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Delay Region for One-to-Many File Transfer in P2P networks

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Problem Statement and Motivation

Motivation

- Motivated by P2P file transfer applications (e.g., BitTorrent) on the Internet.
- Consider the problem of delivering a file from a server to multiple receivers in a P2P network.
- Aim at understanding the optimal delay (download time) region, i.e., the set of all achievable delay vectors.
- Assume that all peers are cooperative in order to focus to the fundamental limit of the delay region.

Previous Work

- Minimizing the maximum delay of the receivers in a P2P network. [Mutualcast,SIGCOMM05]
  - A routing-based scheme achieve the optimal.
  - Peers don't have download limits.
- Minimizing the weighted average delay of the receivers in a P2P network. [DelayRegion,ISIT09]
  - Find the optimal in polynomial time if the downloading order of the receivers is given.
  - A routing-based scheme has performance very close to the optimal.
  - Peers don't have download limits.

System Model

- Consider a full-mesh P2P network with one server and N heterogeneous peers.
- The server with upload bandwidth u_s has a file with size unit 1.
- Each peer with upload bandwidth ui and larger download bandwidth di wants to download the file and has weighting w_i.
- A transmission rate allocation is characterized by r_{ij} and r_{si}.
- Object: Find the optimal static rate allocation to minimize the weighted average delay.

Optimal Solution, Lower Bound, LT-Code-Based Scheme and Routing-Based Scheme

Optimal Rate Allocation

- For a given rate allocation r_{ij} and r_{si}, the download rate of each peer is limited by the min-cut bound.
- By network coding theory, we can achieve the min-cut bound for each peer simultaneously.
  - In other words, there exists a linear network codes such that peer i can download r_i independent packets if the min-cut bound for peer i is r_i.
- **Theorem:** Convex optimization can find the optimal rate allocation to achieve the minimum weighted delay.

$$\text{minimize} \quad \sum_{k=1}^{N} w_k \cdot \frac{1}{r_k}$$

$$\text{subject to} \quad f_{kj}^{(k)} \le r_{kj}, \forall k, j; \;\; f_{kj}^{(k)} = 0, \forall j \ne k$$

$$f_{ij}^{(k)} \ge 0, \forall i, j, k; \;\; \sum_{j=1}^{N} f_{jk}^{(k)} = r_k, \forall k$$

$$\sum_{j=1}^{N} f_{ji}^{(k)} = \sum_{j=1, j\ne i}^{N} f_{ij}^{(k)}, \forall i \ne k$$

$$r_{ij} \ge 0, \forall i, j; \;\; \sum_{j=1}^{N} r_{ji} \le d_i, \forall i$$

$$\sum_{j=1, j\ne i}^{N} r_{ij} \le u_i, \forall i; \;\; \sum_{i=1}^{N} r_{ii} \le u_s$$

- **Corollary:** The optimal solution can be found in polynomial time.

Analytical Lower Bound

- Relaxing the constraints in the convex optimization.
- The sum of the download rates of peers is limited by the total upload bandwidth resource.
- The download rate of each peer is limited by its download bandwidth and the server upload bandwidth.

$$\text{minimize} \quad \sum_{k=1}^{N} w_k \cdot \frac{1}{r_k}$$

$$\text{subject to} \quad \sum_{k=1}^{N} r_k \le u_s + \sum_{k=1}^{N} u_k$$

$$0 \le r_k \le \min(d_k, u_s), \forall k$$

- This convex optimization has a water-filling-type solution.

$$r_k = \begin{cases} \sqrt{w_k} \cdot R & \text{if } \sqrt{w_k} \cdot R < \min(d_k, u_s) \\ \min(d_k, u_s) & \text{if } \sqrt{w_k} \cdot R \ge \min(d_k, u_s) \end{cases}$$

where R is chosen such that

$$\sum_{k=1}^{N} r_k = \min(u_s + \sum_{k=1}^{N} u_k, \sum_{k=1}^{N} \min(d_k, u_s)).$$

- **Theorem:** This lower bound can be achieved by routing-based scheme when all weights are equal.
- **Corollary:** This lower bound can be achieved by routing-based scheme when

$$u_s + \sum_{k=1}^{N} u_k \ge \sum_{k=1}^{N} \min(d_k, u_s).$$

LT-Code-Based Scheme

- The server can use simple tree structures to distribute the content.
- Step 1: The server sends LT coded packets to peers directly.
- Step 2: Each peer copy and forward the LT-coded packets it received to other peers.
- The rate allocation for the LT-code-based scheme has some more constraints.

$$r_{ij} \le r_{ii}, \forall i, j, \quad \text{and} \quad r_k = \sum_{j=1}^{N} r_{jk}, \forall k.$$

- We provide a fast algorithm to find a suboptimal rate allocation for LT-code-based scheme.

Routing-Based Scheme

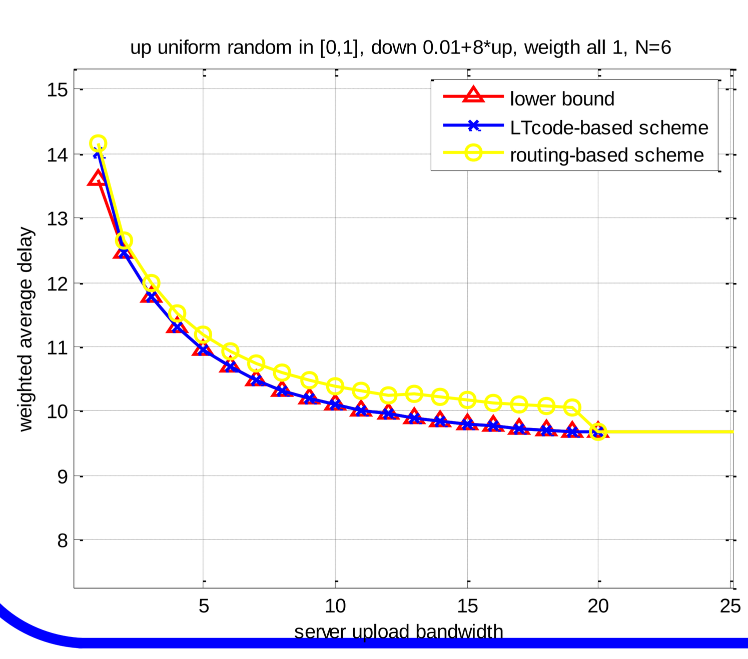
- Type-1 Tree: The server sends some packets to peers directly and these packets are not forwarded.
- Type-2 Tree: The server sends some packets to peers, and each peer copy and forward the packets it received to other peers.
- The rate allocation for the routing-based scheme has some more constraints than that of the LT-code-based scheme.
- We provide a fast algorithm to find a suboptimal rate allocation for routing-based scheme.

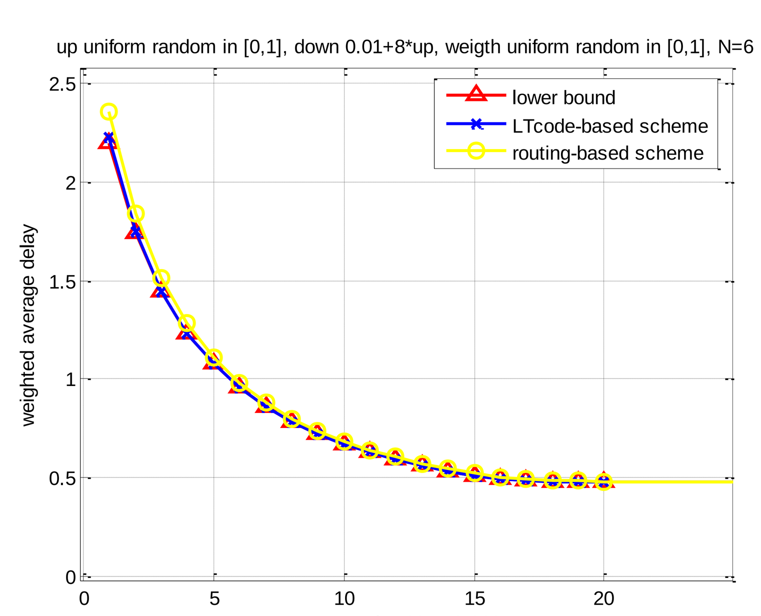
Simulation Results and Conclusion

Simulation Setting

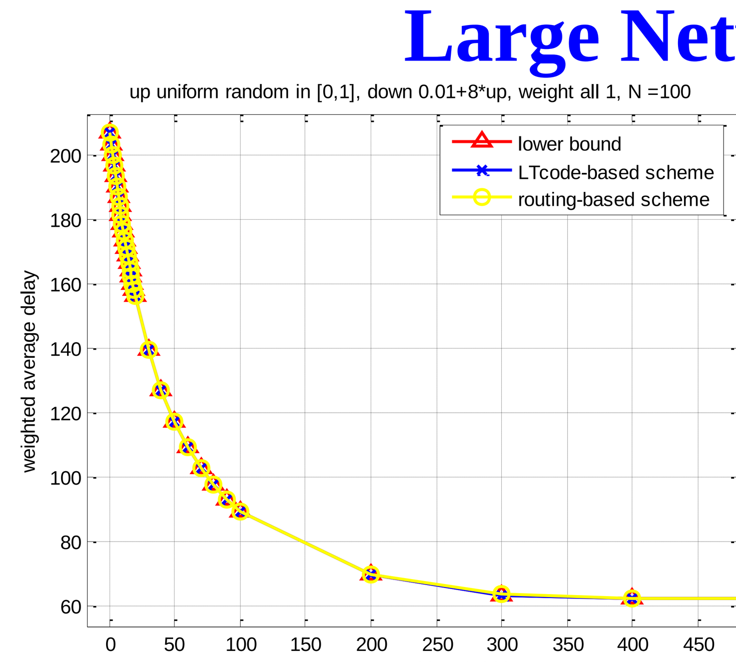
- In all the simulations, each peer's upload bandwidth is uniformly randomly picked in [0,1].
- Each peer's download bandwidth is the sum of 8 times its upload bandwidth and 0.01.

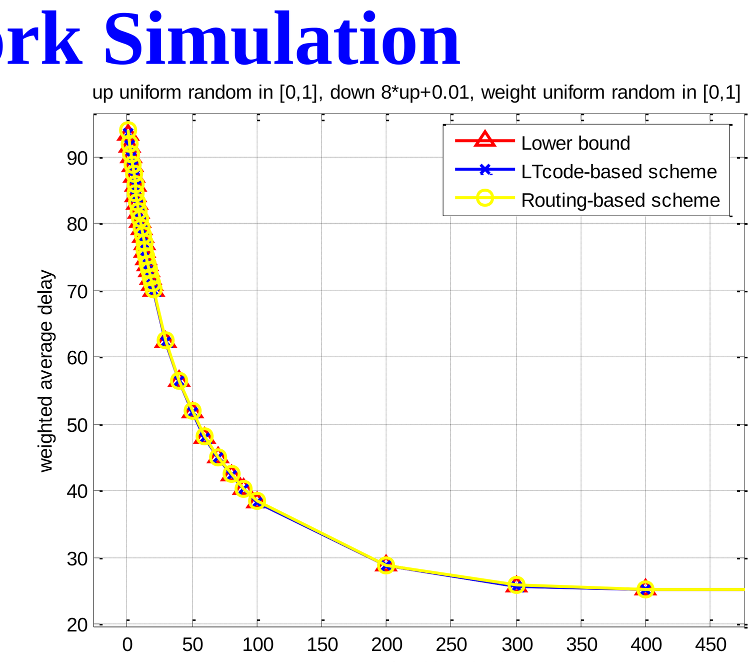
Small Network Simulation





Large Network Simulation





Conclusion and Future Work

- LT-code-based schemes and routing-based schemes can achieve performance very close to the optimal.
- Future Work: Extend to the dynamic network solution, i.e., the rate allocation can be updated when one or multiple peers finish downloading.