Linear Compressive Networks

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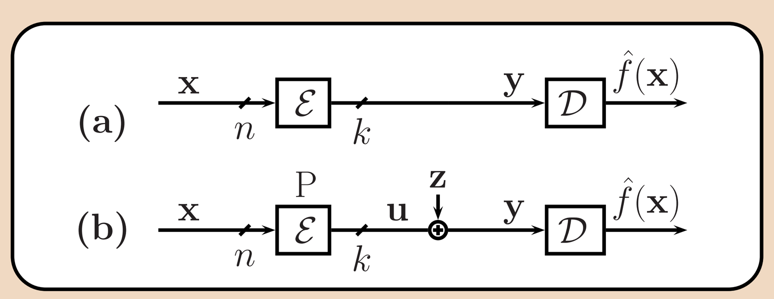
Overview

- **Linear subspace projections** of Gaussian signals are transmitted through a linear compressive network (LCN).
- The LCN is a graph of source sensors, relay nodes, and decoders communicating through noisy analog channels.
- Each **power-constrained** node in an LCN may **compress** incident signal vectors and transmit a reduced number of linear projections to other nodes.
- **Quadratic programs** optimize encoding and decoding matrices iteratively for all nodes.
- A **generalization** of the **Karhunen-Loève Transform** to noisy multi-layer networks.

Single Layer Networks

- An elementary network: (a) ***ideal*** (b) ***noisy***

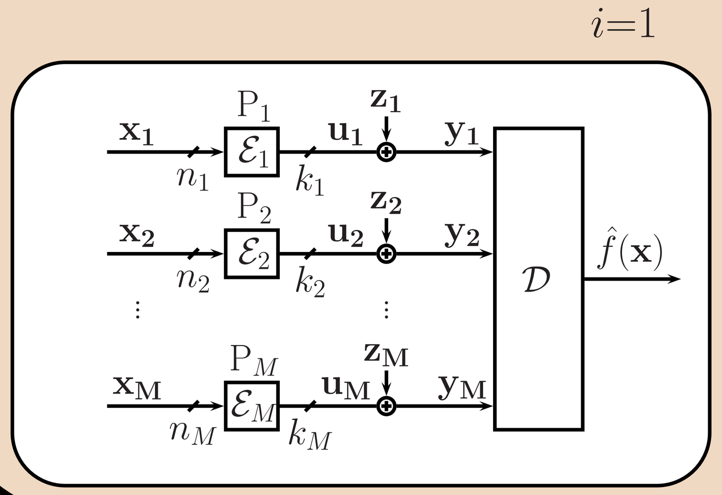


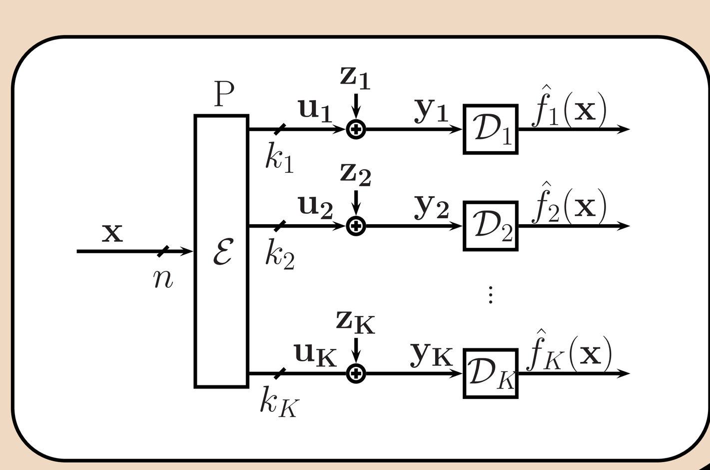
- Decoding performance depends on LCN α compression ratios and SNR signal-to-noise ratios:

$$\alpha_{E-LCN} = \frac{k}{n} \qquad SNR_{E-LCN} = \frac{E\left[\|\mathbf{u}\|_2^2\right]}{E\left[\|\mathbf{z}\|_2^2\right]} = \frac{Tr\left[\Sigma_{\mathbf{u}}\right]}{Tr\left[\Sigma_{\mathbf{z}}\right]}$$

- The goal is to minimize the sum distortion of K decoders. Decoders reconstruct *linear functions* of high-dimensional decentralized sources.

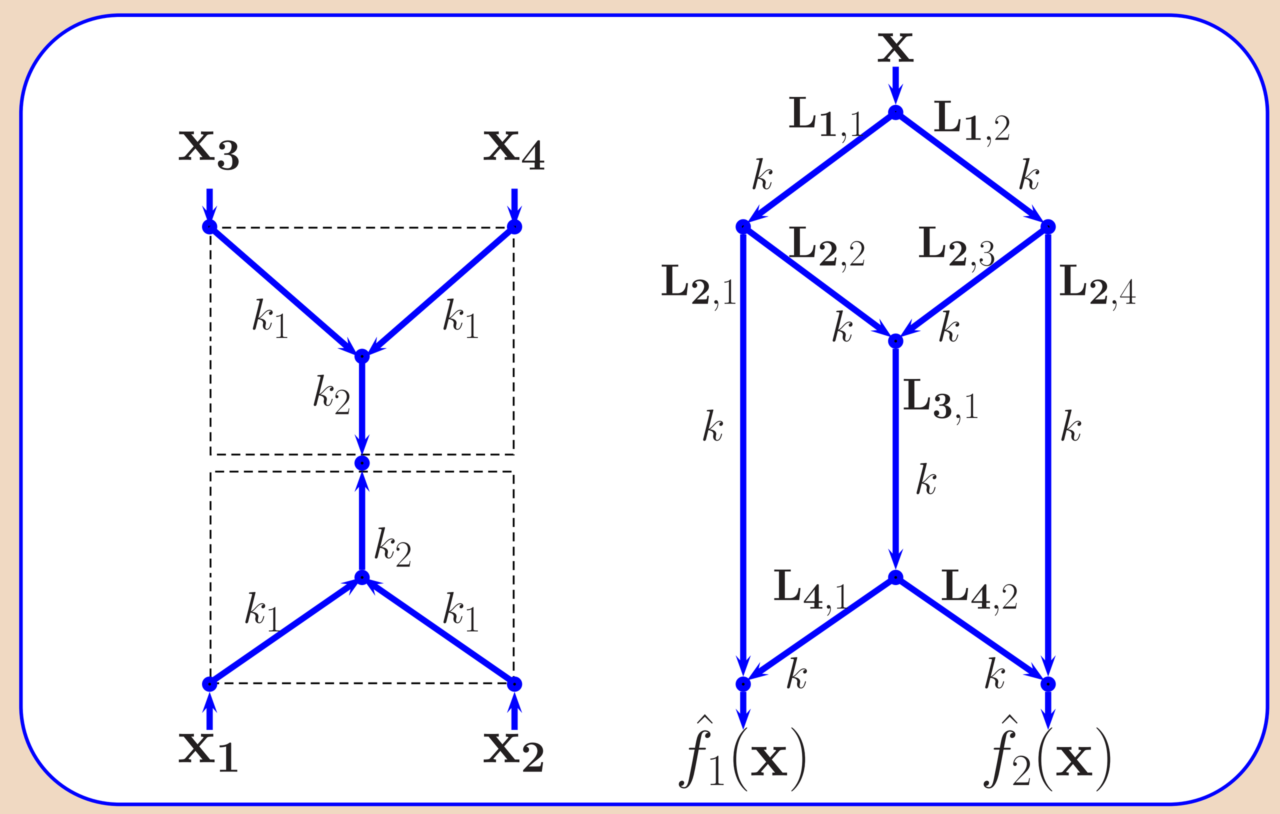
$$D_{MSE,\,K} = \sum_{i=1}^{K} E\left[\|f_i(\mathbf{x}) - \hat{f}_i(\mathbf{x})\|_2^2\right]$$





Multi-Layer Networks

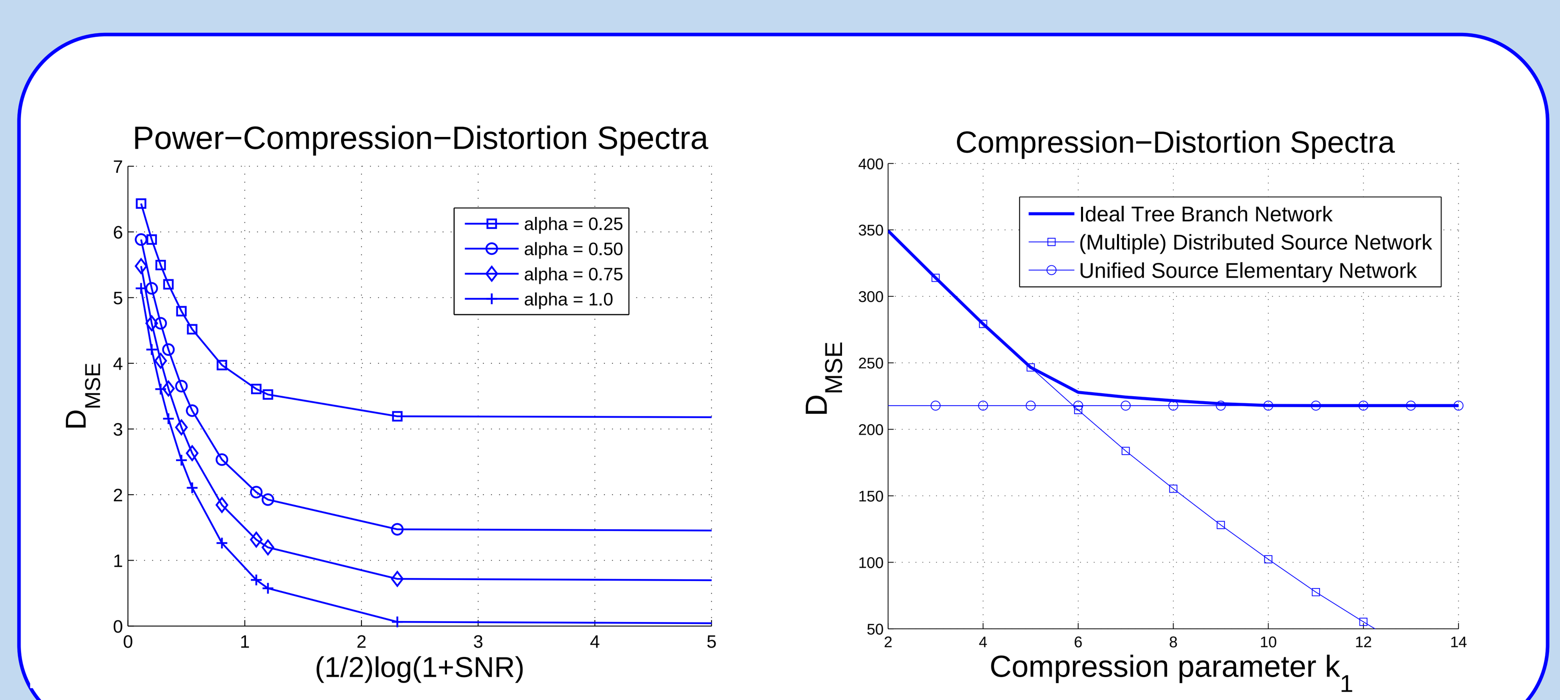
- Single layer distributed processing units are cascaded to form **multi-layer** networks.
- p-layer LCNs include **tree structures** ($p = 2$, *left*) and **butterfly networks** ($p = 4$, *right*).



- Both *ideal* and *noisy* acyclic networks are analyzed.
- In larger networks, concepts such as **min-cut capacity**, **multi-cast limits**, and **delay** emerge.

Power-Compression-Distortion Spectra

- The optimization of linear transforms in networks is measured by **power-compression-distortion spectra**.
- The spectra for a multiple-source single layer noisy LCN (*left*) and an ideal multi-layer tree branch LCN (*right*):



Iterative QCQP and QP Convex Optimization

- In single layer networks, optimizing the encoding layer $\mathbf{L_1}$, given a *fixed* decoding layer $\mathbf{B}$, is possible through convex optimization:

$$\begin{aligned} &\min_{\mathbf{L_1}} \; E[\|\mathbf{x} - \mathbf{B}(\mathbf{L_1}\mathbf{x} + \mathbf{z})\|^2] \\ &s.t. \quad Tr[\mathbf{L_1}\Sigma_{\mathbf{x}}\mathbf{L_1}^T] \le P. \end{aligned}$$

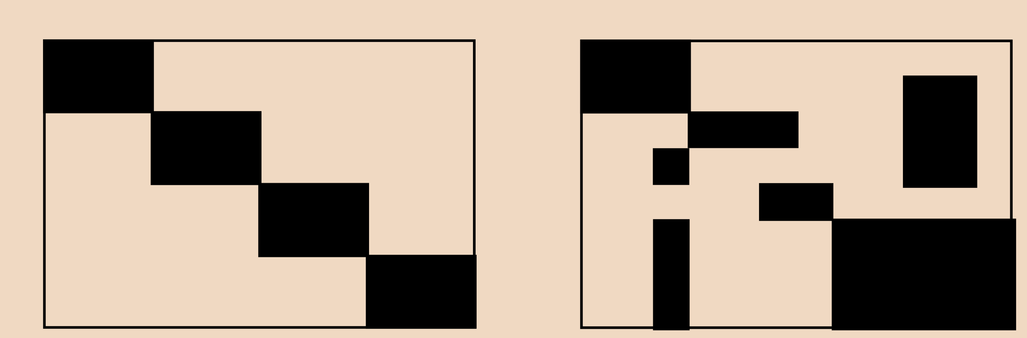
  The above minimization is reduced to an **equivalent** quadratically constrained quadratic program (**QCQP**):

$$\begin{aligned} &\min_{\tilde{\mathbf{c}}} \; \tilde{\mathbf{c}}^T\mathbf{Q_0}\tilde{\mathbf{c}} + \mathbf{q}^T\tilde{\mathbf{c}} + r \\ &s.t. \quad \tilde{\mathbf{c}}^T\mathbf{Q_1}\tilde{\mathbf{c}} \le P. \end{aligned}$$

  where $\mathbf{Q_0} \succeq \mathbf{0}$, $\mathbf{Q_1} \succeq \mathbf{0}$, $\mathbf{q}$, and r are calculated.
- Fixing encoding layer $\mathbf{L_1}$, the decoding layer $\mathbf{B}$ is optimized via a quadratic program (**QP**).
- An **iterative** algorithm initializes, then refines both encoding and decoding layers step by step.
- Distortion $D_{MSE,K}$ is **non-increasing** over iterations.
- Framework **generalizes** to noisy multi-layer networks.

Connection to Network Coding, Sparse PCA

- As in **network coding**, non-trivial processing at intermediary nodes (e.g. in the butterfly network) improves decoding performance. The LCN also includes noisy channels which is different from network coding.
- The iterative QCQP algorithm allows optimization of **distributed** (block-diagonal) transforms as well as **sparse** (random structure) transforms:



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