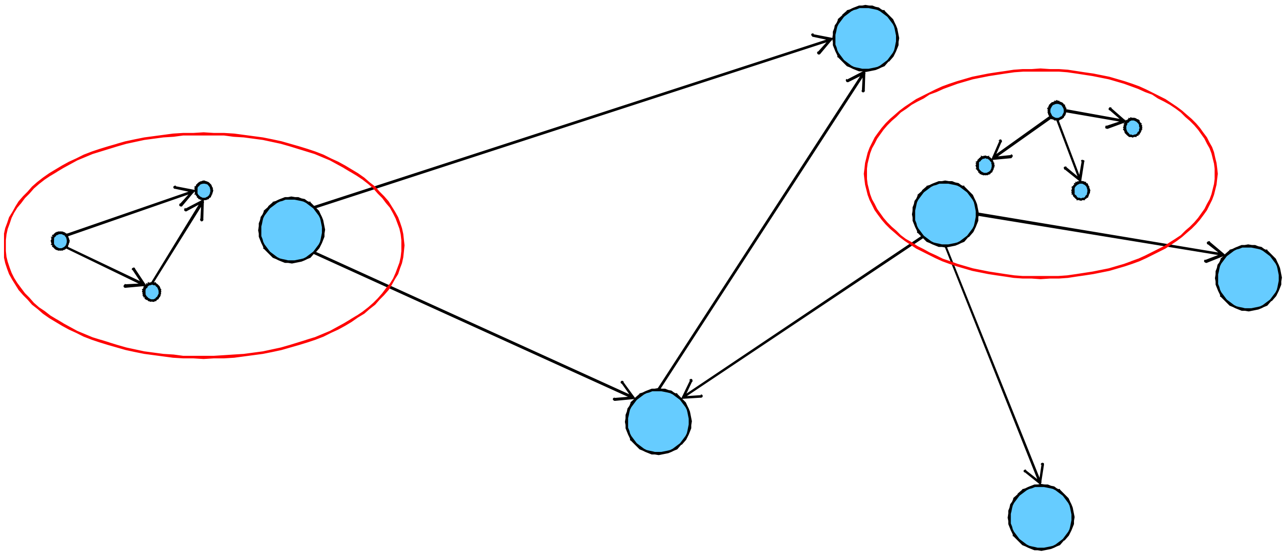
Interference Channels with a Local View

Vaneet Aggarwal (Princeton), Youjian Liu (Colorado), Ashutosh Sabharwal (Rice)



Nodes have limited knowledge about

topology & channel gains

Mismatched in their knowledge

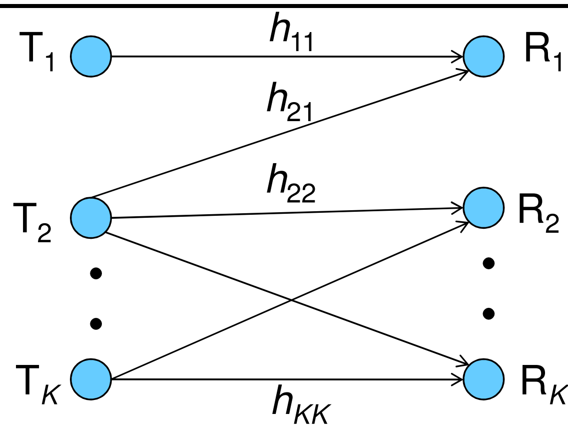
Often simply a local view

Which partial info cases to analyze ?

Performance with partial info ?

Structure of distributed transmission strategies ?

Interference Network

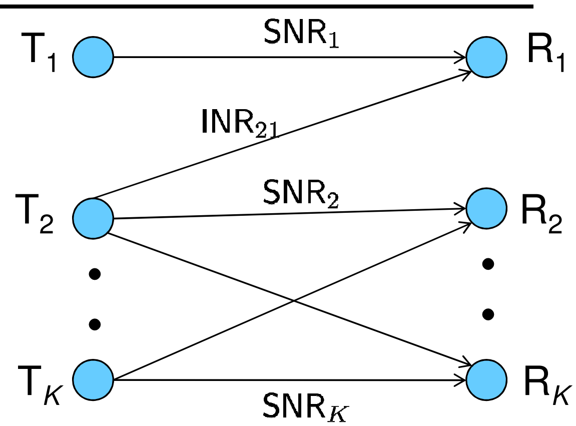


Network matrix $H = \left[h_{ij}\right]$

- Single-hop, $h_{ii} \neq 0$
- Arbitrary h_{ij} and unknown

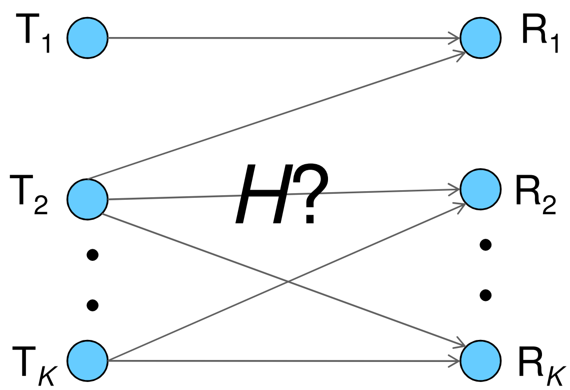
Gaussian Interference Network

$$y_i = \sqrt{\mathsf{SNR}_i} x_i + \sum_{j \neq i} \sqrt{\mathsf{INR}_{ji}} x_j + n_i$$



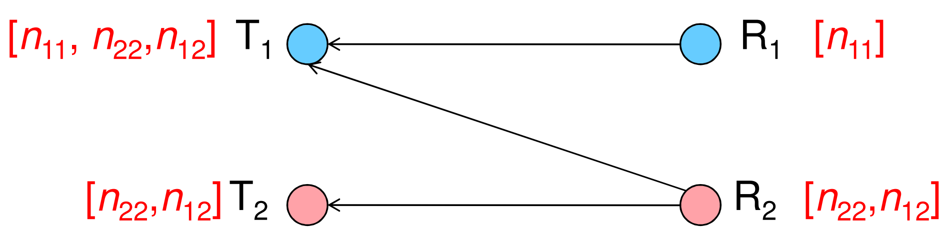
- Super-position of signals in Gaussian noise
- Basic step to analyzing wireless fading channels

At "Startup"



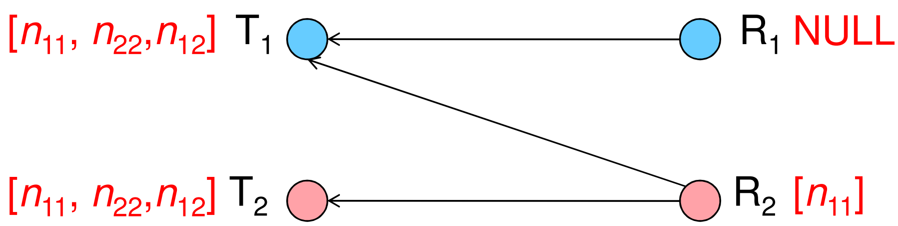
- Transmitters know receivers are reachable in one-hop
- But all channel gains in H are unknown
- Learn the network via local message passing
  - Aggarwal and Sabharwal'09 – two-way fading channels

Z-channel



- Round 1 (forward)
  - Transmitters send $m_{i,1}$ = training (with their ID)
  - Receivers estimate one-hop labeled channels
- Round 1 (reverse)
  - Receivers send what they learnt, $M_{i,1} = H_{[],i}$
  - Transmitters have partial information

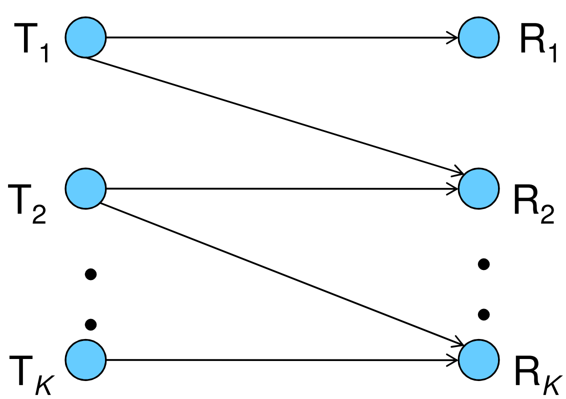
Z-channel

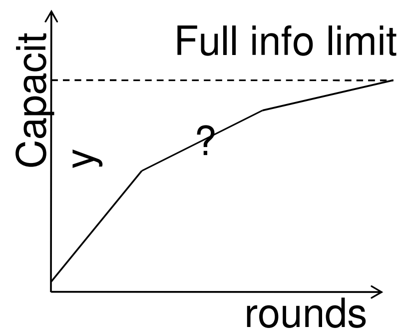


- Round 2 (forward)
  - Transmitters send $m_{i,2} = (M_{1,1} \cup M_{2,1}) \setminus (m_{i,1})$
  - Receivers know the whole network
- Round 2 (reverse)
  - Receivers send what is not known to transmitters

$$M_{i,2} = (m_{1,2} \cup m_{2,2}) \setminus (M_{i,1}) \setminus (m_{1,1} \cup m_{2,1})$$

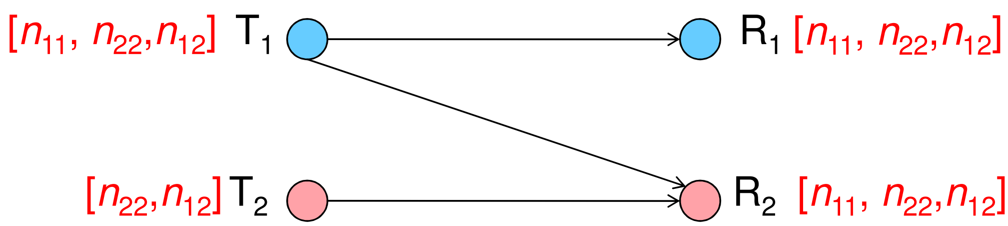
K-user Interference Network





- # of rounds measure extent of information
  - More rounds = more information
- K+1 rounds are sufficient to learn the whole network
- What is the maximum sum-rate after each round ?
  - Interesting cases: less than maximum rounds

Deterministic Z-channel: 1.5 Rounds

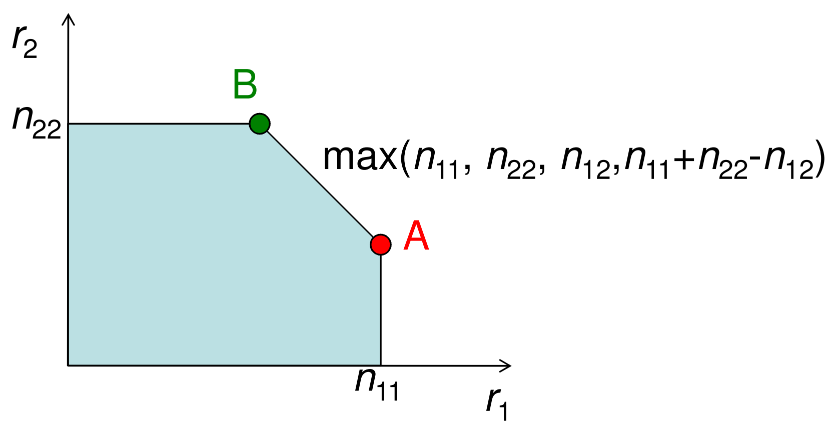


- Receivers know all channels
  - Synchronized with transmitter actions
- T_2 does not know n_{11}

What is the max sum-rate ?

Distributed (rate, code, decoder) selection

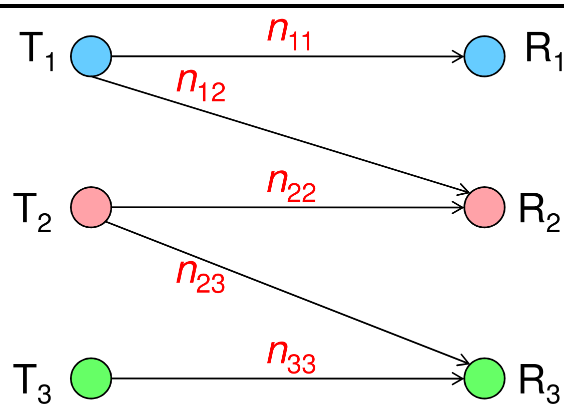
Corner Points Need Less Info



- A: T_1 sends full rate, T_2 backs off
  - Commonly used in Z-channel analysis
- B: T_2 sends full rate, T_1 backs off
  - Achieved with 1.5 rounds of information

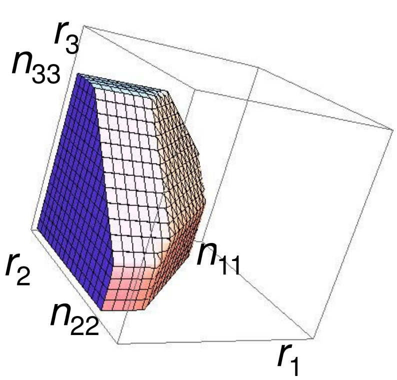
- A: T_1 does not need to know n_{22}
- B: T_2 does not need to know n_{11}

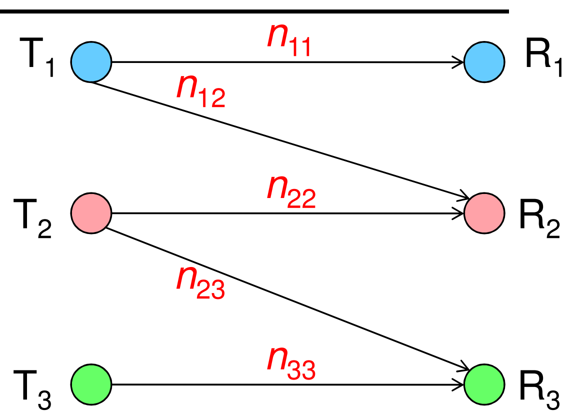
Deterministic Double Z-channel



- Message-passing terminates in 4 rounds
- Interesting cases: 1.5, 2.5, 3.5 rounds

Structure of Capacity Region





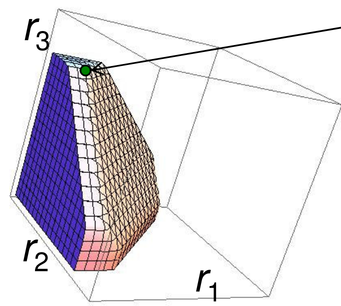
- (r_1,r_2) constraints same as Z-channel
- (r_2,r_3) constraints same as Z-channel
- Two regions combined by constraint on $r_1+r_2+r_3$

Sum-rate: 2.5 Rounds

Theorem: $r_{\text{sum},2.5\text{ rounds}} = r^*_{\text{sum},4\text{ rounds}}$

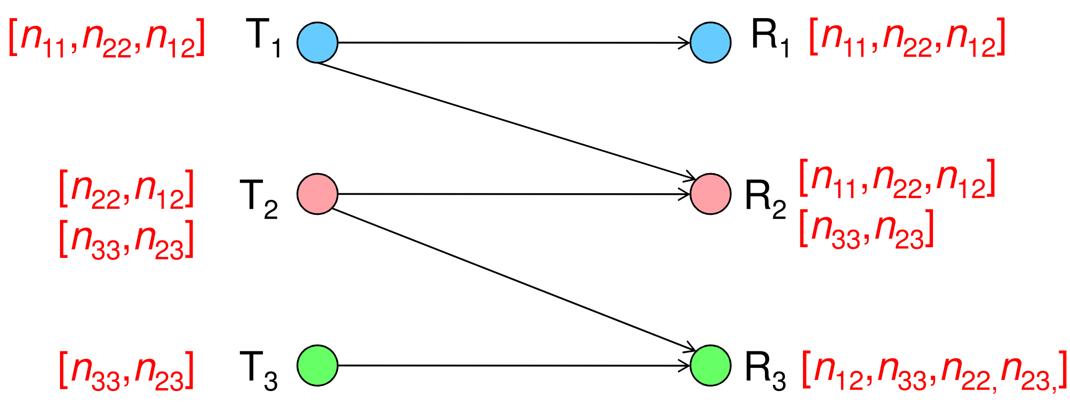
Proof:

- T_1 and T_2 know all, T_3 knows partial topology
- Achieves a corner point on dominant sum-rate face.

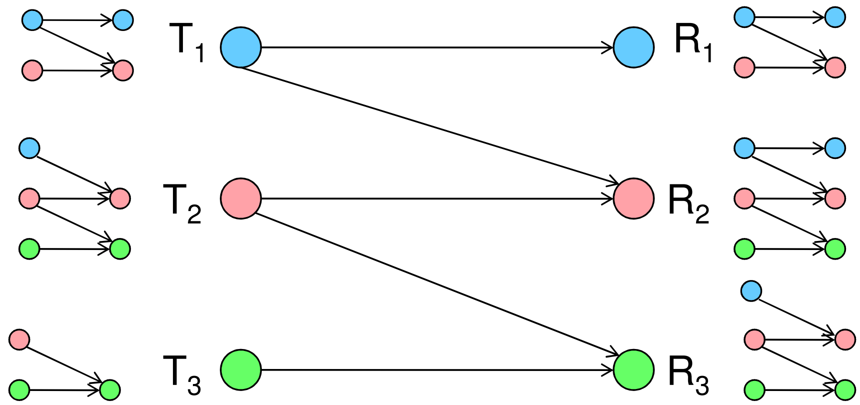


More Interesting case is 1.5 Rounds

Sum-rate: 1.5 Rounds

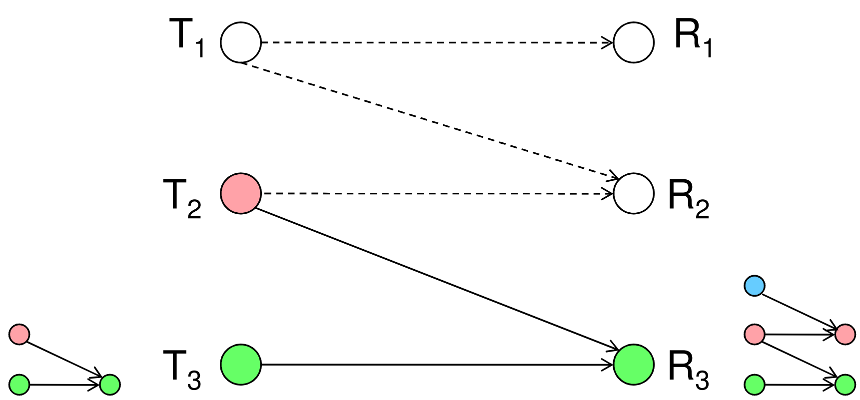


After 1.5 Rounds



- T_1 does not know T_3
- T_2 knows it has a Z-channel with T_1 and T_3
- T_3 but does not know T_1

Iterative Rate Allocation

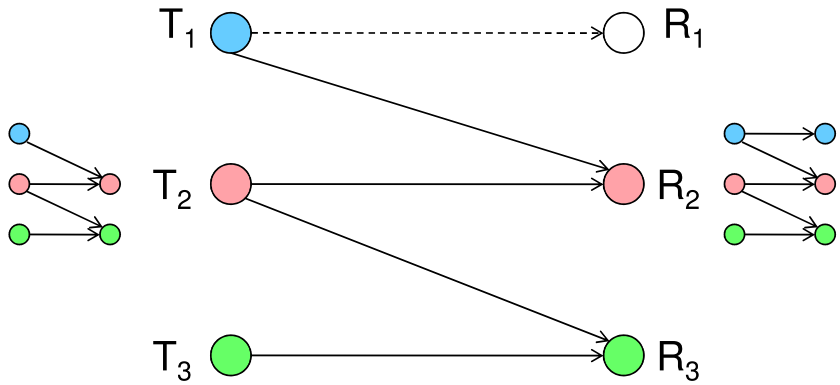


$$r_3 = n_{33}$$

Not enough information about T_2

Hence send full-rate.

Iterative Rate Allocation



If $n_{23} < n_{33}$,

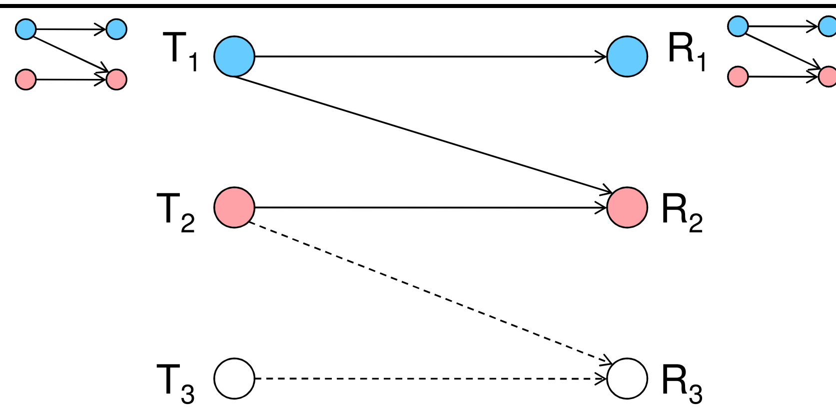
$$r_2 = (n_{22} - n_{23})^+$$

else $$r_2 = \min(\max(n_{22}, n_{23}) - n_{33}, n_{22})$$

Act like a top-node in Z-channel : back-off for T_3

Act like a bottom-node: Ignore T_1

Iterative Rate Allocation



If $n_{12} < n_{22}$,

$$r_1 = (n_{11} - n_{12})^+$$

else $$r_1 = \min(\max(n_{11}, n_{12}) - n_{22}, n_{11})$$

Act like a top-node in Z-channel : back-off for T_2

Does not know that T_2 may be backing off for T_3

In General

- After 1.5 rounds

$$r_{\text{sum}} = \min(\max(n_{22}, n_{23}, n_{33}, n_{22}+n_{33}-n_{23}), n_{22}+n_{33}) + \min(n_{11}, \max(n_{11}, n_{12}) - \min(n_{12}, n_{22})).$$

- (rate, code, decoder) choices are decentralized

- How does it perform ? Any loss ?
  - Yes, can be anywhere from 0 to ∞

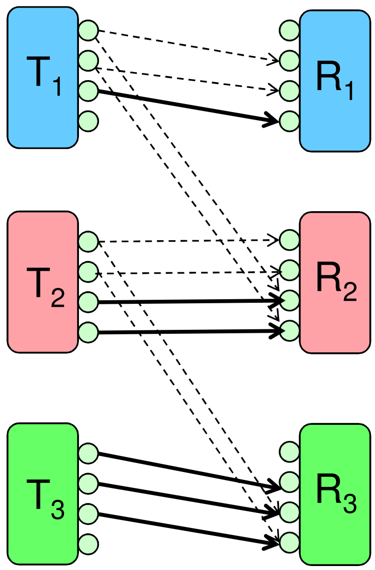
No Loss in Some Cases

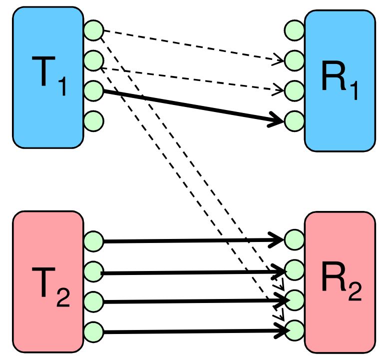
Sufficient condition for no loss: Any of

$$n_{23} \geq n_{22} + n_{33}$$
$$n_{12} \geq n_{11} + n_{22}$$
$$n_{23} \leq n_{33} \textit{ and } n_{22} \geq n_{23} + n_{12}$$

In some regimes, 1.5 rounds suffice

No Loss in Some Cases

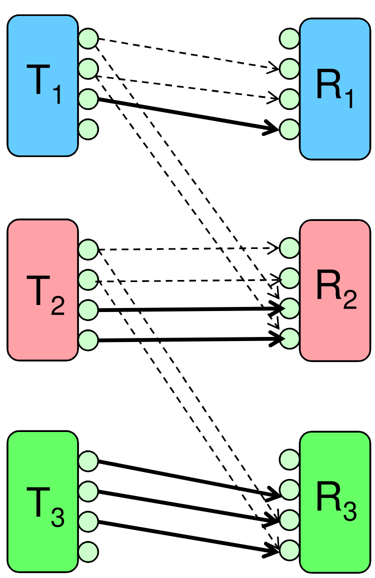




$T_1 T_2$ Z-channel

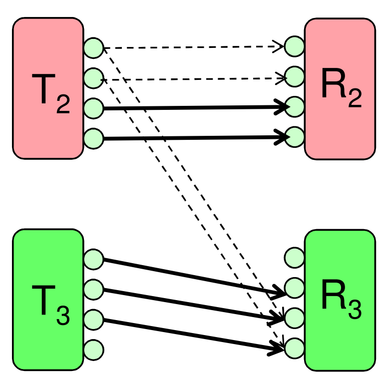
- T_1 gets the same rate
- T_2 gets more

No Loss in Some Cases

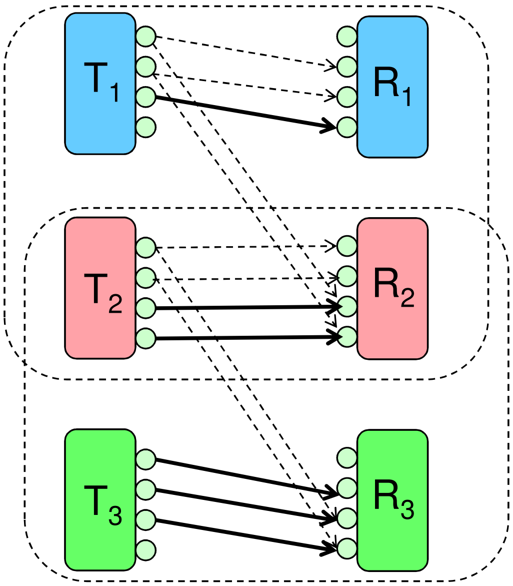


$T_2 T_3$ Z-channel

- T_3 gets the same rate
- T_2 adjusts to T_3

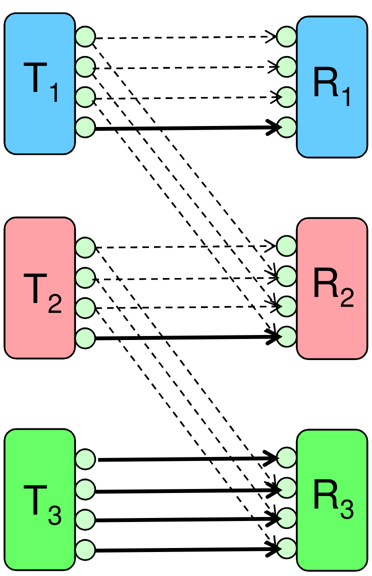


Decoupling Condition

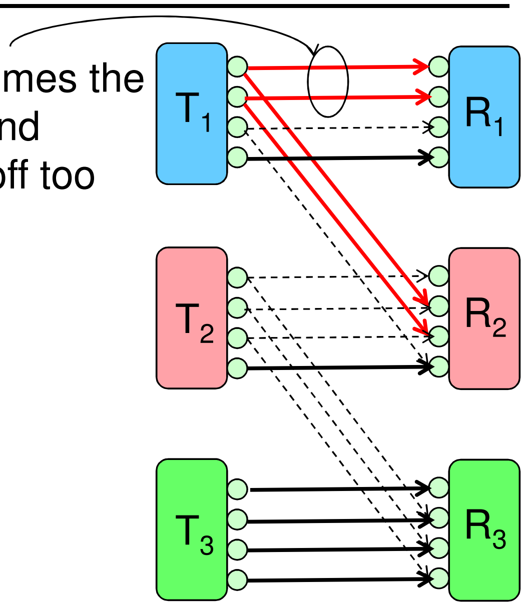


- Decouple into two Z-channels
- Z-channel rate-allocation for T_1 and T_3 is optimal
- T_2 knows what to do

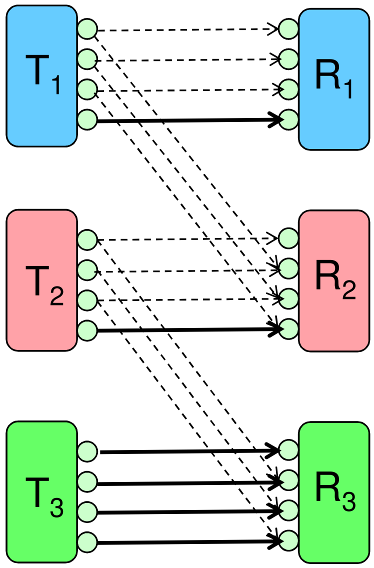
Loss in Spatial Reuse After 1.5 Rounds



T_1 assumes the worst and backs-off too much



Loss in Spatial Reuse After 1.5 Rounds



$$n_{12} = n_{23} = n_c, n_{11} = n_{22} = n_{33} = n_d > n_c$$

$$n_d, n_c \to \infty \text{ with } \alpha = \frac{n_c}{n_d} = \text{ constant}$$

- Symmetric
- Loss of spatial reuse
  - Arbitrarily large for ½ < α< 2 (weak-strong interference regime)
  - Holds for K-user interference channel

Loss is Unavoidable

Theorem: There is no universally optimal strategy with 1.5 rounds of information.

Proof:

Universally optimal: optimal in every channel state

- If there exists one, proposed strategy is also optimal.
- Proposed strategy is not *universally* optimal
- Hence no universally optimal strategy exists

Optimality in Gaussian Double-Z

- 2.5 rounds
  - Deterministic is optimal
  - Gaussian is ≤ 2-bits from global optimal
- 1.5 rounds
  - 2-bit away from optimal with decoupling condition
  - For ½ < α< 2, loss can be arbitrarily large
- Hidden nodes can cause large losses
  - In system capacity
  - (And fairness)

Conclusions

- Message Passing Algorithm
- Analysis of cases with partial message passing
  - Single Z channel
    - 1.5 Rounds: Sum Rate within 2 bits
  - Double Z channel
    - 1.5 Rounds: Can be unbounded
    - 2.5 rounds: Sum Rate within 4 bits

•Aggarwal, Liu and Sabharwal, "Message passing in Distributed Wireless Networks," at ISIT'09

•Aggarwal, Liu and Sabharwal, "Capacity with a Local Network View," in prep for IEEE IT, 2009.