Feedback Communication via Stochastic Approximation

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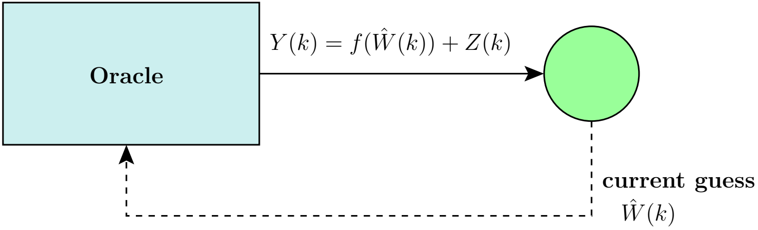
System Model Assumptions

- Point-to-point communication across continuous AWGN channel.
- Messages are pulse-amplitude modulated $\Rightarrow$ Discrete-time equivalent.
- Average power constraint - P_{av}.
- No constraint on bandwidth or peak power.
- Divide unit interval $[0, 1]$ into M disjoint message intervals of equal length.
- Let W be the midpoint of the message interval, representing the corresponding message.

Stochastic Approximation

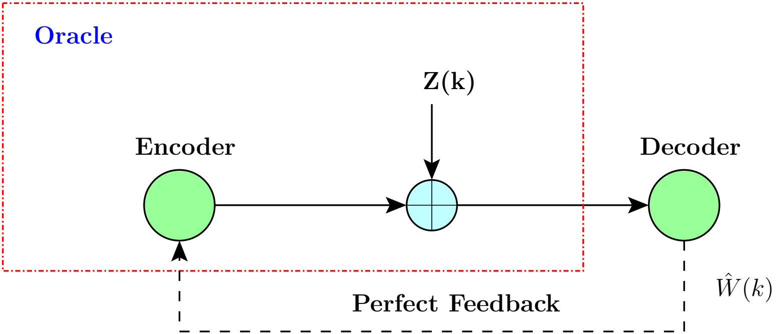
Find zero of unknown function f, if an oracle provides **noisy** values of f at desired points.

Robbins-Munro Algorithm



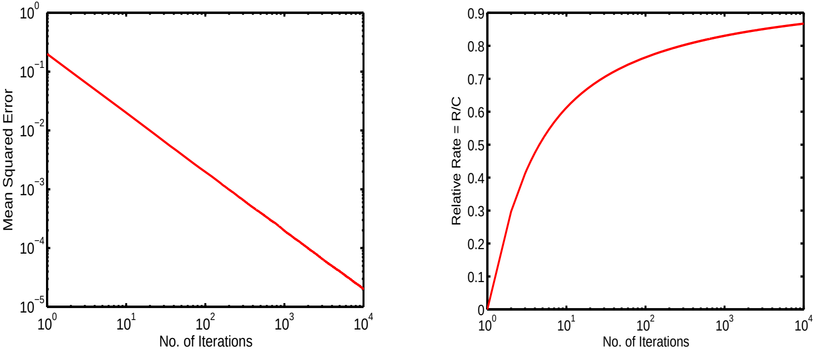
Updates guess as $\hat{W}(k) = \hat{W}(k-1) - \frac{Y(k)}{k}$.

Schalkwijk-Kailath Scheme

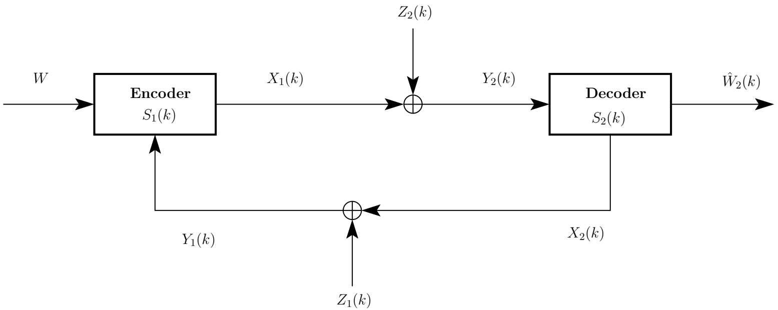


- Decoder calculates zero of the function $f(x) = \alpha(x - W)$.
- Estimate update same as Robbins-Munro.

Performance Curves



Discrete-time Equivalent Channel Model



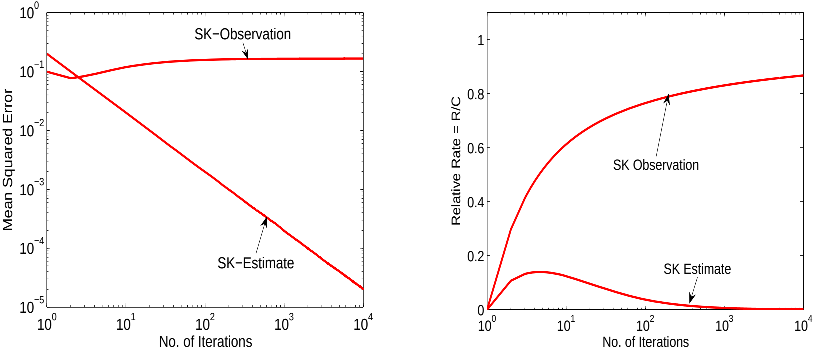
1. X: State variables.
2. Y: Observations.
3. $\hat{W}_2$: Estimate.

Schalkwijk Kailath variants

If feedback is noisy, two strategies

1. **SK-Estimate**
   - Decoder feeds back the estimates.
   - $P_e(n) \longrightarrow 0$ doubly exponentially but!
   - $R(n)/C \longrightarrow 0$.
2. **SK-Observation**
   - Decoder feeds back the observation.
   - $R(n)/C \longrightarrow 1$ but!
   - $P_e(n)$ *performance is very poor.*

Performance Curves



Our Algorithm

- Introduce parameters in addition to α.
  1. γ - Helps decrease the amount of information about feedback noise sent on the forward link.
  2. β - Smoothing of estimates, a second order filter.
- For $\gamma = 0, \beta = 0$, our algorithm reduces to Schalkwijk-Kailath.

Key Differences

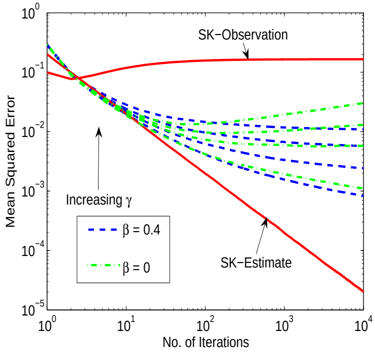
	Schalkwijk-Kailath	Our Scheme
Encoder	$f(.) = \alpha\,(S_1(k) - W)$	$f(.) = \frac{\alpha}{k^\gamma}\,(S_1(k) - W)$
Decoder	$S_2(k) = S_2(k-1) - \frac{Y_2(k)}{\alpha k}$ $\hat{W}_2(k) = S_2(k)$	$S_2(k) = P(k) + Q(k)$ $\hat{W}_2(k) = \frac{(k-1)\hat{W}_2(k-1)}{k} + \frac{S_2(k)}{k}$

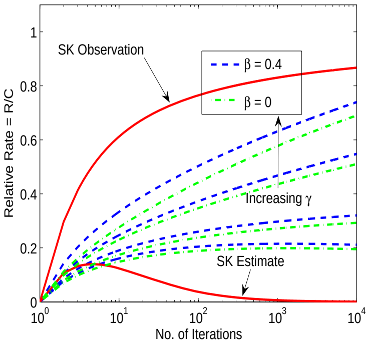
Observation term: $P(k) = S_2(k-1) - \frac{Y_2(k)}{\alpha k^{1-\gamma}}$.

Averaging term: $Q(k) = \frac{\beta}{k^{1-\gamma}}\left(\hat{W}_2(k-1) - S_2(k-1)\right)$.

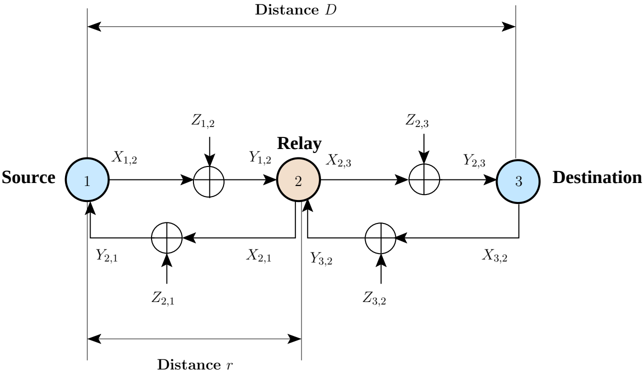
Smoothing: $\hat{W}_2(k)$ equation above.

Numerical Results





System Model



$$Y_{i,j}(k) = a_{i,j}X_{i,j}(k) + Z_{i,j}(k), \quad (1)$$

$$a_{i,j} = bd_{i,j}^{-\eta/2}, \quad (2)$$

Power distribution - $P_{av,1} = \kappa P_{av}$, $P_{av,2} = (1-\kappa)P_{av}$.

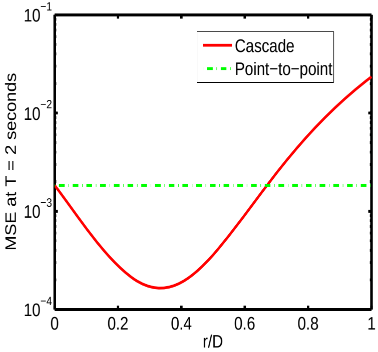
Key Idea : Distributed Stochastic Approximation

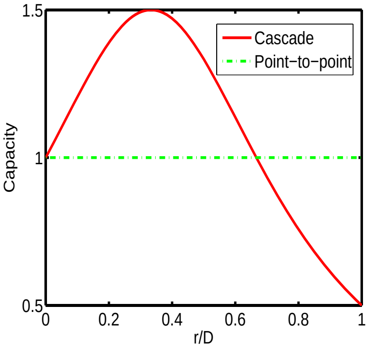
- Consider a vector-valued function $\mathbf{f} : \mathbb{R}^3 \to \mathbb{R}^3$, the i-th component of which is given by

$$\mathbf{f} = \begin{bmatrix} 0 \\ \alpha_1(S_1 - S_2) \\ \alpha_2(S_2 - S_3) \end{bmatrix} \tag{3}$$

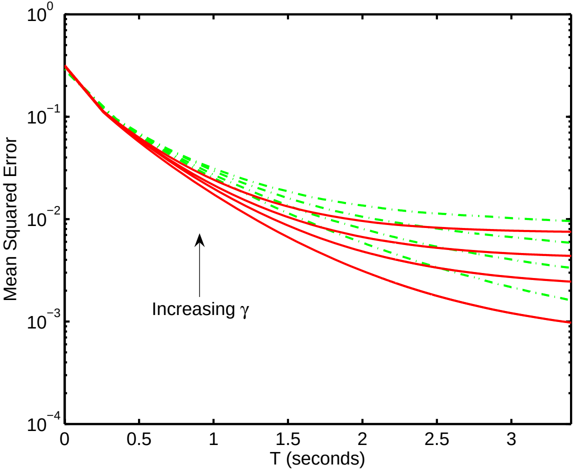
- $S_1(k)$ is fixed at W.
- Distributed stochastic approximation algorithm $\Rightarrow$ $S_2(k) \to W$ and $S_3(k) \to W$.

Performance comparison, $\eta = 2$, $P_{av}D^{-2}/2\sigma^2 = 1$, $\sigma^2 = 0.1$





Cascade with Feedback, $r/D = 0.5, P_{av}D^{-2}/2\sigma^2 = 1, \sigma^2 = 0.1$



Concluding Remarks

- Works for non-Gaussian noise.
- Address noisy feedback scenarios.
- Present new linear schemes that permits us to tradeoff MSE and Rate performance.
- Performance similar for $d > 1$ time steps of delay.