\lambda}\int \mathbf{h}'(\mathbf{g}(\mathbf{x})+\mathbf{n})[\mathbf{x}-\mathbf{h}(\mathbf{g}(\mathbf{x})+\mathbf{n})] f_N(\mathbf{n}) d\mathbf{n}$$

$$\mathbf{h}(\hat{\mathbf{y}}) = \frac{\int \mathbf{x}\, f_X(\mathbf{x})\, f_N[\hat{\mathbf{y}} - \mathbf{g}(\mathbf{x})]\, d\mathbf{x}}{\int f_X(\mathbf{x})\, f_N[\hat{\mathbf{y}} - \mathbf{g}(\mathbf{x})]\, d\mathbf{x}}$$

where $\lambda = \left.\frac{\partial D(g)}{\partial P(g)}\right|_{P(g)=P}$

Algorithm

Iterate encoder-decoder updates till convergence
Encoder equation is not in closed form
gradient descent search :

$$\mathbf{g}_{i+1}(\mathbf{x}) = \mathbf{g}_i(\mathbf{x}) - \mu \nabla \mathbf{J}(\mathbf{g})$$

$\mathbf{g}_{\text{init}}(\cdot)$: previously proposed suboptimal mappings

Example Scalar mapping

Consider the Gaussian mixture source with distribution $f_x(x) = \frac{1}{2\sqrt{2\pi}}\{e^{\frac{-(x-2)^2}{2}} + e^{\frac{-(x+2)^2}{2}}\}$ and unit variance Gaussian noise.
As intuitively expected, the encoder and decoder pairs are close to locally linear.

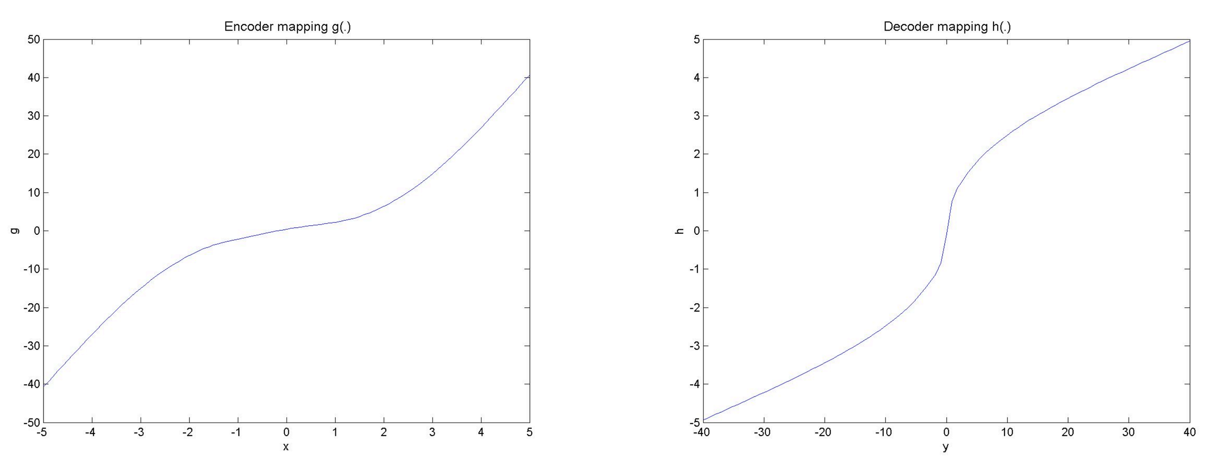


Figure: Example mappings for GMM source, Gaussian channel

Results-Scalar mapping

We compare the performance of the proposed mappings to the linear encoder and optimized decoder

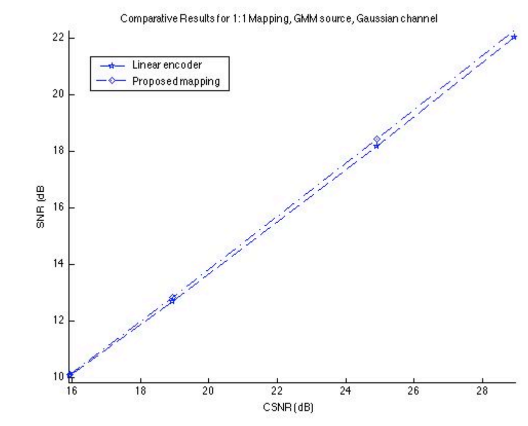


Figure: Comparative results for 1:1 (scalar) mappings, GMM source, Gaussian channel

Results 2:1 mapping

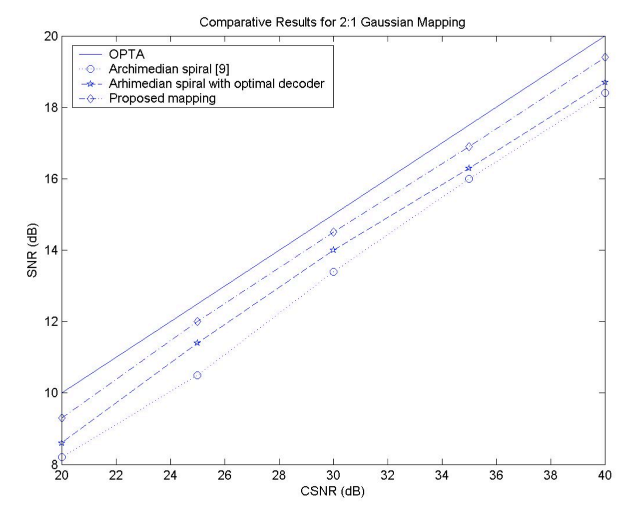


Figure: Comparative results for 2:1 mappings, Gaussian source, Gaussian channel

Results 1:2 mapping

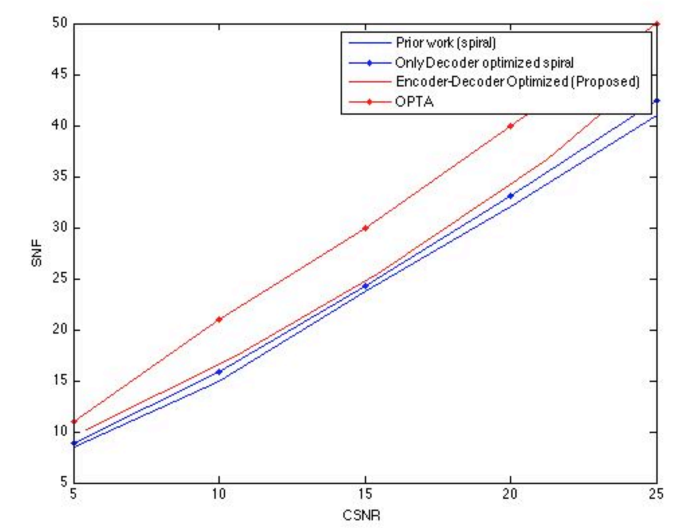


Figure: Comparative results for 1:2 mappings, Gaussian source, Gaussian channel