On Maximizing Mutual Information over Parallel Gaussian Channels with Discrete Input Signaling

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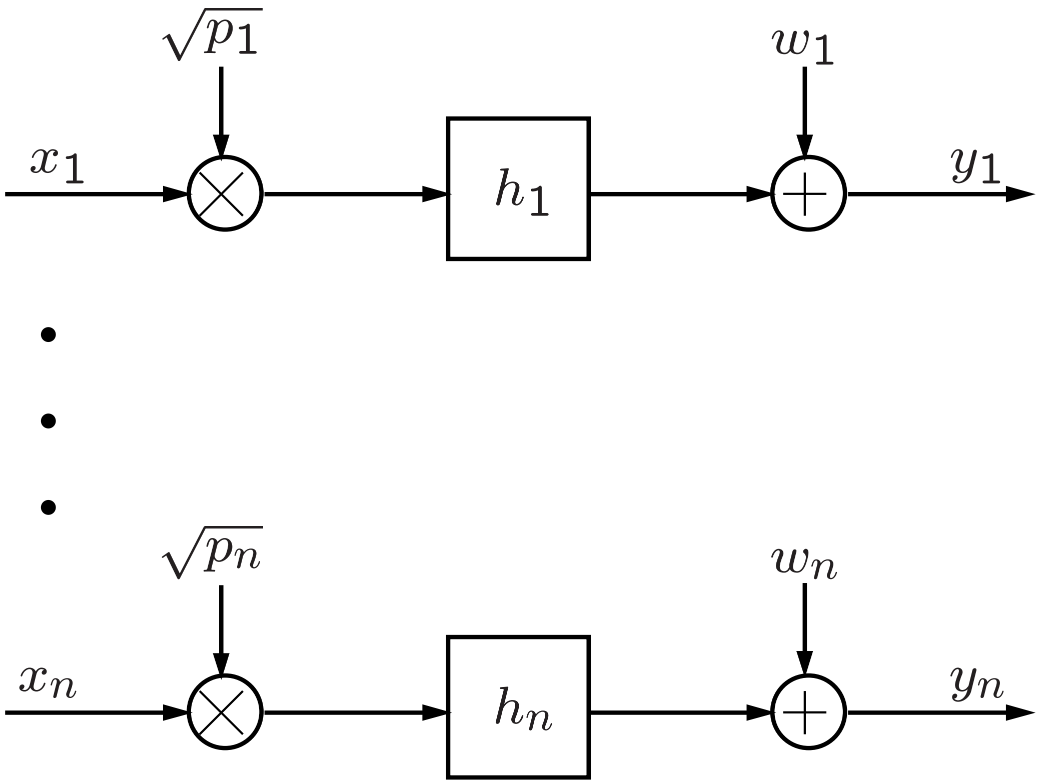
INTRODUCTION

- Often we encounter bank of parallel channels
  - ∗ Multi-carrier transmission : DMT and OFDM
  - ∗ Flat fading channels with power allocation over time
  - ∗ Multi-antenna communications : Diagonalized MIMO channels
- Such channels can be modeled as parallel Gaussian channels
  - ∗ non-interfering channels impaired by independent Gaussian noise
- Spectral efficiency achievable with arbitrary reliability is determined by sum Mutual Information

Parallel Gaussian Channels



- $\{w_i\}$ are mutually independent and $w_i \sim \mathcal{CN}(0, \sigma^2)$
- With CSIT, under an average power constraint $\sum_i p_i \leq P$,

  sum MI $= \sum_i I(y_i; x_i)$ is maximized through optimal power allocation

Power Allocation

- For Gaussian inputs waterfilling maximizes sum MI
- In practice, we use uniformly distributed discrete input symbols
- Mercury/Waterfilling[1]
  - ∗ Optimal power allocation for given arbitrary input constellations
- In high-power regime, with same constellation on all the channels, MWF equalizes received signal powers on all the channels

1. Lozano *et al.*, "Optimum power allocation for parallel Gaussian channels with arbitrary input distributions", *IEEE IT Trans.*, July 2006.

MAXIMIZING MI THROUGH PRECODING (1)

- Coordinate interleaving across parallel channels[2] was shown to achieve higher MI than MWF in high-power regime

  * Coordinates of input symbols, chosen from rotated n-dimensional QAM constellations, are interleaved such that each coordinate goes through a different channel

  * Simple to implement and no need of CSIT

  * Can be viewed as precoding the transmit vector with a rotation matrix

  * Results are based on simulation studies

2. K. V. Srinivas *et al.*, "Coordinate Interleaving across Parallel Gaussian Channels", Proc. CISS 2007.

MAXIMIZING MI THROUGH PRECODING (2)

- Perez-Cruz *et al.*[3] proposed an optimal linear precoder maximizing sum MI
  - Outperforms MWF
  - Shown that, in high-power regime, maximizing minimum distance in the received signal constellation (d_{min}) maximizes MI
  - Need to solve a fixed point equation to determine the precoder
- Motivated by higher MI achieved by coordinate interleaving without CSIT and the optimal linear precoder we investigate precoding for maximizing sum MI in high power regime without CSIT.

3. Perez-Cruz *et al.*, “Optimal Precoding for Digital Subscriber Lines”, Proc. ICC 2008.

Maximizing Sum MI in High-Power Regime

- In MWF, $d_{\text{min}}^{\text{mwf}}$ (and hence, sum MI) is determined by Harmonic mean of channel gains

$$d_{\text{min}}^{\text{mwf}} = \sqrt{\frac{n}{\sum_i \frac{1}{|h_i|^2}}}\, d_{\text{min}}^{\text{tx}}$$

where $d_{\text{min}}^{\text{tx}}$ is minimum distance in transmitted signal constellation

- As $\frac{n}{\sum_i \frac{1}{|h_i|^2}} < n|h_{\text{min}}|^2$,

$$d_{\text{min}}^{\text{mwf}} < \sqrt{n|h_{\text{min}}|^2} d_{\text{min}}^{\text{tx}}$$

Unitary Precoding for Maximizing Sum MI in High-Power Regime

- Precoding transmit symbol vector with a scaled Hadamard matrix results in

$$d_{\text{min}}^{\text{up}} = \sqrt{n|h_{\text{min}}|^2}\, d_{\text{min}}^{\text{tx}}$$

- When $n = 2$,

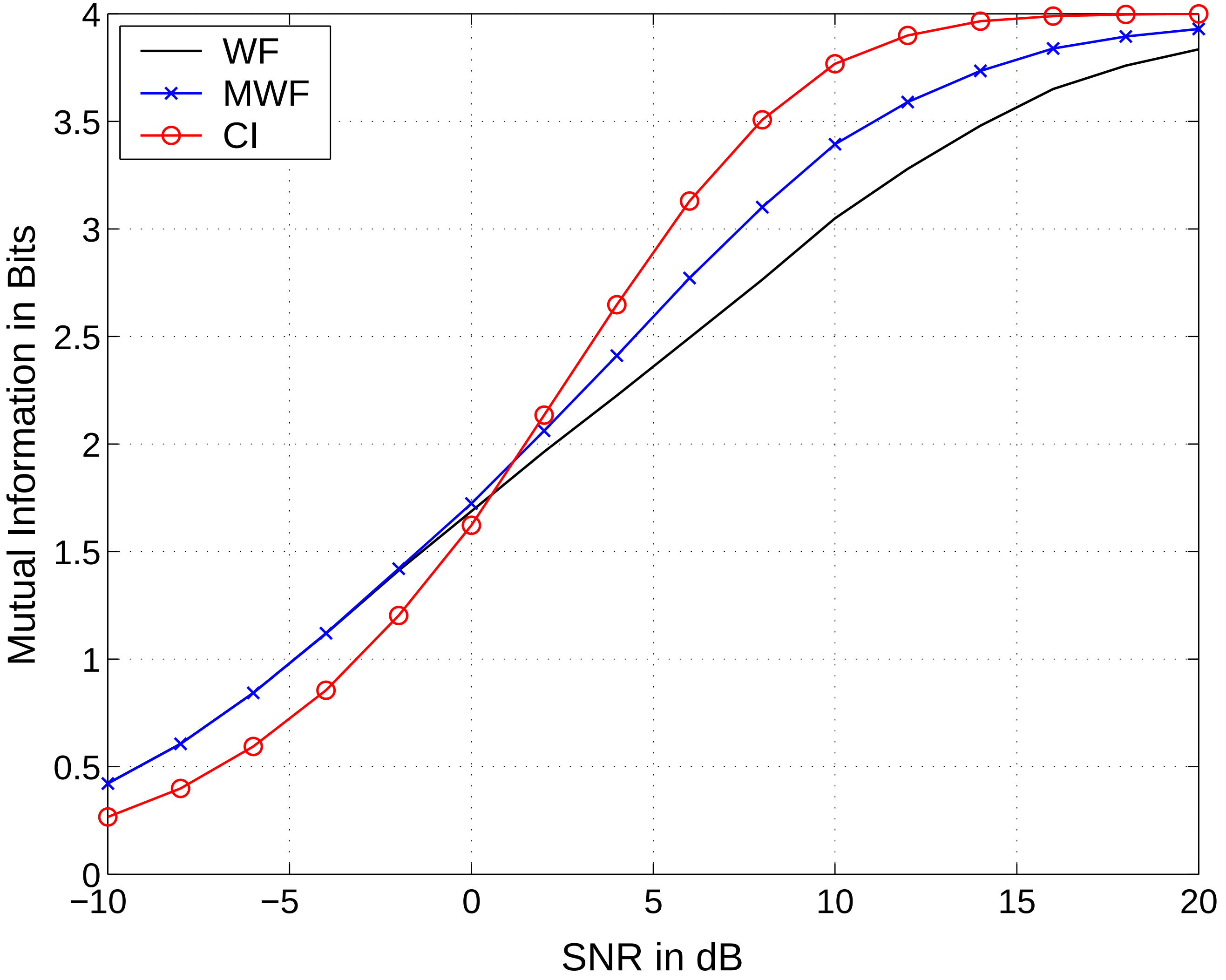
$$\frac{d_{\text{min}}^{\text{up}}}{d_{\text{min}}^{\text{mwf}}} = \frac{|h_{\text{max}}|^2 + |h_{\text{min}}|^2}{|h_{\text{max}}|^2} > 1$$

- In general, for any value of n, $d_{\text{min}}^{\text{up}} \geq d_{\text{min}}^{\text{mwf}}$

Unitary Precoding (contd..)

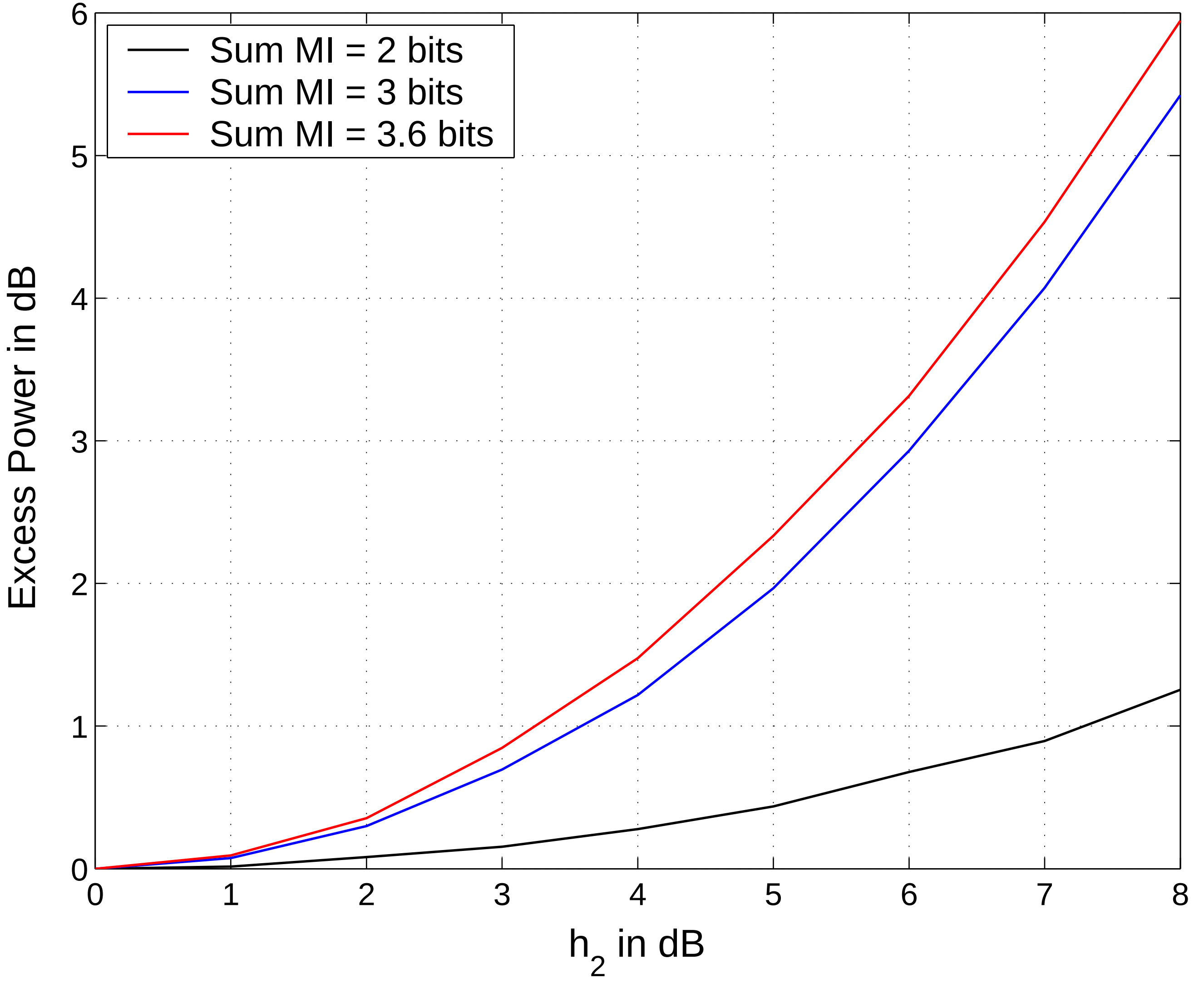
- Requires no CSIT
- Simple to implement
- Limited to even number of channels
- Would be inferior to optimal linear precoder of [3]

SUM MI OF DIAGONALIZED 2X2 MIMO CHANNEL



- $h_1 = \lambda_1, h_2 = \lambda_2$ where λ_1 and λ_2 are eigenvalues of MIMO channel matrix

Excess power required by MWF compared to UP



- h_1 is fixed at 0 dB and 4-QAM is used on both the channels

Summary and Future Work

- Considered maximizing sum MI across parallel Gaussian channels in the high power regime without CSIT

- Shown that, precoding with Hadamard matrix achieves better MI than the optimal Mercury/Waterfilling power allocation

- The proposed unitary precoding is simple to implement but is limited to even number of channels

- Currently working on

  * How does UP perform when compared with MI optimal linear precoder that requires CSIT

  * The precoder when n is an odd integer

THANK YOU !