Energy-efficient Policies for Finite-horizon Scheduling with Causal CSIT

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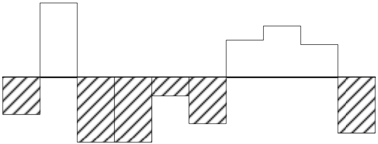
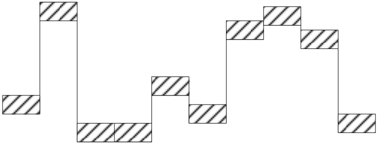
Motivation & Problem

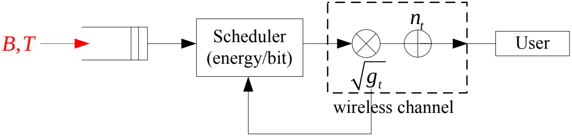
- **Resource Allocation in PHY**





- with perfect CSIT $\Longrightarrow$ waterfilling
- without CSIT $\Longrightarrow$ uniform (equal) resource allocation
  - What if CSIT is available (sequentially) causally?

- **Problem Description**



- No other packet
- Hard deadline

- Objective: Find scheduler that minimizes transmitted (expected) energy

Regular Packet Arrivals and Hard Deadlines

- Transmission of a single packet
  - Fu, Modiano, and Tsitsiklis (2006)
    - Considered the same problem and formulated as a dynamic program.
    - Obtained a closed-form solution when
      - Energy cost is a linear function of bits.
      - Channel state g_t is restricted to integer multiples of some constant.
  - Zafer and Modiano (2007)
    - Considered a continuous-time scheduling (not slotted time).
    - Formulated with a general cost function.
      - Provided an ODE solution for monomial cost function.
    - Provided a closed-form solution for a constant drift channel.
  - Negi and Cioffi (2002)
    - Considered the dual problem: maximize rate in finite # slots with finite amount of power

- Our work
  - Energy is an exponential function of bits (Gaussian channel)
  - Develop low-complexity, asymptotically-optimal scheduling policies.

Problem Formulation

- t: time index in descending order (represents # of remaining slots)
  - β_t: unserved bits (queue size)
  - g_t: channel state (in power unit)
    - iid across slots
    - perfect causal CSIT: at t, g_t known but $g_{t-1}, \cdots, g_1$ unknown
  - b_t: allocated bits ($0 \leq b_t \leq \beta_t$, $b_t \in \mathbb{R}$) $\leftarrow$ scheduling policy

- AWGN channel & transmission at capacity: $y_t = \sqrt{g_t} x_t + n_t$

$$b_t = \log(1 + g_t E_t) \quad \Longrightarrow \quad \boxed{E_t(b_t, g_t) = \frac{e^{b_t} - 1}{g_t}}$$

- Hard deadline constraint (no outage is allowed): i.e., $b_1 = \beta_1$.

- Problem: find scheduler that minimizes expected energy

$$\{b_t\}_{t=1}^{T} = \arg \min_{b_T, \cdots, b_1} \mathbb{E}\left[\sum_{t=1}^{T} E_t(b_t, g_t)\right]$$

Optimal Scheduler

- **Optimal Scheduler for $T = 1$ and $T = 2$**
  - $t = 1$: $b_1^{\text{opt}}(\beta_1, g_1) = \beta_1 \implies \bar{J}_1(\beta_1) = \left(e^{\beta_1} - 1\right) \mathbb{E}\left[\frac{1}{g_1}\right]$
  - $t = 2$: only know g_2 (causal CSI)

$$\min_{0 \le b_2 \le \beta_2} \left\{ \underbrace{\frac{e^{b_2} - 1}{g_2}}_{\text{current}} + \underbrace{\bar{J}_1(\beta_2 - b_2)}_{\text{future (assuming optimal used)}} \right\}$$

  - By taking derivative and set to zero, the optimal solution is given by

$$b_2^{\text{opt}} = \left\langle \frac{1}{2}\beta_2 + \frac{1}{2}\log\left(\frac{g_2}{\eta_2}\right) \right\rangle_0^{\beta_2}, \quad \eta_2 = \frac{1}{\mathbb{E}\left[\frac{1}{g_1}\right]}$$

- **Optimal Scheduler for $T > 2$**
  - At $t = 3$, need to solve $\min_{0 \le b_3 \le \beta_3} \left\{ \frac{e^{b_3} - 1}{g_3} + \bar{J}_2(\beta_3 - b_3) \right\}$
  - Solve by setting $\frac{d}{db_3}\left(\frac{e^{b_3} - 1}{g_3} + \bar{J}_2(\beta_3 - b_3)\right) = 0$
  - But, the cost-to-go function $\bar{J}_2$ is too complex.
  - Should rely on numeral method (which gives little insight)

Boundary-relaxed Scheduler

- The optimal scheduler is obtained by solving

$$b_t^{\text{opt}}(\beta_t, g_t) = \arg \min_{\color{red}{0 \le b_t \le \beta_t}} \left(\underbrace{\frac{e^{b_t} - 1}{g_t}}_{\text{current energy cost}} + \underbrace{\bar{J}_{t-1}^{\text{opt}}(\beta_t - b_t)}_{\text{expected future cost}} \right).$$

- **Idea:** Relax the constraint "$0 \le b_t \le \beta_t$"
  - Cost-to-go function takes on simple form for all t
- Optimal solution:

$$b_t^{\text{relax}} = \left\langle \frac{1}{t}\beta_t + \frac{t-1}{t} \log\left(\frac{g_t}{\eta_t^{\text{relax}}}\right) \right\rangle_0^{\beta_t}, \qquad \eta_t^{\text{relax}} = \frac{1}{(\nu_{t-1}\nu_{t-2}\cdots\nu_1)^{\frac{1}{t-1}}}.$$

- (cf) Fractional moments : a measure $\|\cdot\|_{\frac{1}{m}}$

$$\nu_m = \left(\mathbb{E}\left[\left(\frac{1}{g}\right)^{\frac{1}{m}}\right]\right)^m, \qquad m = 1, 2, \ldots$$

Remark on the Structure

- Opportunistic Communication vs. Delay-limited Communication

$$b_t(\beta_t, g_t) = \underbrace{\frac{1}{t}}_{\text{weight}} \overbrace{\beta_t}^{\text{remaining bits (1)}} + \underbrace{\frac{t-1}{t}}_{\text{weight}} \underbrace{\log \frac{g_t}{\eta_t}}_{(2)}$$

  - When t is large, (2) is important: when the deadline is far away, be very opportunistic.
  - When t is small (near deadline), (1) becomes important: when the deadline approaches, want to be opportunistic but have to be aware of the approaching deadline.

- Waterfilling Interpretation

	At each t, perform IWF over	Remark
Equal-bit	$g_t, \underbrace{g_t, g_t, \cdots, g_t}_{t-1}$	no CSIT
Boundary-relaxed	$g_t, \frac{1}{\nu_{t-1}}, \frac{1}{\nu_{t-2}}, \cdots, \frac{1}{\nu_1}$	causal CSIT
Non-causal IWF	$g_t, g_{t-1}, g_{t-2}, \cdots, g_1$	perfect CSIT

One-Shot Threshold Scheduler

- The optimal scheduler is obtained by solving ($t \geq 2$)

$$b_t^{\mathsf{opt}}(\beta_t, g_t) = \arg \min_{0 \leq b_t \leq \beta_t} \left(\underbrace{\frac{e^{b_t} - 1}{g_t}}_{\text{current energy cost}} + \underbrace{\bar{J}_{t-1}^{\mathsf{opt}}(\beta_t - b_t)}_{\text{expected future cost}} \right).$$

- **Idea:** Replace $0 \leq b_t \leq \beta_t$ with $b_t \in \{0, \beta_t\}$. Then, equivalently

$$J_t^{\mathsf{one}}(B, g_t) = \min \left\{ \frac{e^B - 1}{g_t}, \ \bar{J}_{t-1}^{\mathsf{one}}(B) \right\}, \quad t \geq 2$$

$\Longrightarrow$ Optimal stopping problem

- May not want to split bit allocation across multiple time slots.
- In this setting, serve entire packet in one slot.

- Solution is a threshold policy: transmit ($b_t = B$) once $g_t > \frac{1}{\omega_t}$

$$\omega_t = \mathbb{E}\left[\frac{1}{g} \middle| \frac{1}{g} < \omega_{t-1} \right] \Pr\left\{ \frac{1}{g} < \omega_{t-1} \right\} + \omega_{t-1} \Pr\left\{ \frac{1}{g} \geq \omega_{t-1} \right\}$$

- Thresholds $1/\omega_t$ increasing (w.r.t. t) and independent of B.

Delay-constrained Ergodic Scheduler

- **Ergodic Scheduler:** constrained with average number of bits ($\bar{b}$)

$$\bar{E}^{\text{erg}}(\bar{b}) = \min_{b(g)} \quad \mathbb{E}_g\left[\frac{e^{b(g)}-1}{g}\right]$$
$$\text{subject to} \quad \mathbb{E}_g[b(g)] \geq \bar{b}, \quad b(g) \geq 0$$

  - The optimal solution is given by WF:

$$b^{\text{erg}}(\bar{b}, g) = \left\langle \log\left(\frac{g}{\eta^{\text{erg}}}\right)\right\rangle_0^{\infty} = \begin{cases} \log\left(\frac{g}{\eta^{\text{erg}}}\right), & g \geq \eta^{\text{erg}}, \\ 0, & \text{else}, \end{cases}$$

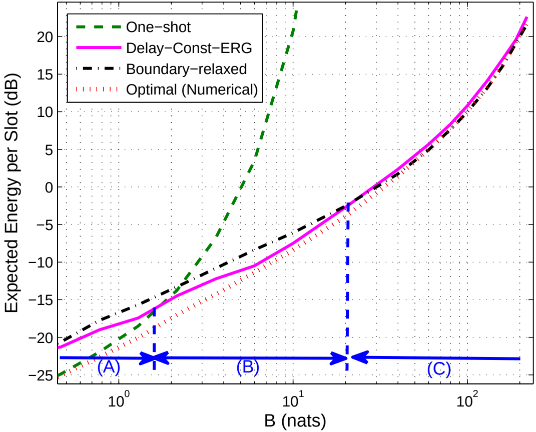
$\Longrightarrow$ when T large, the ergodic scheduler is expected to perform well

- **Delay-constrained Ergodic Scheduler**
  - Use ergodic scheduler except in last slot ($t = 1$)

$$b_t^{\text{constrained-erg}}\left(\frac{B}{T}, g_t; \delta\right) = \begin{cases} b^{\text{erg}}\left(\frac{B}{T}+\delta, g_t\right), & t = T, T-1, \cdots, 2, \\ \beta_1, & t = 1, \end{cases}$$

for some $\delta > 0$

Simulation: Per Slot Energy Consumption for $T = 50$



- **Observation:**
  - Region A : one-shot scheduler outperforms
  - Region B : delay-constrained ergodic scheduler outperforms
  - Region C : boundary-relaxed scheduler outperforms

Large B and Finite T

Asymptotic Optimality of Boundary-relaxed Scheduler

Theorem (Policy Convergence)

Let the PDF f of g_t be continuous on a support of $[g_{\min}, g_{\max}]$ with $g_{\min} > 0$ and $g_{\max} < \infty$. Then,

$$b_t^{relax}(\cdot, g_t) \to b_t^{opt}(\cdot, g_t) \quad \textit{uniformly on} \;\; [g_{\min}, g_{\max}],$$

as $\beta \to \infty$ for $t = 1, 2, \cdots, T$.

Theorem (Cost Convergence)

Let the PDF f of g_t be continuous on a support of $[g_{\min}, g_{\max}]$ with $g_{\min} > 0$ and $g_{\max} < \infty$. Then,

$$\lim_{B \to \infty} \left[\bar{J}_T^{relax}(B) - \bar{J}_T^{opt}(B) \right] = 0.$$

for any T.

Small B and Finite T

Asymptotic Optimality of One-shot Scheduler

Theorem (Policy Convergence)

$$\lim_{\beta \to 0} \sup\{g : b_t^{opt}(\beta, g) = 0\} = \lim_{\beta \to 0} \inf\{g : b_t^{opt}(\beta, g)\} = \frac{1}{\omega_t}$$

- **Implication**
  - The optimal policy becomes a threshold policy as $\beta \to 0$.
  - The threshold value converges to the threshold of the one-shot scheduler.

Theorem (Cost Convergence)

$$\lim_{B \to 0} \frac{\bar{J}_T^{one}(B)}{\bar{J}_T^{opt}(B)} = 1.$$

- **Intuition**
  - Cost function $e^b - 1$ roughly linear for small b

Large T

Asymptotic Optimality of the Delay-constrained Ergodic Scheduler

Theorem (Per-Slot Cost Convergence)

$$\lim_{T\to\infty} \frac{1}{T} \bar{J}_T^{\textit{constrained-erg}}(\bar{b}T) = \lim_{T\to\infty} \frac{1}{T} \bar{J}_T^{\textit{opt}}(\bar{b}T) = \bar{E}^{\textit{erg}}(\bar{b}).$$

- Average of $\bar{b} = \frac{B}{T}$ bits/slot
- When T large, the effect of the hard-deadline becomes inconsequential
  - the channel realizations over the deadline horizon closely match the fading distribution.

Conclusion

- Provided a framework for delay-constrained scheduling
  - balance opportunistic communication with delay-limited communication
    - large t: wait for a good channel
    - small t: less opportunistic

- Found asymptotically optimality schedulers for Shannon energy-cost
  - Large B and finite T: Boundary-relaxed scheduler
  - Small B and fintie T: One-shot scheduler
  - Large T: Delay-constrained ergodic scheduler

- Quantified scheduling gain

$$\lim_{B\to\infty} \Delta_T^{\mathsf{opt}}(B) = \lim_{B\to\infty} \frac{\bar{J}_T^{\mathsf{eq}}(B)}{\bar{J}_T^{\mathsf{relax}}(B)} = \frac{\nu_1}{\mathbb{G}(\nu_T, \nu_{T-1}, \cdots, \nu_1)}$$

$$\lim_{B\to 0} \Delta_T^{\mathsf{opt}}(B) = \lim_{B\to 0} \frac{\bar{J}_T^{\mathsf{eq}}(B)}{\bar{J}_T^{\mathsf{one}}(B)} = \frac{\nu_1}{\omega_{T+1}}$$