Delay-minimal Transmission for Average Power Constrained Multi-access Communications

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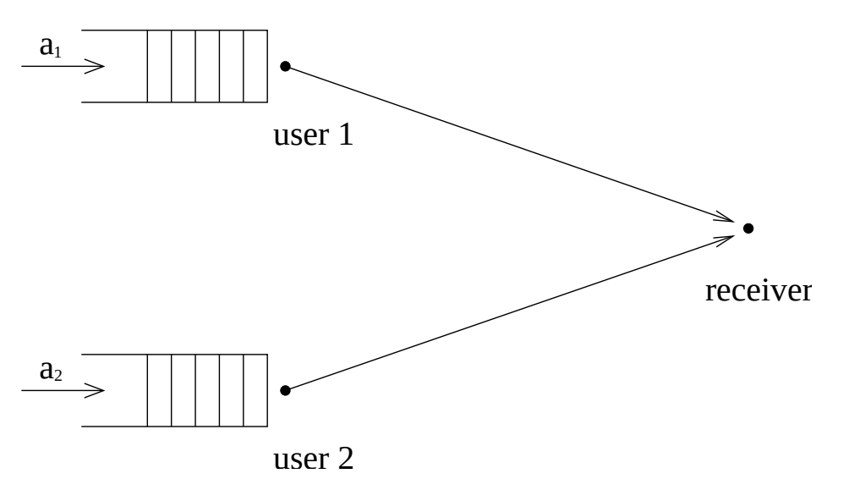
Our Contribution

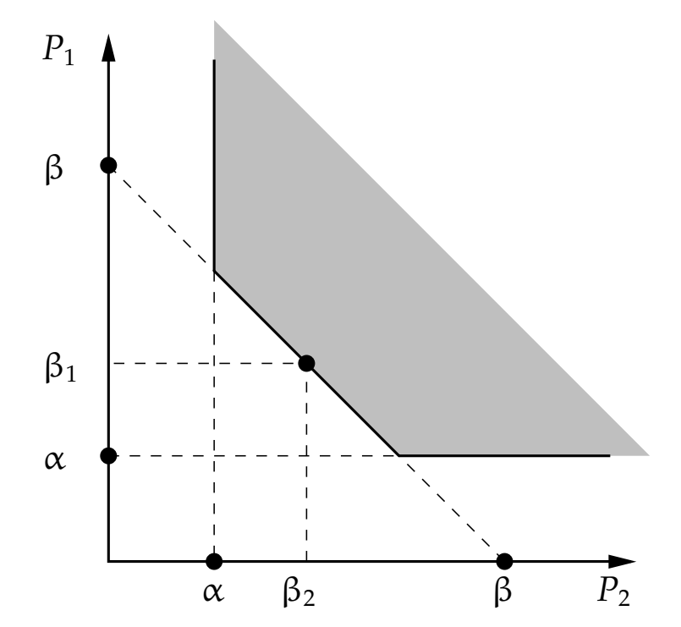
- Our objective is to minimize the average delay in a two-user multi-access system with average power constraints at the transmitters.
- We formulate the problem as a constrained optimization problem.
- We transform the problem into a linear programming problem, and develop a two-step scheme to find the optimal solution analytically.

System Model

- A non-fading discrete-time AWGN MAC system

$$Y = X_1 + X_2 + Z$$





$$q_1[n+1] = (q_1[n] - d_1[n])^+ + a_1[n]$$
$$q_2[n+1] = (q_2[n] - d_2[n])^+ + a_2[n]$$

$$P_i \geqslant \sigma^2(2^{2R} - 1) \triangleq \alpha$$
$$P_1 + P_2 \geqslant \sigma^2(2^{4R} - 1) \triangleq \beta$$

- When $\mathbf{q} \neq (0,0)$, three possible departure rate pairs: $d^1 = (1,0), d^2 = (0,1)$ or $d^3 = (1,1)$
- Define probabilities of choosing each pair as $g_{ij}^1, g_{ij}^2, g_{ij}^3$, then $g_{ij}^1 + g_{ij}^2 + g_{ij}^3 = 1$.
- g_{ij}^ks are the main parameters we aim to choose optimally.

The Optimization Problem

We obtain a Markov chain, whose the state space consists of all possible pairs of queue lengths: $\mathbf{q} \triangleq (q_1, q_2)$.

$$\begin{aligned} \min_{\mathbf{g},\beta_1,\beta_2} \quad & \frac{1}{2\theta} \sum_{i,j} \pi_{ij}(i+j) \\ \text{s.t.} \quad & \sum_{i,j} \pi_{ij}(g_{ij}^1 \alpha + g_{ij}^3 \beta_1) \leqslant P_{avg} \\ & \sum_{i,j} \pi_{ij}(g_{ij}^2 \alpha + g_{ij}^3 \beta_2) \leqslant P_{avg} \\ & \boldsymbol{\pi}\mathbb{P} = \boldsymbol{\pi} \\ & \boldsymbol{\pi}\mathbf{1} = 1 \end{aligned}$$

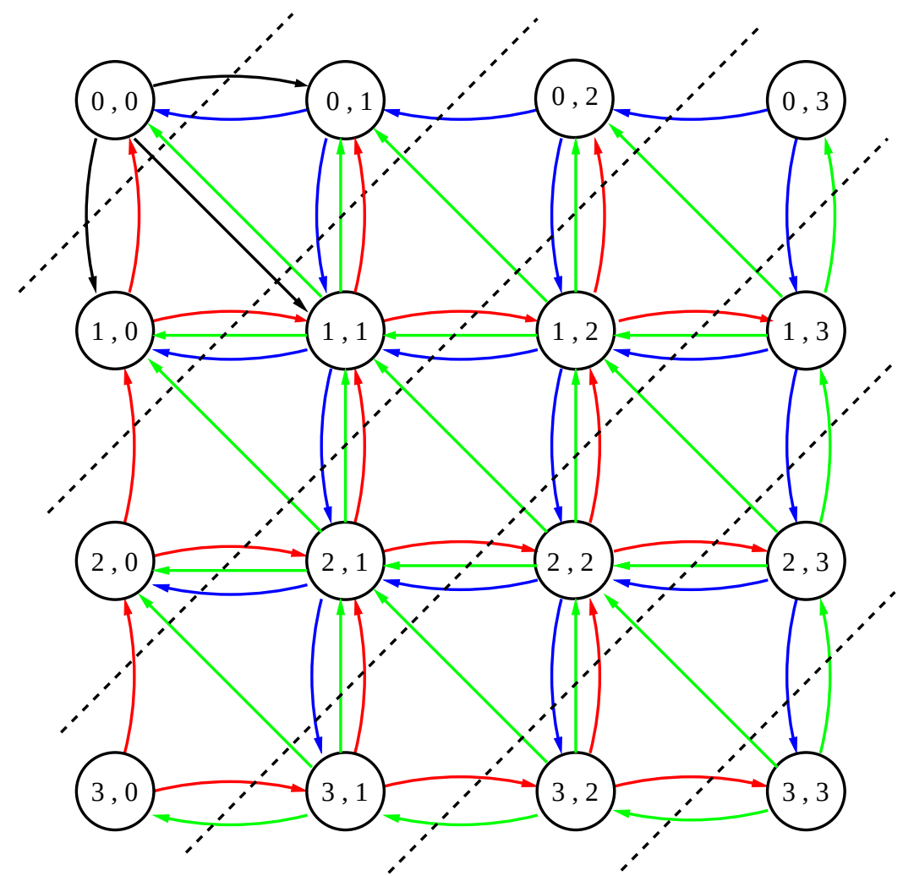
- Define $x_{00} = \pi_{00}$, $x_{ij}^k = \pi_{ij} g_{ij}^k$.
- When both power constraints are tight, we transform our optimization problem into

$$\begin{aligned} \min_{\mathbf{x}} \quad & \sum_{i,j} \left(\sum_{k=1}^{3} x_{ij}^k (i+j) \right) \\ \text{s.t.} \quad & x_{00} = 1 - \frac{2\theta(\beta - \alpha) - 2P_{avg}}{\beta - 2\alpha} \\ & \sum_{i,j} x_{ij}^1 = \frac{\theta\beta - 2P_{avg}}{\beta - 2\alpha} \\ & \sum_{i,j} x_{ij}^2 = \frac{\theta\beta - 2P_{avg}}{\beta - 2\alpha} \\ & \sum_{i,j} x_{ij}^3 = \frac{2P_{avg} - 2\theta\alpha}{\beta - 2\alpha} \\ & \mathbb{Q}\mathbf{x} = \mathbf{0} \end{aligned}$$

A Linear Programming problem!

Group Along the Diagonals

The transitions between diagonal groups when $N = 3$.



- Define the sum for the nth diagonal, as

$$y_n = \sum_{i=0}^{n} (x_{i,n-i}^1 + x_{i,n-i}^2), \quad t_n = \sum_{i=0}^{n} x_{i,n-i}^3$$

- The objective function is

$$\sum_{n=1}^{2N} (y_n + t_n) n = \frac{1}{2(1-\theta)} \sum_{n=1}^{2N} y_n n + C$$

where

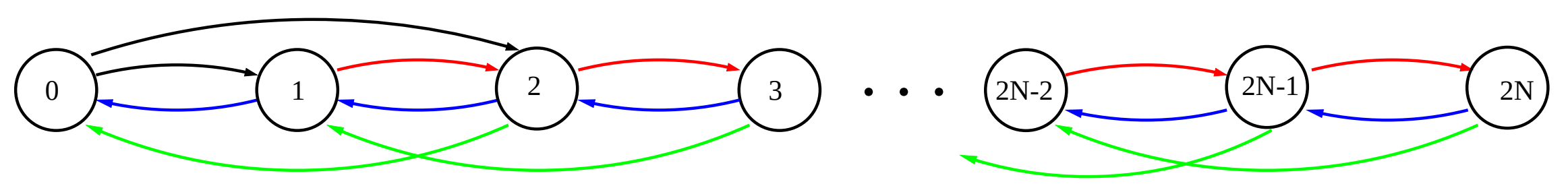
$$\sum_{n=1}^{2N} y_n = \sum_{i,j} (x_{ij}^1 + x_{ij}^2) \triangleq \Psi$$

Transitions among groups: For $n = 0, 1$,

$$x_{00} \left(\theta^2 + 2\theta(1-\theta)\right) = (y_1 + t_2)(1-\theta)^2$$
$$(x_{00} + y_1)\theta^2 = (y_2 + t_3)(1-\theta)^2 + t_2(1-\theta^2)$$

and for $n = 2, 3, \ldots, 2N-2$,

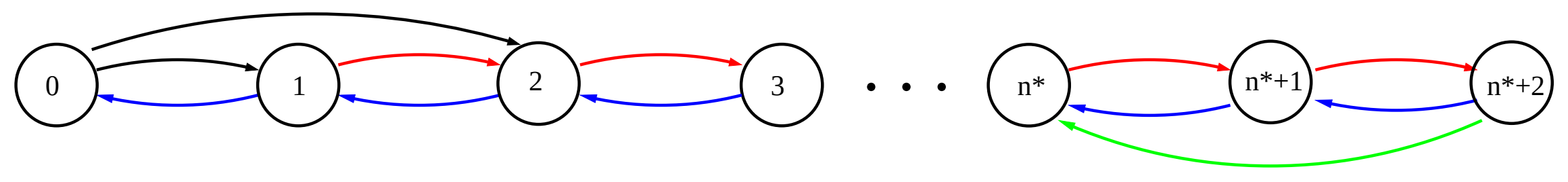
$$y_n \theta^2 = (y_{n+1} + t_{n+2})(1-\theta)^2 + t_{n+1}\left(1-\theta^2\right)$$
$$y_{2N-1}\theta^2 = t_{2N}\left(1-\theta^2\right)$$



The First Step: Allocation among Groups

$\min_{\mathbf{y},\mathbf{t}} \sum_{n=1}^{2N} y_n n$, with $\sum_{n=1}^{2N} y_n = \Psi$, plus $2N$ group transition equations.

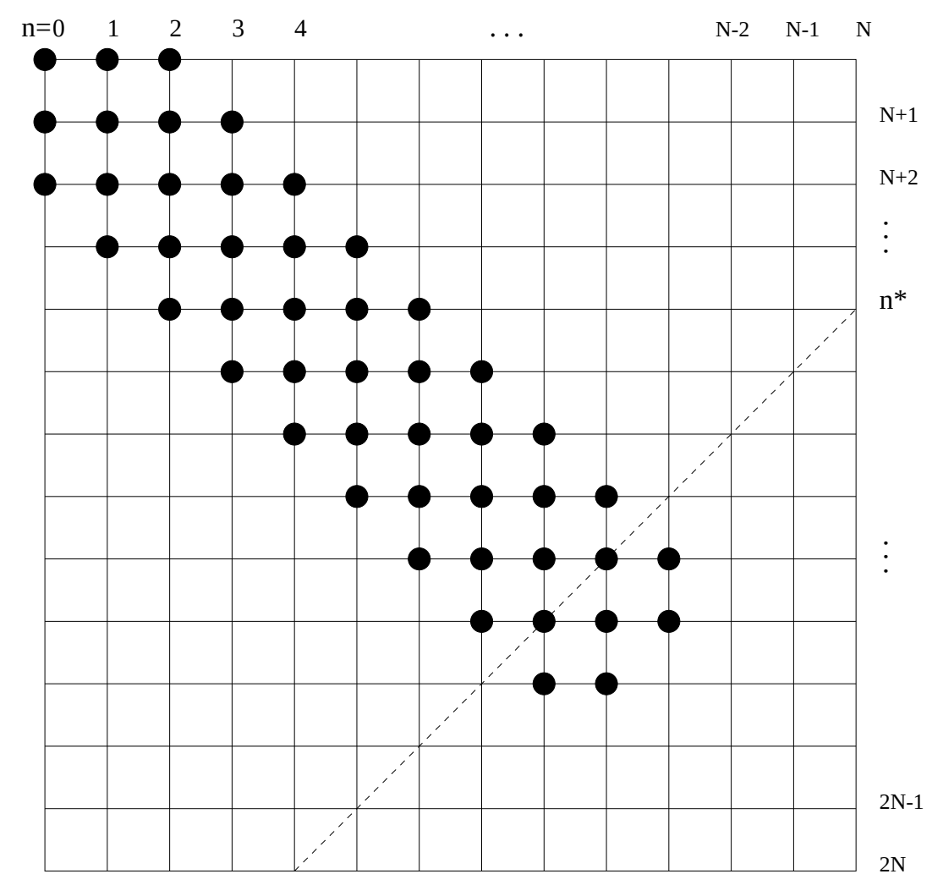
- The optimization requires us to assign larger values to y_ns with smaller indices n as much as possible.
- The allocation gives us $\{y_n\}_{n=1}^{n^*+1}$ and $t_{n^*+2} \neq 0$
- Transitions among groups after the allocation:



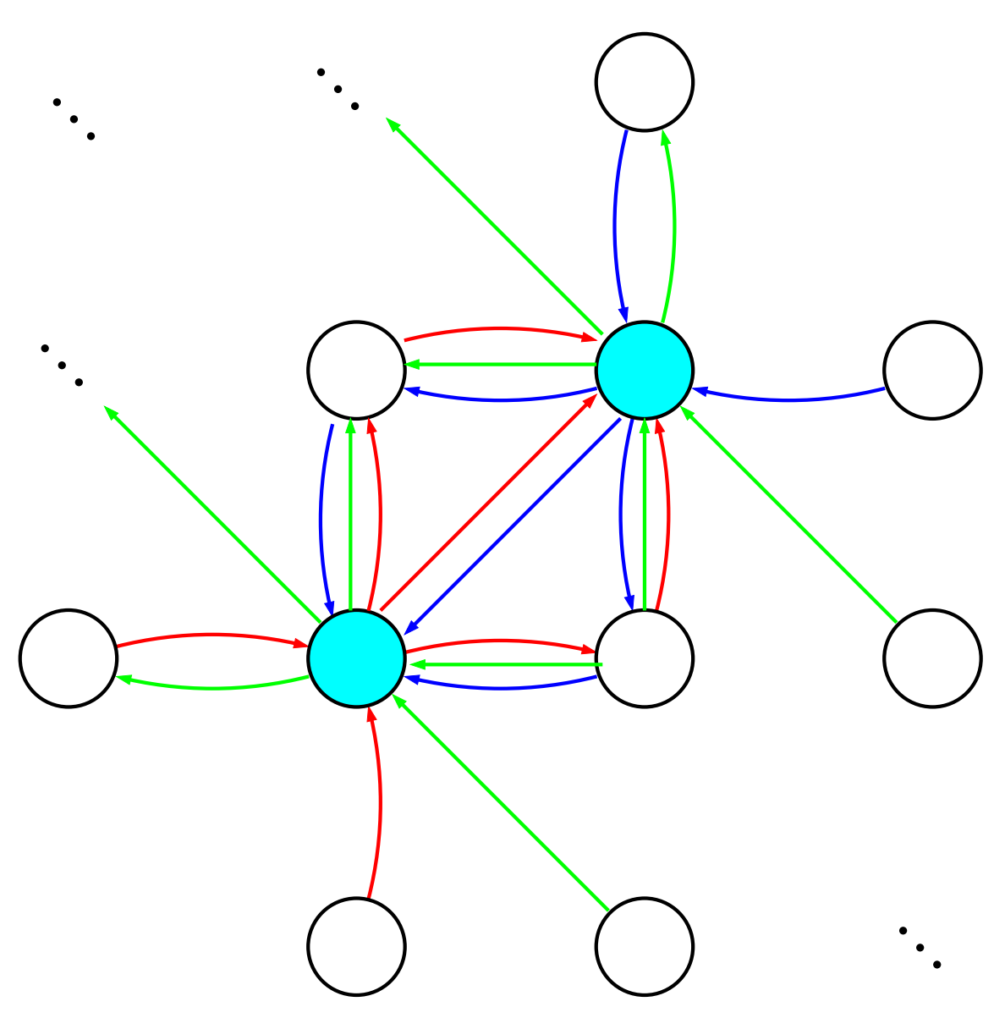
The Second Step: Allocation within Groups

Allocate y_ns and t_ns obtained from the first step to x_{ij}^ks.

- A valid allocation in the second step proves the feasibility of the solution found in the first step.
- Only allocate nonzero values to a small number of states (dotted states).
- All of the remaining states are transient states with zero steady state probability.
- The values of nonzero x_{ij}^ks are determined by the transition equations.



Determining the Values of x_{ij}^ks

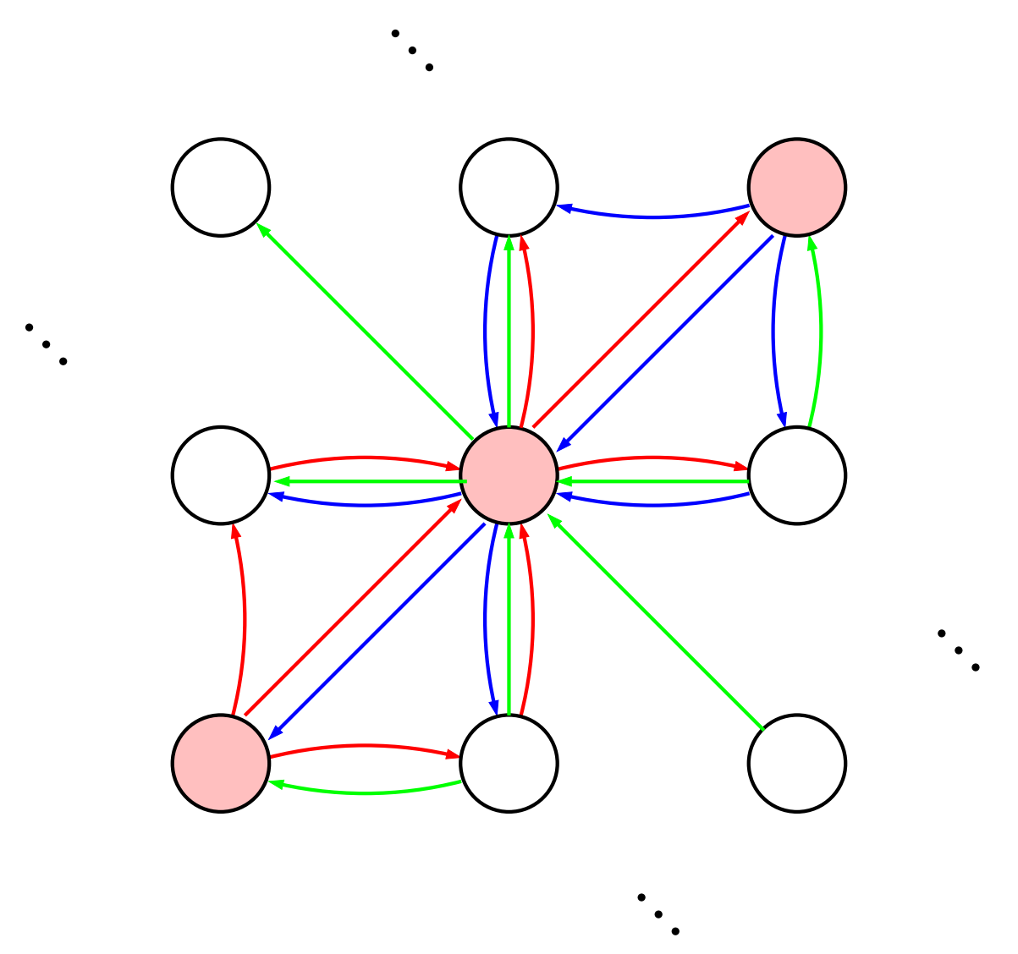


When n is ODD,

- Symmetric setting allows a symmetric allocation.
- Let

$$x_{\frac{n+1}{2},\frac{n-1}{2}}^1 = x_{\frac{n-1}{2},\frac{n+1}{2}}^2 = y_n/2$$
$$x_{\frac{n+1}{2},\frac{n-1}{2}}^3 = x_{\frac{n-1}{2},\frac{n+1}{2}}^3 = t_n/2$$

- The symmetric allocation for even-indexed groups guarantees that the transition equations are satisfied.



When n is EVEN,

- Keep the allocation symmetric
- Analyzing the transition equations for individual states, we have

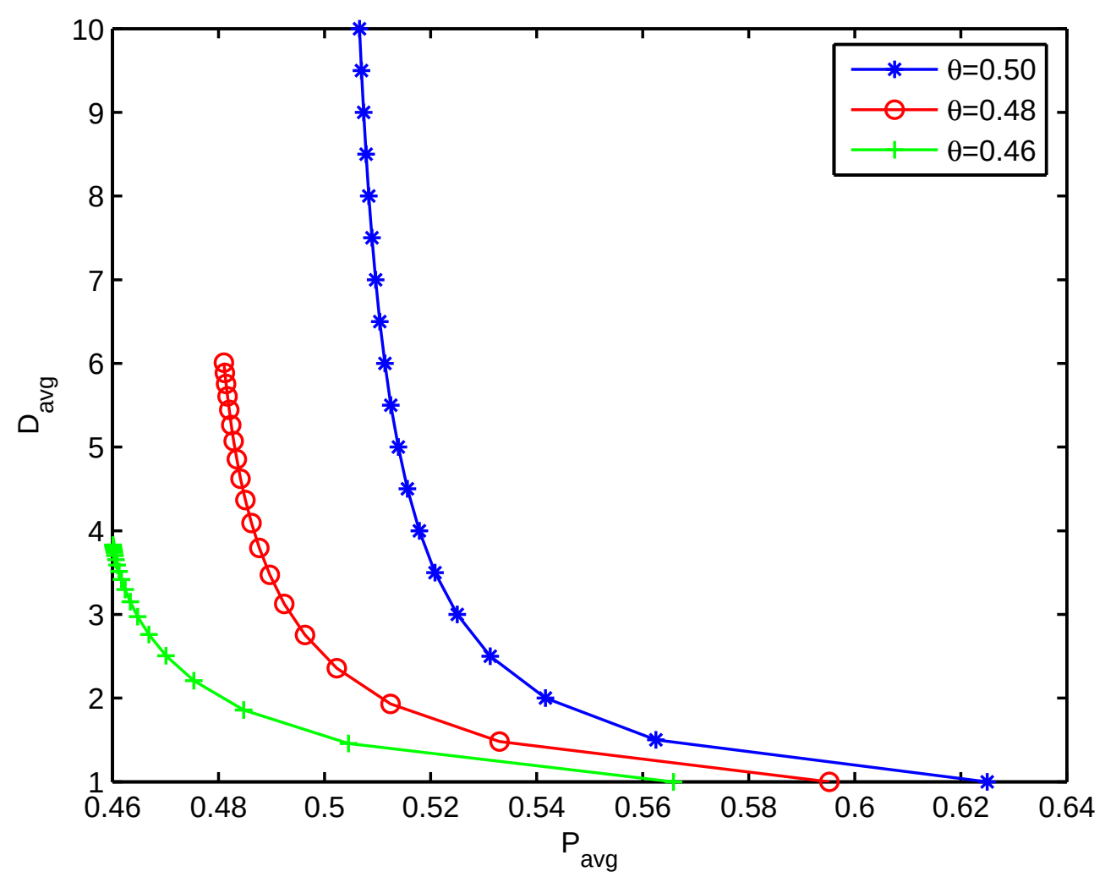
$$x_{\frac{n}{2},\frac{n}{2}}^3 = t_n$$
$$x_{\frac{n}{2}+1,\frac{n}{2}-1}^1 = 1/2(y_n + t_{n+1})\theta(1-\theta)$$
$$x_{\frac{n}{2},\frac{n}{2}}^2 = 1/2 y_n - 1/2(y_n + t_{n+1})\theta(1-\theta)$$

- The transition equations are satisfied, and x_{ij}^ks are nonnegative as well.

The Transmission Probabilities g_{ij}^ks

- Compute the transmission probabilities g_{ij}^k according to $g_{ij}^k = \frac{x_{ij}^k}{\sum_{k=1}^{3} x_{ij}^k}$.
  - Let $\bar{n} = \max\{n : y_n \neq 0\}$.
    - If $i + j < \bar{n}$, then, when $i > j$, $g_{ij}^1 = 1$; when $i < j$, $g_{ij}^2 = 1$; when $i = j$, $g_{ii}^1 = g_{ii}^2 = 1/2$ and $g_{ii}^3 = 0$.
    - If $i + j > \bar{n}$, $g_{ij}^3 = 1$.
- The optimal policy has a THRESHOLD structure:
  - If the sum of the queue lengths is greater than $\bar{n}$, both users transmit during the slot.
  - If the sum of the queue lengths is less than $\bar{n}$, only the longer queue transmits; if in this case both queues have the same length, each queue transmits while the other one keeps silent with probability $1/2$.

Simulation results



- The average power vs average delay curve is convex, and piecewise linear.
- Each linear segment corresponds to the same threshold value.
- The average delay decreases as average power increases.