The Gaussian Interference Channel with a Potent Relay

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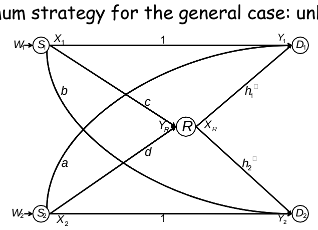
Motivation

- Cooperation and interference play crucial roles in wireless networks.
- Interference relay channel: characterize both cooperation and interference.
- Optimum strategy for the general case: unknown.



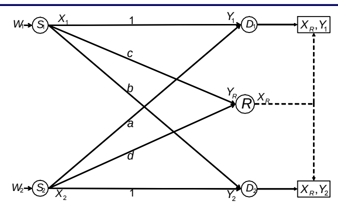
Known Results

- Achievable rate using DF [Sahin-Erkip 2007]
- Cognitive relaying with one-sided interference [Sahin-Erkip 2008]
- Interference channel with an infrastructure relay [Sahin-Erkip 2009]
- Interference forwarding [Maric-Goldsmith 2008]
- For the general (fully connected) case, finding a good (tight) outerbound has eluded the community.

Methodology

- Assume the relay has ample amount of power.
- Relay power $\rightarrow \infty \Rightarrow$ obtain an outerbound for the channel.
- How good is it?
- Potent relay model is equivalent as SIMO interference channel where two receivers share a common antenna.
- Are recently investigated techniques for the interference channel useful?
- Consider weak & strong interference cases.

Channel Model



$$Y_1 = X_1 + aX_2 + Z_1$$
$$Y_2 = bX_1 + X_2 + Z_2$$
$$Y_R = cX_1 + dX_2 + Z_R$$

Weak Interference

- Use "smart" and "useful" genie [Annapureddy-Veeravalli 2008].

$$S_1 = bX_1 + bN_1, S_2 = aX_2 + aN_2$$
$$S_R = cX_1 + cN_3, T_R = dX_2 + dN_4$$

- Give minimum but sufficient amount of information to receivers such that i.i.d. Gaussian inputs maximize the capacity.
- "Useful" in the sense that i.i.d. Gaussian distributed inputs satisfying the power constraints maximize the sum rate.

Weak Interference (contd.)

$$\begin{aligned}
&n(R_1+R_2)\\
&\le I(X_1^n;Y_1^n,X_R^n)+I(X_2^n;Y_2^n,X_R^n)\\
&\le I(X_1^n;Y_1^n,X_R^n,Y_R^n)+I(X_2^n;Y_2^n,X_R^n,Y_R^n)\\
&= I(X_1^n;Y_1^n,Y_R^n)+I(X_2^n;Y_2^n,Y_R^n)\\
&\le I(X_1^n;Y_1^n,Y_R^n,S_1^n,S_R^n)+I(X_2^n;Y_2^n,Y_R^n,S_2^n,T_R^n)\\
&\le nI(X_1;Y_1,Y_R,S_1,S_R)+nI(X_2;Y_2,Y_R,S_2,T_R)
\end{aligned}$$

- "Smart" in the sense that the genie information does not increase the capacity too much --- guarantee the "minimum amount of information".

$$\begin{aligned}
&I(X_1;Y_1,Y_R,S_1,S_R)+I(X_2;Y_2,Y_R,S_2,T_R)\\
&= I(X_1;Y_1,Y_R)+I(X_2;Y_2,Y_R)
\end{aligned}$$

Weak Interference (contd.)

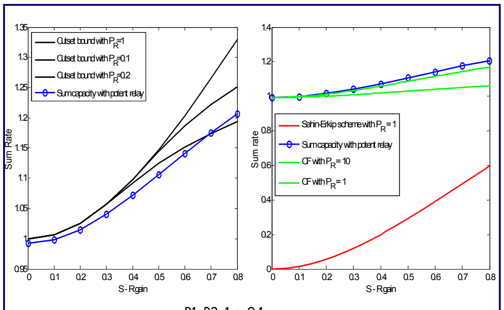
- To guarantee the existence of such a genie, we need

$$\frac{\rho_1^2}{(1+a^2P_2)^2}+\frac{c^2\rho_3^2}{(1+d^2P_2)^2}\ge\frac{b^2}{1-\rho_2^2}+\frac{c^2}{1-\rho_4^2}$$
$$\frac{\rho_2^2}{(1+b^2P_1)^2}+\frac{d^2\rho_4^2}{(1+c^2P_1)^2}\ge\frac{a^2}{1-\rho_1^2}+\frac{d^2}{1-\rho_3^2}$$
$$\rho_1,\rho_2,\rho_3,\rho_4\in[0,1]$$

- Symmetric case shows us this condition implies weak interference.
- The sum capacity upperbound is

$$C_\Sigma=\frac{1}{2}\log\Big(1+\frac{(ac-d)^2P_1P_2+P_1+c^2P_1}{(a^2+d^2)P_2+1}\Big)+\frac{1}{2}\log\Big(1+\frac{(bd-c)^2P_1P_2+d^2P_2+P_2}{(c^2+b^2)P_1+1}\Big)$$

Performance under Weak Interference



P1=P2=1 a=0.1
Symmetric case

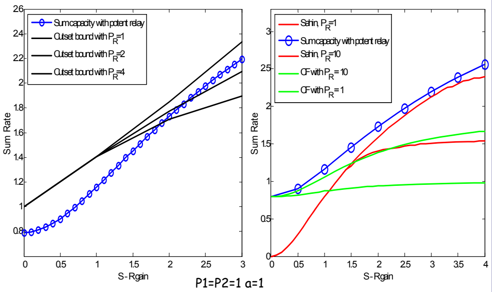
Strong Interference

- When interference is strong, i.e., $a\ge 1, b\ge 1$.
- The channel has the same capacity as a compound SIMO MAC with two antennas at each receiver, where one common one is shared between the two receivers.
- The capacity of the compound SIMO MAC is

$$R_1\le\frac{1}{2}\log(1+P_1+c^2P_1)$$
$$R_2\le\frac{1}{2}\log(1+P_2+d^2P_2)$$
$$R_1+R_2\le\frac{1}{2}\min\{\log(1+(ac-d)^2P_1P_2+(c^2+1)P_1+(d^2+a^2)P_2),$$
$$\log(1+(bd-c)^2P_1P_2+(d^2+1)P_2+(c^2+d^2)P_1)\}$$

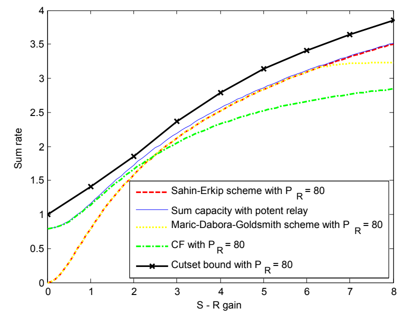
- Can serve as an outerbound for the Gaussian interference relay channel.

Performance under Strong Interference



P1=P2=1 a=1
Symmetric case

Performance when relay has large power



Conclusions

- The outerbound obtained is tighter than the cutset bound for GIFRC in the following scenarios:
  - When source-relay gains are weak.
  - When relay's power is large.
- CF scheme performs better than DF scheme when source-relay gain is weak.
- When relay's power is large, CF scheme is near optimal under weak source-relay gain.
- When relay's power is large, DF scheme is near optimal under strong source-relay gain.

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