Optimal Spectrum Management In Multiuser Interference Channels

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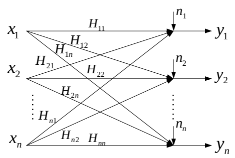
System Model

Multiuser Interference Channel



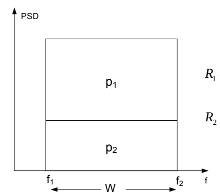
- Users share a common band
- Continuous frequency channel
- Normalized channel gains and noise power: $\begin{cases} \alpha_{ji}(f) \triangleq \frac{|H_{ji}(f)|^2}{|H_{ii}(f)|^2} \\ N_i(f) \triangleq \frac{\sigma_i(f)}{|H_{ii}(f)|^2} \end{cases}$
- Assumption: Treat interference as noise
- Individual user's rate: $R_i = \int_{f_1}^{f_2} \log(1+\frac{P_i(f)}{N_i(f)+\sum_{j\neq i} P_j(f)\alpha_{ji}(f)})df$

Intuitions in the Form of OSM

- The effect of coupling strength: Strong -> FDMA; Weak -> freq. sharing
- The effect of power budget: Large -> FDMA; Small -> freq. sharing

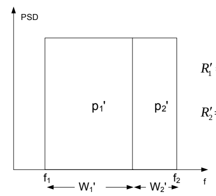
Two Basic Co-existence Strategies and One Basic Transformation

- Flat Frequency Sharing vs. Flat FDMA
- Flat FDMA power re-allocation $W_1' = \frac{p_1}{p_1+p_2}W,\ W_2' = \frac{p_2}{p_1+p_2}W,\ p_1' = p_2' = p_1 + p_2$



$R_1 = W\log\left(1+\frac{p_1}{n_1+p_2\alpha_{21}}\right)$

$R_2 = W\log\left(1+\frac{p_2}{n_2+p_1\alpha_{12}}\right)$



$R_1' = \frac{p_1}{p_1+p_2}W\log\left(1+\frac{p_1+p_2}{n_1}\right)$

$R_2' = \frac{p_2}{p_1+p_2}W\log\left(1+\frac{p_1+p_2}{n_2}\right)$

- Why all flat? Because they are the building blocks of all non-flat cases

Strong Interference Scenario: the Conditions for the Optimality of FDMA

Objective

- The Condition on Channel Parameters under which FDMA is guaranteed to be optimal regardless of power constraints.

Interpretations / Properties of the Condition

- Is this condition *universal*?

  Does this condition apply to achieving *All* n-user Pareto optimal points?
- Is this condition "*pairwise*" ?

  Does whether two users should be orthogonalized only depends on the channel parameters between themselves?
- Is this condition on channel parameters the *weakest possible* ?

The Condition for the Optimality of FDMA within Strongly Coupled Users

- ***Theorem 1***. (2-user flat channel case)

  As long as $\alpha_{12} \geq \frac{1}{2}, \alpha_{21} \geq \frac{1}{2}$, we have $R_1' \geq R_1$ *and* $R_2' \geq R_2$

  i.e., by a flat FDMA re-allocation of a flat frequency sharing, *both* users' rates will increase (or stay unchanged.)
- n-user flat channel case

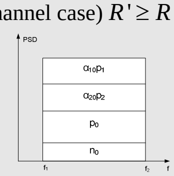
  As long as $\alpha_{ji} \geq \frac{1}{2}, \forall i \neq j$, we have $R_i' \geq R_i$, *for all* i
- n-user frequency selective channel case: As long as $\alpha_{ji}(f) \geq \frac{1}{2}, \forall i \neq j, \forall f \in (f_1, f_2)$, FDMA between all users are preferred by all users. (Universal, but Group-wise condition)

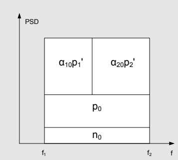
FDMA within a Subset of Users Benefits All Other Users

- 2 interferers (1, 2) flat channel, do a flat FDMA re-allocation
- ***Theorem 2***. (2-interferer flat channel case) $R' \geq R$

$$R = W\log\left(1+\frac{p_0}{\alpha_{10}p_1+\alpha_{20}p_2+n_0}\right)$$

$$R' = \frac{p_1}{p_1+p_2}W\left(1+\frac{p_0}{\alpha_{10}(p_1+p_2)+n_0}\right) + \frac{p_2}{p_1+p_2}W\left(1+\frac{p_0}{\alpha_{20}(p_1+p_2)+n_0}\right)$$





i.e., by a flat FDMA re-allocation of user 0's two interferers (user 1 and 2), user 0's rate will increase (or stay unchanged,) regardless of all the channel/coupling conditions.

- Also true in n-interferer flat channel case and n-interferer frequency selective channel case.

Conclusions

- Among all the existing users 1, 2, 3, … , n, for any two users i and j, within any sub-band (f_1', f_2') , if $\alpha_{ji}(f) \geq \frac{1}{2}$ *and* $\alpha_{ij}(f) \geq \frac{1}{2}, \ \forall f \in (f_1', f_2')$

  then no matter from which user's perspective (1, 2, 3, … , n,) an FDMA between user i and j within this sub-band is always preferred.

--- *Universal, Pairwise and the Weakest* (not only sufficient, but also necessary)

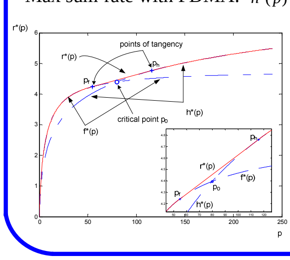
General Interference Scenario

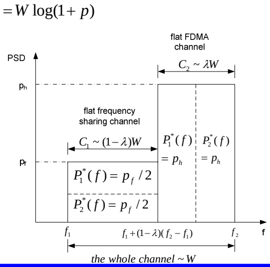
Sum-rate maximization in two-user symmetric channels with equal power constraint

- Flat channel cases $\max_{P_1(f),P_2(f)} R_1 + R_2,\ \ s.t. \int_{f_1}^{f_2} P_i(f)df \leq \frac{P}{2},\ \ P_i(f) \geq 0,\ i = 1, 2.$

Max sum-rate with flat frequency sharing: $f^*(p) = 2W\log\left(1+\frac{p/2}{1+\alpha p/2}\right)$

Max sum-rate with FDMA: $h^*(p) = W\log(1+p)$





$r^*(p)$ is defined to be the convex hull of $f^*(p)$ and $h^*(p)$. ***Theorem 3***: $r^*(p)$ is the optimal value of the sum-rate maximization in two-user symmetric flat channels.

- Frequency selective channel cases: primal domain convex optimization

$$\max_{\tilde{p}(f)} \int_{f_1}^{f_2} r^*(\tilde{p}(f); f)df,\ \ s.t.\ \int_{f_1}^{f_2} \tilde{p}(f)N(f)df \leq P,\ \ \tilde{p}(f) \geq 0, \forall f \in (f_1, f_2)$$

The General Problem

- Problem formulation: $\max_{P_i(f), i=1,2,\ldots,n} \sum_{i=1}^{n} w_i R_i,\ \ s.t. \int_{f_1}^{f_2} \mathbf{P}(f)df \leq \mathbf{P},\ \ \mathbf{P}(f) \geq 0, \forall f \in (f_1, f_2)$
- Define rate density function $r(\mathbf{P}(f); f) \triangleq \sum_{i=1}^{n} w_i \log(1+\frac{P_i(f)}{N_i(f)+\sum_{j\neq i} P_j(f)\alpha_{ji}(f)})$

  and its convex hull $r^*(\mathbf{P}(f); f)$
- Original problem $(a)\ \max_{P_i(f), i=1,2,\ldots,n} \int_{f_1}^{f_2} r(\mathbf{P}(f); f)df$ $s.t. \int_{f_1}^{f_2} \mathbf{P}(f)df \leq \mathbf{P},\ \ \mathbf{P}(f) \geq 0, \forall f \in (f_1, f_2)$ $\Leftrightarrow$ The relaxed problem $(\hat{a})\ \max_{P_i(f), i=1,2,\ldots,n} \int_{f_1}^{f_2} r^*(\mathbf{P}(f); f)df$ $s.t. \int_{f_1}^{f_2} \mathbf{P}(f)df \leq \mathbf{P},\ \ \mathbf{P}(f) \geq 0, \forall f \in (f_1, f_2)$
- Alternative proof of the zero duality gap theorem

$\{Strong\ Duality\ for\ (\hat{a}) \Rightarrow \hat{p}^* = \hat{d}^*;\ \hat{g}(\boldsymbol{\lambda}) \geq g(\boldsymbol{\lambda}), \forall \boldsymbol{\lambda} \geq 0 \Rightarrow \hat{d}^* \geq d^*; Weak\ Duality\ for\ (a) \Rightarrow\ d^* \geq p^*\} \Rightarrow \hat{p}^* = \hat{d}^* \geq d^* \geq p^*,$

$Equivalence\ of\ (a)\ and\ (\hat{a})\ \Rightarrow\ \hat{p}^* = p^*\ \Rightarrow\ \hat{p}^* = \hat{d}^* = d^* = p^*$