Impact of Network Topology Knowledge on Fairness: A Geometric Approach

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Motivation

- Throughput-fairness is a commonly desired objective in network resource allocation.
- Fair allocations often depend on the topology (state) of the network.
- Wireless topology is difficult to know precisely/accurately.

Question: How much fairness can be lost when there are errors in network topology knowledge?

Outline of Approach

1. Model topology-dependent resource allocation problem as a knowledge-dependent mapping from "topology" space to throughput space.
2. Determine geometric properties of fairness measure.
3. Specify a geometric error model.
4. Analyze.

1. Network Model: Scheduling in Multiuser Uplink

- Scheduled multiple-access/single-clique
- Fixed code rate vector $\mathbf{r}$
- Infinite backlog

Topology fully described by service rate vector: $\mathbf{q}$
Topology knowledge (service rate vector estimate): $\hat{\mathbf{q}}$

Fair Allocation:
Find $\mathbf{t}^\star$ such that $\mathbf{1}^\top \mathbf{t}^\star = 1$ and $t_i^\star \hat{q}_i r_i = k$ for all i and some $k > 0$, i.e.

$$t_i^\star(\hat{\mathbf{q}}, \mathbf{r}) = \left(\hat{q}_i r_i \sum_{j=1}^{N} \frac{1}{\hat{q}_j r_j} \right)^{-1}.$$

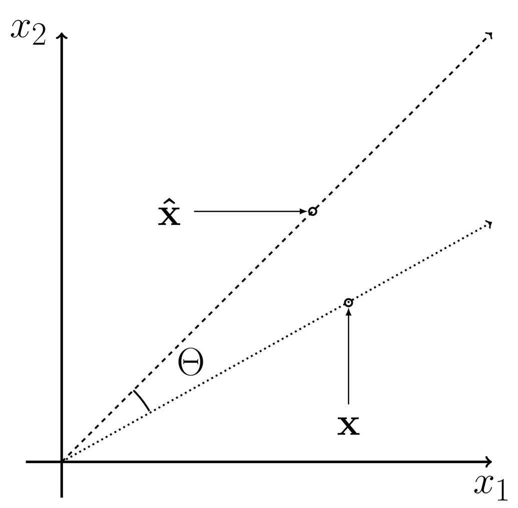
Estimated Throughput Vector: $\hat{\mathbf{x}} = k\mathbf{1}$

Realized Throughput Vector: $\mathbf{x} = \begin{bmatrix} t_1^\star q_1 r_1 \\ \vdots \\ t_N^\star q_N r_N \end{bmatrix}$

2. Geometry of Fairness Measure

Jain's Index:

$$J = \left(\sum_{i=1}^{N} x_i \right)^2 \Big/ \left(N \sum_{i=1}^{N} x_i^2 \right) = \left\langle \frac{\mathbf{x}}{\|\mathbf{x}\|}, \frac{\mathbf{1}}{\|\mathbf{1}\|} \right\rangle^2 = \cos(\Theta)^2.$$



3. Error Model

Error vector:

$$\Delta\mathbf{q} = \mathbf{q} - \hat{\mathbf{q}}.$$

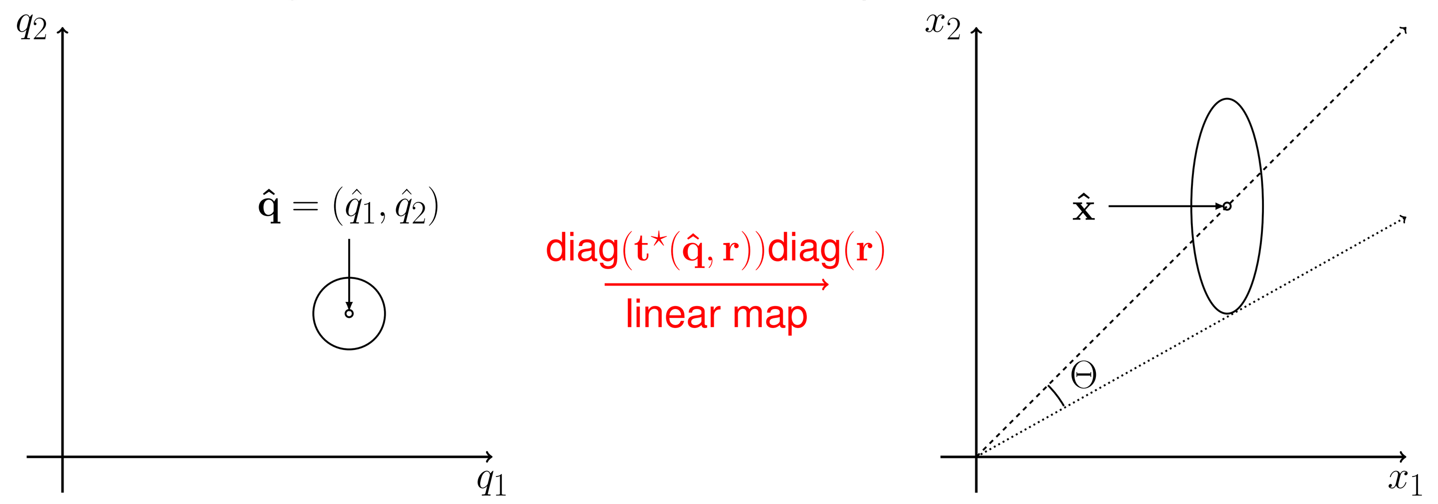
Bounded error magnitude:

$$\|\Delta\mathbf{q}\| \le \delta.$$

Fairness loss:

$$\epsilon(\delta, \hat{\mathbf{q}}) = 1 - \min_{\Delta\mathbf{q} \text{ s.t. } \|\Delta\mathbf{q}\| \le \delta} J(\Delta\mathbf{q}, \hat{\mathbf{q}}).$$

Diagram of error model and search space in two domains



4. Main Results

Theorem 1 [Two-Users] *Let the topology error be bounded by δ, and let a topology estimate $\hat{\mathbf{q}} = (\hat{q}_1, \hat{q}_2)$, with $\hat{q}_1 > \hat{q}_2$ be given. The worst case error, $\Delta\mathbf{q}^{(WC)}$, in a two-user network is*

$$\Delta\mathbf{q}^{(WC)} = -\left(\frac{\delta}{\|\hat{\mathbf{q}}\|}\right)^2 \begin{bmatrix} \hat{q}_1 \\ \hat{q}_2 \end{bmatrix} + \frac{\delta}{\|\hat{\mathbf{q}}\|} \sqrt{1 - \left(\frac{\delta}{\|\hat{\mathbf{q}}\|}\right)^2} \begin{bmatrix} \hat{q}_2 \\ -\hat{q}_1 \end{bmatrix},$$

and the fairness loss is

$$\epsilon(\delta, \hat{\mathbf{q}}) = \frac{\left(\frac{\hat{q}_1}{\hat{q}_2} + \frac{\hat{q}_2}{\hat{q}_1}\right)^2}{\left(2\gamma - \left[\frac{\hat{q}_1}{\hat{q}_2} - \frac{\hat{q}_2}{\hat{q}_1}\right]\right)^2 + \left(\frac{\hat{q}_1}{\hat{q}_2} + \frac{\hat{q}_2}{\hat{q}_1}\right)^2},$$

where

$$\gamma = \sqrt{\left(\frac{\|\hat{\mathbf{q}}\|}{\delta}\right)^2 - 1}.$$

Theorem 2 [N-Users] *The loss in fairness in an N-user scheduled network is given by the optimization problem*

$$\epsilon(\delta, \hat{\mathbf{q}}) = \max_{\mathbf{u}} \frac{\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left(\frac{u_i}{\hat{q}_i} - \frac{u_j}{\hat{q}_j} \right)^2}{\left(N\gamma + \sum_{i=1}^{N} \frac{u_i}{\hat{q}_i} \right)^2 + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left(\frac{u_i}{\hat{q}_i} - \frac{u_j}{\hat{q}_j} \right)^2}.$$

where $\mathbf{u}$ is such that $\sum_{i=1}^{N} \frac{u_i}{\hat{q}_i} \le 0$, $\|\mathbf{u}\| = \|\hat{\mathbf{q}}\|$ and $\langle \mathbf{u}, \hat{\mathbf{q}} \rangle = 0$.

Both results were originally presented in [1].

Label	$\hat{q}_1$	$\hat{q}_2$	$\|\hat{\mathbf{q}}\|$
ASYM-LO	0.1000	0.4123	0.4243
ASYM-HI	0.2000	0.8246	0.8485
SYMM-LO	0.3000	0.3000	0.4243
SYMM-HI	0.6000	0.6000	0.8485

Table 1: *Topology estimates for two-user simulation*

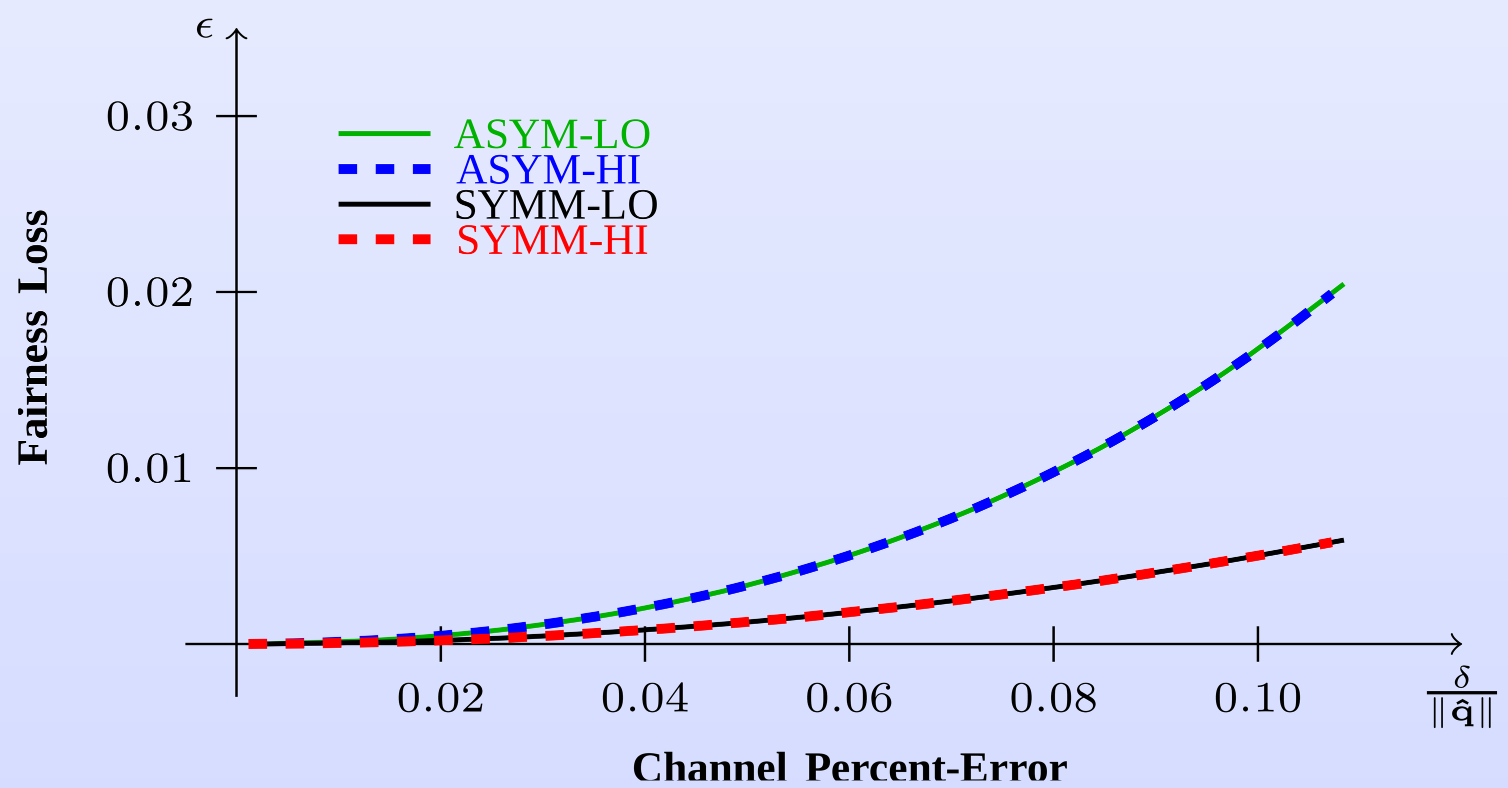


Figure 1: *Fairness loss versus normalized error in a two-user network with topology estimates as listed in Table 1.*

Remarks

Remark 1: Fixed coding rates have no impact on fairness loss — they are not an unknown and can be accounted for perfectly.
Remark 2: Worst-case error places more emphasis on the users with weaker channels, therefore it is more important to have precise measurements for weaker users.
Remark 3: If the channel is more asymmetric, the loss in fairness is exaggerated.
Remark 4: Fairness loss only dependent on normalized error; thus, in order to achieve the same level of fairness, weaker channels require more precise measurement.

Extensions

- Other allocation problems
- Other fairness measures [2]
- Other error models (e.g., stochastic)

Related Work

[1] D. T.-H. Kao and A. Sabharwal, "Impact of network topology knowledge on fairness: A geometric approach". in *Proc. IEEE INFOCOM Mini-Conference*, 2009.

[2] T. Lan, D. Kao, M. Chiang, and A. Sabharwal, "An Axiomatic Approach to Fairness in Network Resource Allocation", submitted to *IEEE INFOCOM Conference*, 2010.

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