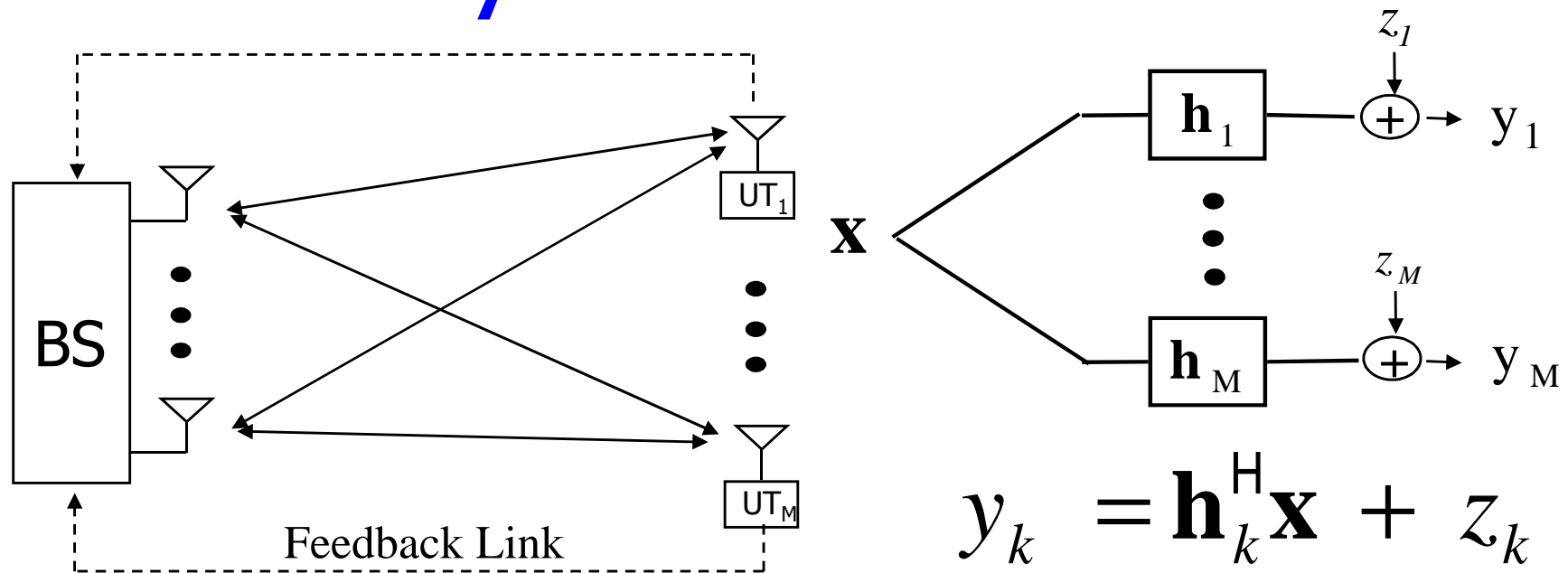


MIMO Downlink Systems with Limited Feedback of Channel Information

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System Model



- M antennas at Base Station (BS)
- 1 antenna per User Terminal (UT)
- \mathbf{h}_k is the $M \times 1$ channel to UT k , Rayleigh block fading
- AWGN z_k with unit variance, transmit power constraint P
- Downlink transmission from BS to UTs.

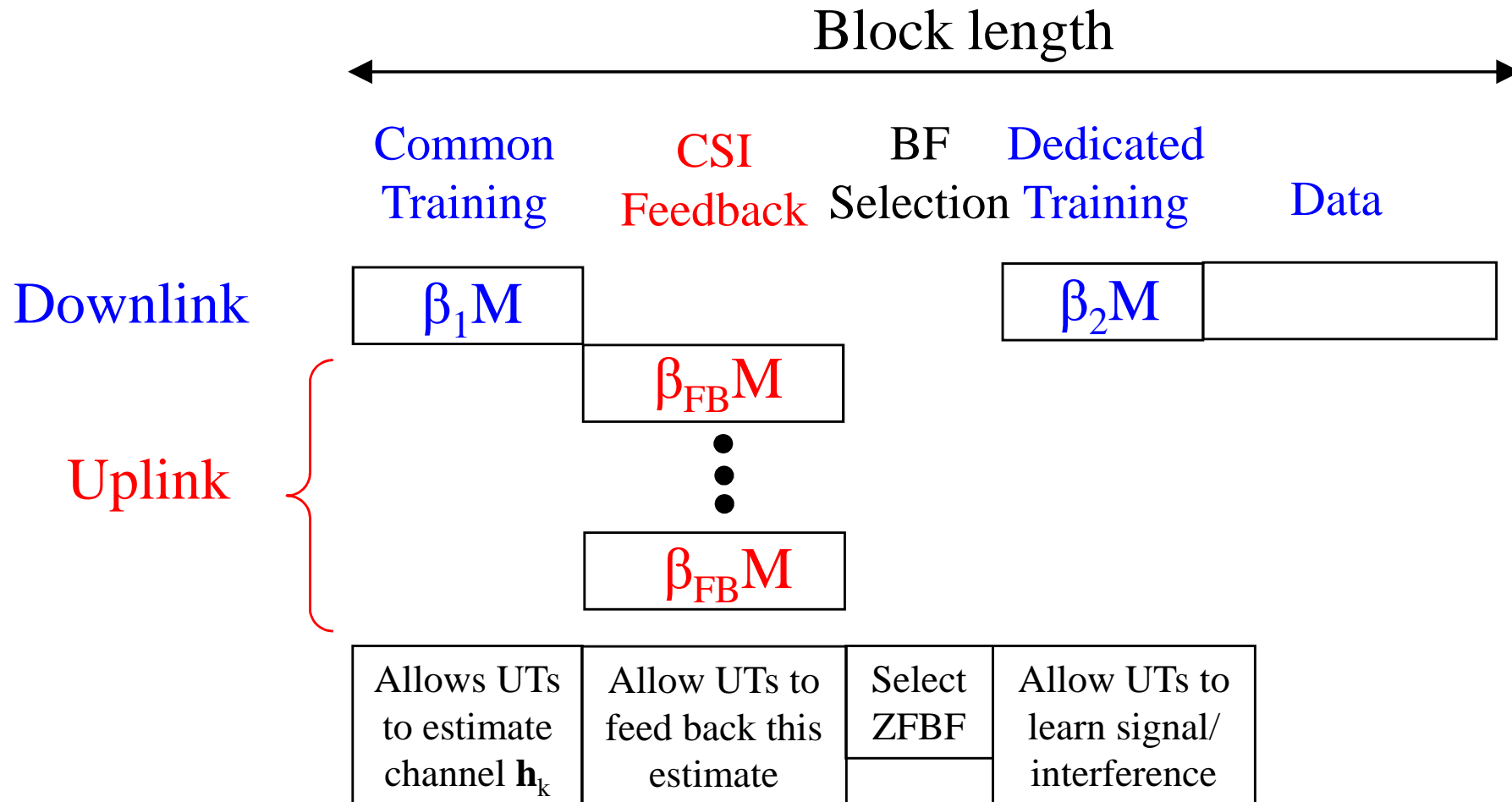
MIMO Downlink Capacity

- Downlink (broadcast channel) sum capacity:
 - With channel state information (CSI) at BS [Caire, Shamai, ...] $\sim M \log P$
 - Without CSI at BS (TDMA optimal) $\sim \log P$
- **Large sum capacity gap** motivates *noisy* or *quantized* CSI feedback from UTs to BS
- **Challenges:**
 - Practical, near-capacity dirty paper codes not yet known
 - What happens to the achievable rate when:
 - CSI is quantized
 - CSI is imperfectly known at the UT
 - There are errors on the feedback channel
 - The feedback is delayed
 - How do you allocate feedback resources for maximum rate?

Zero Forcing

- Example low complexity scheme: Zero Forcing (ZF)
 - Achieves rate $\sim M \log P$ with perfect CSI [Yoo, Goldsmith]
- Set $\mathbf{x} = \sum_{k=1}^M \mathbf{v}_k u_k$. Choose beamformers (BF): for $j \neq k$, $\mathbf{h}_j^H \mathbf{v}_k = 0$
 - No multi-user interference
- Imperfect channels at UT or BS \rightarrow residual multi-user interference
 - Can we still preserve multiplexing gain?

Channel Training & Feedback Model



Feedback methods (1)

- **Analog Feedback:**

- UTs transmits channel coefficients directly over feedback link
- Rate gap (relative to perfect CSI) **bounded** as $\text{SNR} \rightarrow \infty$
(Full multiplexing gain is achieved)

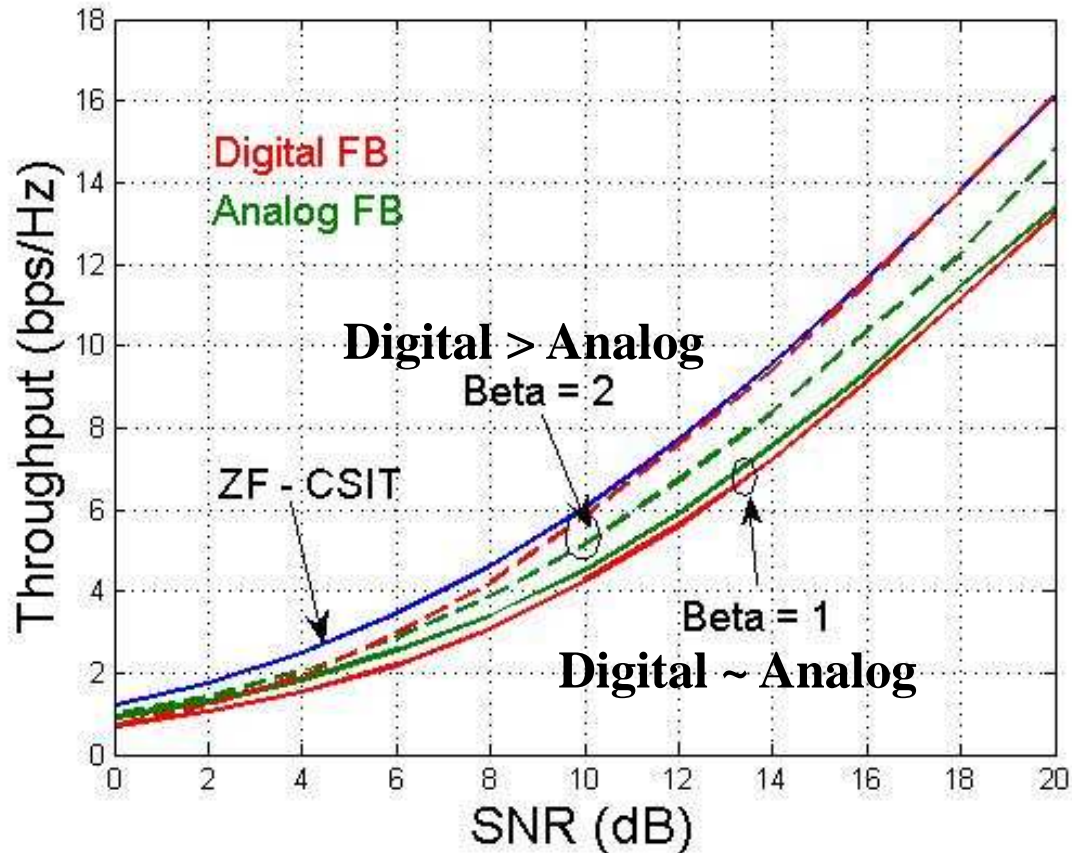
$$\Delta R_{\text{analog}} \leq \log_2 \left(1 + \frac{1}{\beta_1} + \frac{1}{\beta_{FB}} + \frac{1}{\beta_2 M} \right)$$

- **Digital Feedback:**

- UTs quantize channel 'direction' to B bits
- Random quantization codebooks used for analysis
- Assume maximum capacity of feedback link $B = \beta_{FB} M \log_2(P)$
- **Superior** to analog feedback for $\beta_{FB} > 1$
 - Holds even when considering **feedback errors**

$$\Delta R_{\text{digital}} \leq \log_2 \left(1 + \frac{1}{\beta_1} + P 2^{-\frac{B}{M}} + \frac{1}{\beta_2 M} \right) = \log_2 \left(1 + \frac{1}{\beta_1} + P^{1-\beta} + \frac{1}{\beta_2 M} \right)$$

Feedback methods (2)



- M=4 System
- For $\beta_{FB} > 1$, rate gap for digital feedback goes to zero with SNR P.
- For $\beta_{FB} = 1$, rate gap does not go to zero, but is bounded.

Effect of Feedback Delay (1)

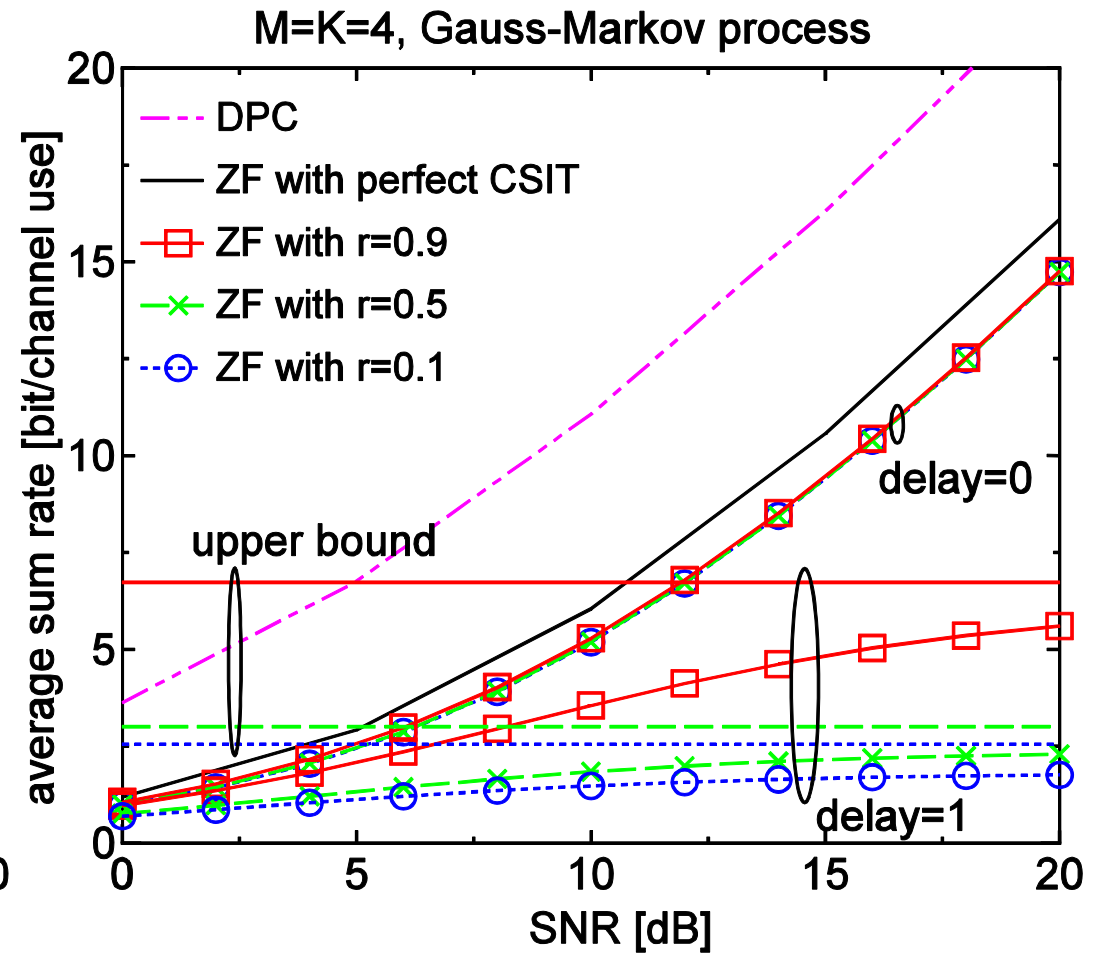
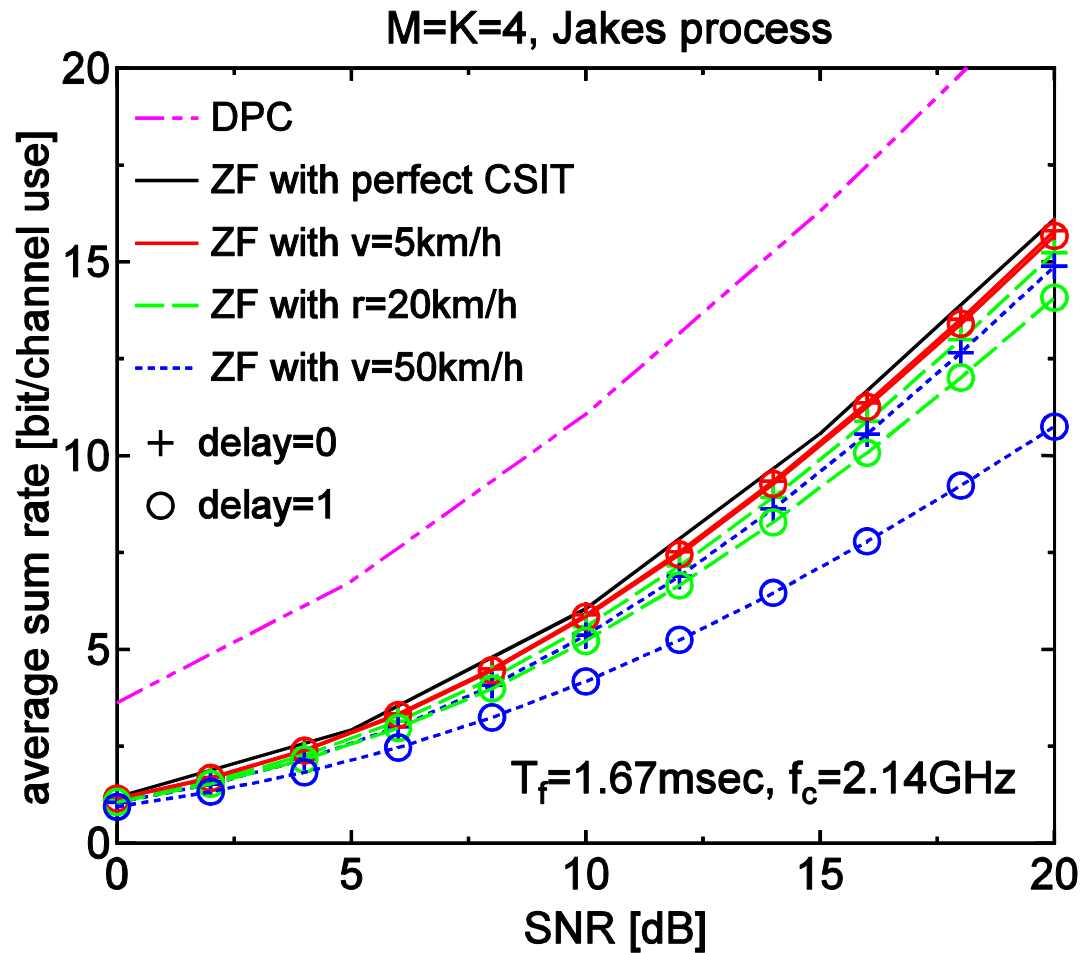
- **1-block delay** in feedback
- **Fading process band-limited** in $[-F, F]$ for $F < 1/2$
 - “Predict” future CSI at UT based on past observations
 - Achieves multiplexing gain of $(1-2F)M$ rather than M
 - Egs.: UT velocity = 50km/h, Carrier freq. = 2 GHz,
Block length = 1ms, $M = 4$, $M(1-2F) = 3.26$ (instead of 4)

- **Regular processes** (AR-1)

$$h(t + 1) = r \cdot h(t) + \sqrt{1 - r^2} w(t + 1)$$

- **Throughput bounded** (zero multiplexing gain)
- Effect of feedback delay can be **critical!**
 - Need to “predict” future CSI to regain multiplexing gain

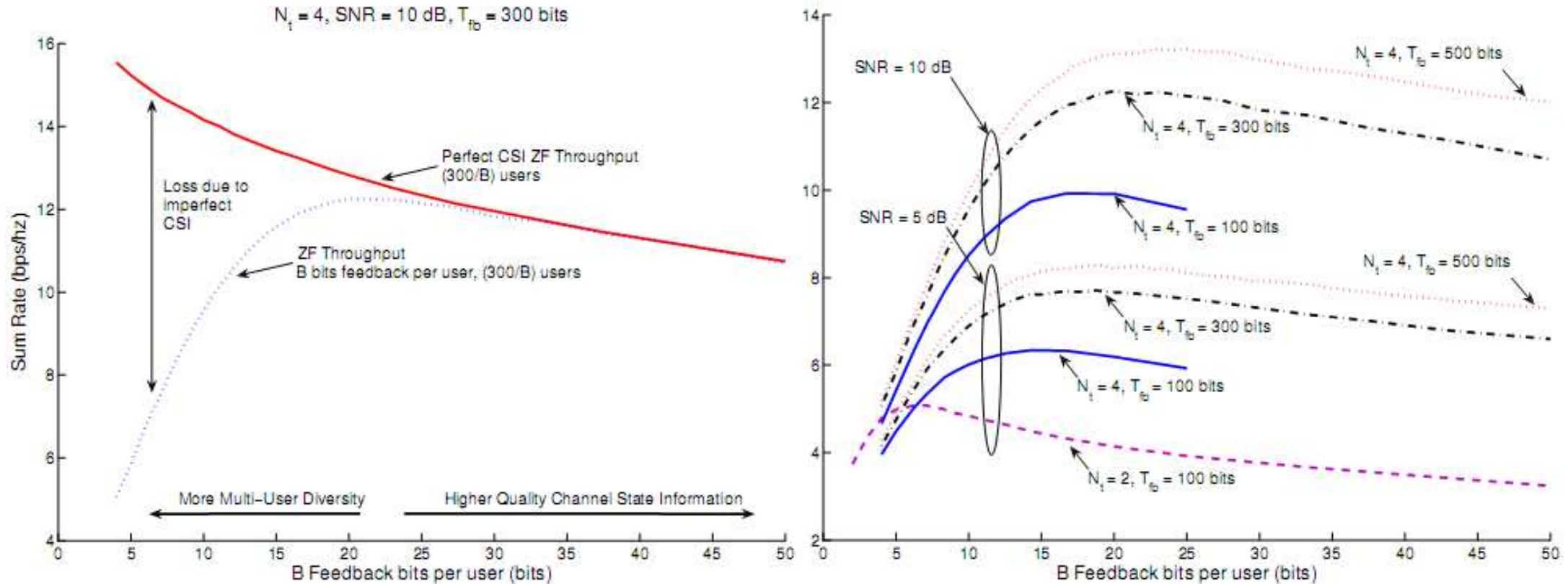
Effect of Feedback Delay (2)



Limited feedback with many UTs

- *ZF with selection* : With many UTs ($> M$), BS selects up to M “best” UTs for transmission (Greedy selection [[Dimic-Sidiropoulos](#)])
 - First select UT with largest channel norm/SINR
 - Next select UT with largest projection on nullspace of 1st user...
- **Multi-user diversity** (more UTs): BS can select UTs with
 - large channel magnitude
 - near-orthogonal channel directions, small quantization error
- **Accurate CSI** (more feedback bits per UT):
 - Reduce multi-user interference due to imperfect CSI
- Fix **total feedback budget** (T bits)
 - Randomly select (T/B) UTs who feed back B bits each
- **Fundamental design question:** Should
 - *More users feed back fewer bits* (more multiuser diversity), or
 - *Less users feed back more bits* (better CSIT) ?

MUD vs CSI



- Can show strong preference for accurate CSI
- Fix T: ZF with optimized feedback load > low-rate feedback methods (PU²RC, Sharif-Hassibi Random beamforming)

Conclusions

- Quality of CSI **critical** for MIMO downlink
- Multiplexing gain *can* be preserved with intelligent design of feedback
 - Imperfect CSI at UT → non-negligible but bounded rate loss
 - Quantization of CSI (digital feedback) > Analog feedback
 - Multiplexing gain preserved even with feedback errors
- Multiplexing gain may not be preserved with feedback **delay**
 - Need to exploit time correlation to achieve any gain
- Fix feedback budget (T bits):
 - First use budget to obtain **accurate CSI**
 - Then allow more users to feedback for **multi-user diversity**