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WINLAB | Wireless Information
Network Laboratory

Optimizing LT Codes for Multicast in Heterogeneous Networks†

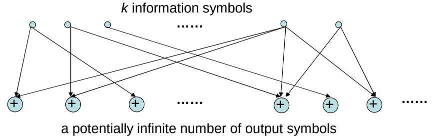
Yao Li
yaoli@winlab.rutgers.edu

Advisors
Emina Soljanin, Bell Labs
Predrag Spasojevic, Rutgers University

Motivation

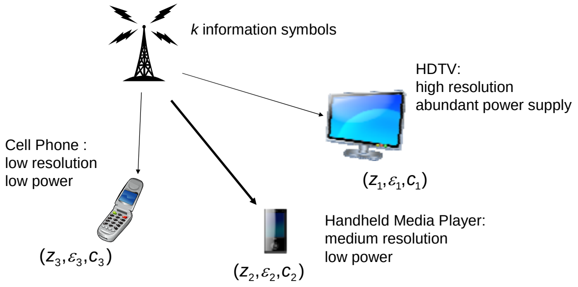
- Optimize LT rateless codes for multicast in a heterogeneous network of
  - **Diverse Source-Sink Channel Conditions**
  - **Non-Uniform Demand**
  - **Both Decoding and Non-Decoding Sinks**

Degree Distribution of LT Codes



- Degree of an output symbol = the number of information symbols selected to be linearly combined.
- $p_1^{(k)}$ = *Prob* [output symbol is of degree *i*], *i*=1,2,…,*k*
- Conventionally the "ideal soliton distribution" or its variations are used.
- Suboptimal iterative "belief propagation" (BP) decoding

System Model: A Heterogeneous Network



- *k*: the total number of information symbols
- Degree distribution of LT symbols encoded by the source

$$P^{(k)}(x)= p_1^{(k)}x+p_2^{(k)}x^2+\ldots+p_k^{(k)}x^k$$

- Sink clusters characterized by $(z_i, c_i, \varepsilon_i)$, size n_i, *i*=1,2,…,*l*
  - z_i: the fraction of information symbols the sink intends to recover
  - c_i: = 1 for a BP decoding and = 0 for a non-decoding sink
  - ε_i: erasure rate of the BEC link connecting the source to the sink
  - t_i: latency, the number of output symbols transmitted over the link until the demand of the sink is satisfied, normalized by *k*

Optimization in the Asymptotic Regime

Performance Measures

- Min-Max Latency: $\text{minimize}_{P,}\max_i t_i$
  - minimize the total latency at the source side
- Max-Min Throughput: $\text{maximize}_{P,}\min_i z_i/t_i$
  - Throughput measures the actual channel degradation and provides reference for the pricing of services.
- Max-Min Channel Utilization: $\text{maximize}_P \min_i z_i/(t_i(1-\varepsilon_i))$
  - Expect worse service on worse channels
- Min Average Latency $\text{minimize}_{P,t_i} \dfrac{\sum n_i t_i}{\sum n_i}$
  - Optimize the average performance of all sink nodes

Optimization Constraints

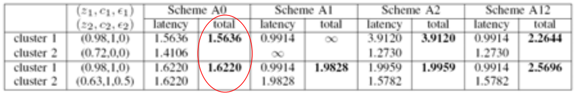
- *Ripple*
  - the set of output symbols of (induced) degree-1 during BP decoding of LT codes
  - The decoding process halts when the ripple becomes empty
- Optimization constraints:
  keep ripple size>0 until the unrecovered fraction goes down to 1-z
  - For a decoding sink cluster *i*: $t_i(1-\varepsilon_i)P'(1-u)+\ln u>0,\ 1-z_i<u\le 1$
  - For a non-decoding sink cluster *i*:. $t_i(1-\varepsilon_i)p_1+\ln u>0,\ 1-z_i<u\le 1$
- Derivation of the Optimization Constraints
  - **Theorem**[Maatouk et al.'09] (the expected ripple size (*rk*) as a function of the number of unrecovered information symbols (*uk*) and the number of output symbols collected (*wk*))

$$r(u)=wu(P^{(k)}{}'(1-u)+\frac{1}{w}\ln u)+O(\frac{1}{k})$$

  - As $k \to \infty$, $P^{(k)}(u) \to P(u)$, $\quad r(u)=wu(P'(1-u)+\frac{1}{w}\ln u)$

Source Latency Comparison

- Total number of output symbols transmitted by the source



	(z_1,c_1,ϵ_1) (z_2,c_2,ϵ_2)	Scheme A0		Scheme A1		Scheme A2		Scheme A12	
		latency	total	latency	total	latency	total	latency	total
cluster 1	(0.98,1,0)	1.5636	**1.5636**	0.9914	∞	3.9120	**3.9120**	0.9914	**2.2644**
cluster 2	(0.72,0,0)	1.4106		∞		1.2730		1.2730	
cluster 1	(0.98,1,0)	1.6220	**1.6220**	0.9914	**1.9828**	1.9959	**1.9959**	0.9914	**2.5696**
cluster 2	(0.63,1,0.5)	1.6220		1.9828		1.5782		1.5782	

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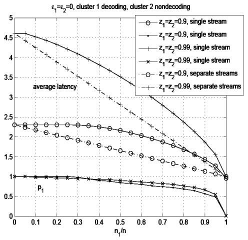
Schemes
A0-A2: the source sends **a single stream** to all sinks
A0: minimize the **max latency**.
A1: minimize the **latency of cluster 1**.
A2: minimize the **latency of cluster 2**.
A12: The source sends **two independent streams** to the clusters, each minimizing the **latency of the targeted cluster**.

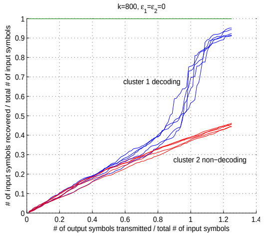
Minimizing Average Latency



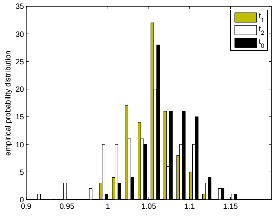
Minimum average latency and achieving p_1 versus the fraction of decoding sinks, $z_1 = z_2$.

- Perfect channels
- 2 clusters, one decoding, one non-decoding
- Uniform demand
- ▪ Minimum average latency drops as the fraction of decoding sinks grows larger
- ▪ p_1 of the optimized degree distribution falls slowly as the fraction of decoding sinks grows larger

Finite-Length Simulations



Decodable fraction v.s. # of output symbols collected; z_1=0.8, z_2=0.4, perfect channel, *k*=800; 5 sample runs



Histograms for t_1, t_2 and $t_0=\max\{t_1,t_2\}$; 100 sample runs, mean of t_0 is1.0718, standard deviation 0.0300; optimization result t_0=1.0473

Conclusions and Future Work

- **We have optimized LT codes for multicast in a highly heterogeneous network**
- **Optimization is through the design of the code degree distribution**
- **Optimized performance shows significant savings in the total bandwidth consumption compared to sending separately encoded streams to different types of sink nodes**
- **Finite-length simulations confirm the usability of the asymptotic results to guide the finite-length code design**
- **Future Work: More complex network topologies**
  - **Cooperation between sinks?**
  - **Multi-hop relay?**

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