On the Capacity of Deterministic Interference Channels Beyond Two Users

Suvarup Saha, Randall Berry

COMNET Group

EECS Department

Northwestern University

Interference Channels and Deterministic Models

Capacity of a Gaussian Interference Channel

- A long standing open problem
- Recent Advancements made by using the 'deterministic model'
- Capacity of the canonical 2 user IC has been determined upto 1 bit ***[Etkin, Tse, Wang '07]***

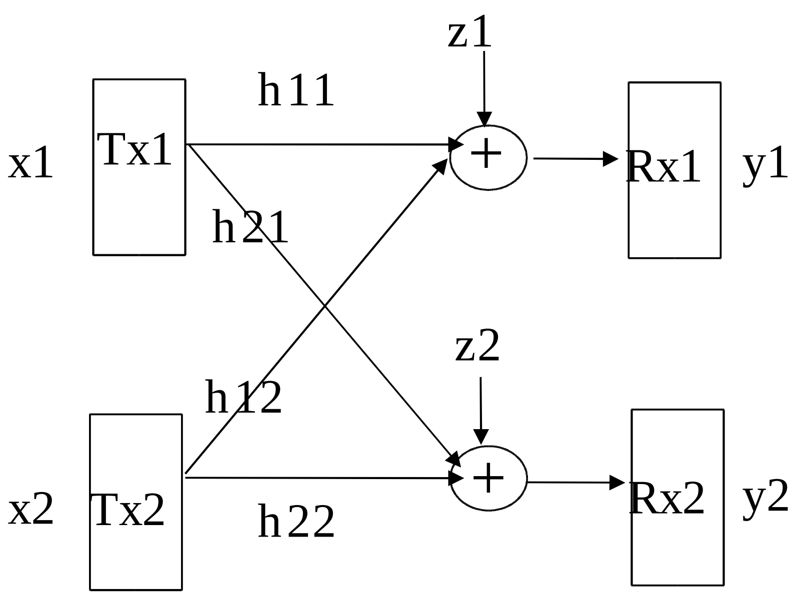


Fig 1. Two User Gaussian Interference Channel

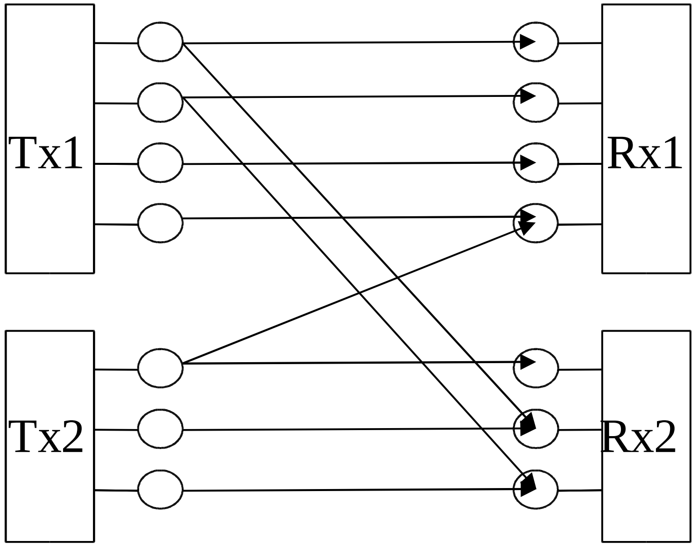


Fig 2. Two User Deterministic Interference Channel

Deterministic Model

- First proposed in the context of Gaussian Relays [***Avestimehr, Diggavi, Tse '07***]
- Naturally models the Gaussian Channel
- Simpler to analyze
- Provides intuition towards 'capacity achieving' schemes in the Gaussian Case

Known Capacity Results for Deterministic IC

Capacity In 2 User Case

- Known for a General Class of Deterministic IC due to [***El-Gamal, Costa, '82***]
- Alternatively derived for a more special case by [***Bresler, Tse '08***]
- Can be achieved by the Han-Kobayashi schemes [***Han, Kobayashi '81***],

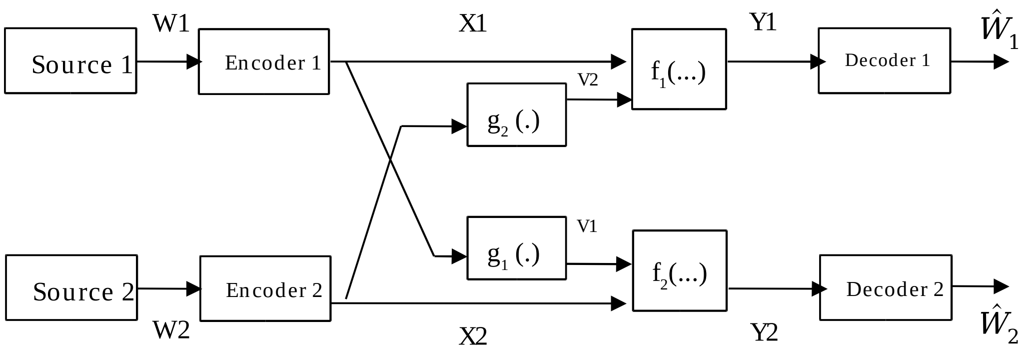


Fig 3. A General Deterministic I C

$$R_1 \le H(Y_1 | V_2)$$
$$R_2 \le H(Y_2 | V_1)$$
$$R_1 + R_2 \le H(Y_1 | V_1\, V_2) + H(Y_2)$$
$$R_1 + R_2 \le H(Y_2) + H(Y_2 | V_1\, V_2)$$
$$R_1 + R_2 \le H(Y_1 | V_1) + H(Y_2 | V_2)$$
$$2R_1 + R_2 \le H(Y_1) + H(Y_1 | V_1\, V_2) + H(Y_2 | V_2)$$
$$R_1 + 2R_2 \le H(Y_1 | V_1) + H(Y_2) + H(Y_2 | V_1\, V_2)$$

Capacity Region for the Model in Figure 3.

Beyond 2 Users

- Capacity is known for Many-to-One and One-to-Many Channels [***Bresler, Parekh, Tse, 08***]
- Three User Lattice Coding strategies for Gaussian Case [***Sridharan, Jafrian, Viswanath, Jafar, Shamai '08***]

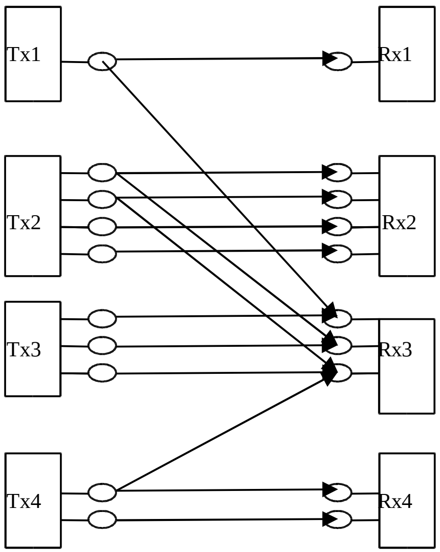
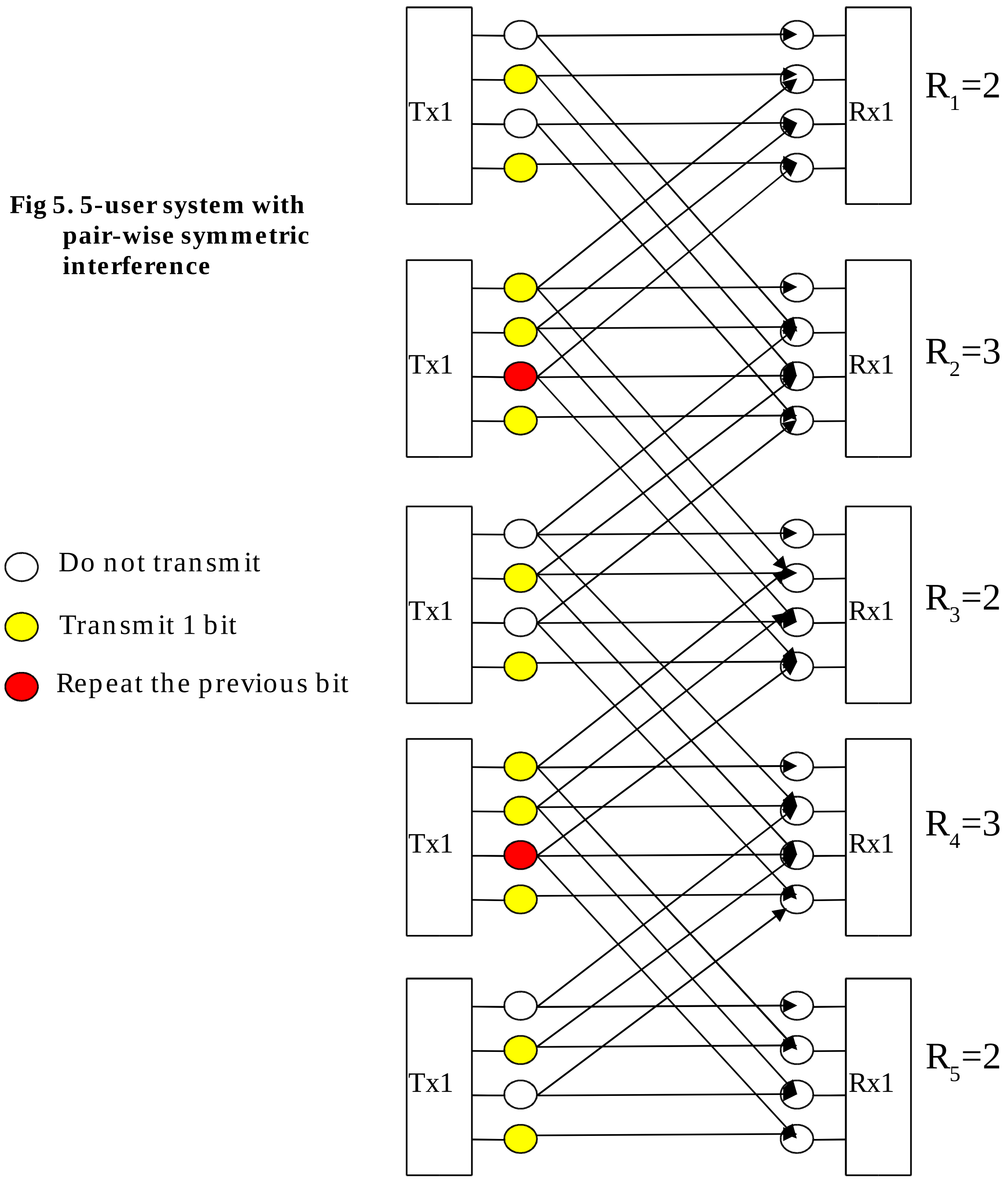


Fig 4. Many -to – One Deterministic IC

New Capacity Results for 'n- user' Pairwise Symmetric Interference

Fig 5. 5-user system with pair-wise symmetric interference



Lemma: For an 'n' user symmetric pairwise interference channel, the sum capacity can be achieved by directly extending the two-user symmetric interference case.

Key Observation : Considering three adjacent users at a time, due to the symmetry in the interference pattern, the achievable two-user rates are also achievable.

In this example, we have $n_{11}=n_{22}=n_{33}=n_{44}=n_{55}=4$ while the symmetry conditions are given by $n_{12}=n_{21}=n_{23}=n_{32}=n_{34}=n_{43}=n_{45}=n_{54}=3$

$R_1=2, R_2=3$ achieves capacity for a two user system with this interference pattern ***[Berry, Tse '08]***

Three User Pairwise Interference System

Motivation

- simplest extension beyond 2 user system motivated by the previous example.

- one user only gets interfered by its two nearest users and thus is just one step beyond a two user system that captures the peculiarity of an 'n' user sytem

- models the interference pattern of cellular base stations in a linear, say highway, network.

A General 3 User Deterministic Interference Model

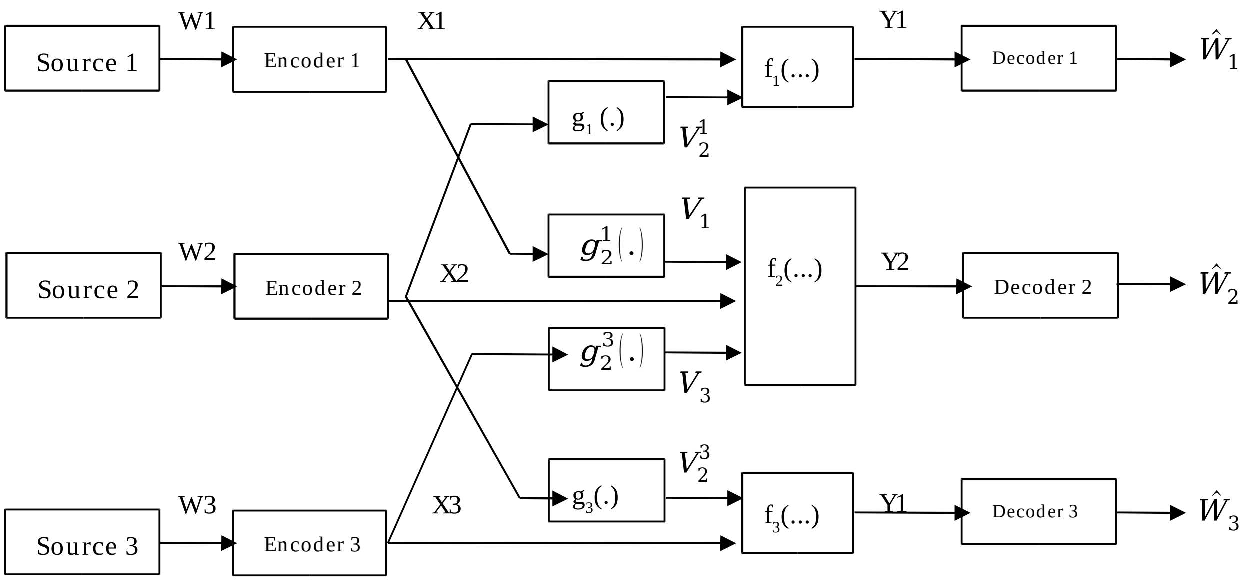


Fig 6. A General 3 user pairwise interfering Deterministic I C

Sum Rate Upper Bounds

Bounds

(1) $R_1 \leq H(Y_1 | V_2)$
(2) $R_2 \leq H(Y_2 | V_1)$
(3) $R_3 \leq H(Y_3 | V_1)$

} Individual rate bounds

(4) $R_1+R_2+R_3 \leq .5*(H(Y_1)+H(Y_1 | V_1\, V_2^1)+H(Y_2 | V_2^1)+H(Y_3)+H(Y_3 | V_3\, V_2^3)+H(Y_3 | V_2^3))$

} Comes from Two-user Sum Rate Bounds

(5) $R_1+R_2+R_3 \leq H(Y_1 | V_1\, V_2^1)+H(Y_2)+H(Y_3 | V_3\, V_2^3)+H(V_1\, V_3)-H(g(V_1\, V_3))$
(6) $R_1+R_2+R_3 \leq H(Y_1)+H(Y_2 | V_1\, V_3\, V_2^1\, V_2^3)+H(Y_3)-I(V_2^1 ; V_2^3)$
(7) $R_1+R_2+R_3 \leq H(Y_1 | V_1)+H(Y_2 | V_2^1\, V_2^3)+H(Y_3 | V_3)+H(V_1\, V_3)-H(g(V_1\, V_3))$
(8) $R_1+R_2+R_3 \leq H(Y_1 | V_1\, V_2^1)+H(Y_2 | V_1\, V_3\, V_2^1\, V_2^3)+H(Y_3)+H(V_1)+H(V_2^1)+H(V_2^1 | V_2^3)-H(V_2^3)$
(9) $R_1+R_2+R_3 \leq H(Y_1)+H(Y_2 | V_1\, V_3\, V_2^1\, V_2^3)+H(Y_3 | V_3\, V_2^3)+H(V_3)+H(V_2^3)+H(V_2^3 | V_2^1)-H(V_2^1)$
(10) $R_1+R_2+R_3 \leq H(Y_1 | V_1\, V_2^1)+H(Y_2)+H(Y_3)+H(V_1)-H(V_2^3)-H(g(V_1\, V_3))$
(11) $R_1+R_2+R_3 \leq H(Y_1)+H(Y_2)+H(Y_3 | V_3\, V_2^3)+H(V_3)-H(V_2^1)-H(g(V_1\, V_3))$

} Three-user Sum Rate Bounds

Observations

1. Sum rate bounds for this model differs from the two user model in terms of the presence of the function g(.) that models the interaction between the interferences from users 1 and 3 at the receiver of user 2.

2. Unlike the 2 user case, uniform input distributions may not maximize the upper bounds in the RHS of all the above inequalities

Scenarios where bounds can be tight - I

A low Interference Scenario

$$n_{11} > n_{12} + n_{21,} \quad n_{22} > max(n_{21,} n_{23}) + max(n_{12}, n_{32}),$$
$$n_{33} > n_{23} + n_{32,}$$

Assuming $n_{21} > n_{23}, n_{12} > n_{32}$
the bound (7) can be reformulated as

$$R_1 + R_2 + R_3 \leq max(n_{12,} (n_{11} - n_{21})^+) + max(n_{32,} (n_{33} - n_{23})^+)$$
$$+ max(max(n_{21}, n_{23}), (n_{22} - max(n_{12,} n_{32}))^+) + min(n_{21,} n_{23})$$
$$= n_{11} - n_{21} + n_{33} - n_{23} + n_{22} - n_{12} + n_{23}$$
$$= n_{11} - n_{21} + n_{33} + n_{22} - n_{12}$$

An achievable scheme: Transmit from all the levels at Tx 1 and Tx3. Transmit from the free levels at Tx 2 which are neither interfered with nor do interfere. This gives a sum rate of

$$n_{11} + n_{33} + (n_{22} - max(n_{12,} n_{32}) - max(n_{21,} n_{23}))^+$$
$$= n_{11} + n_{33} + n_{22} - n_{12} - n_{21}$$

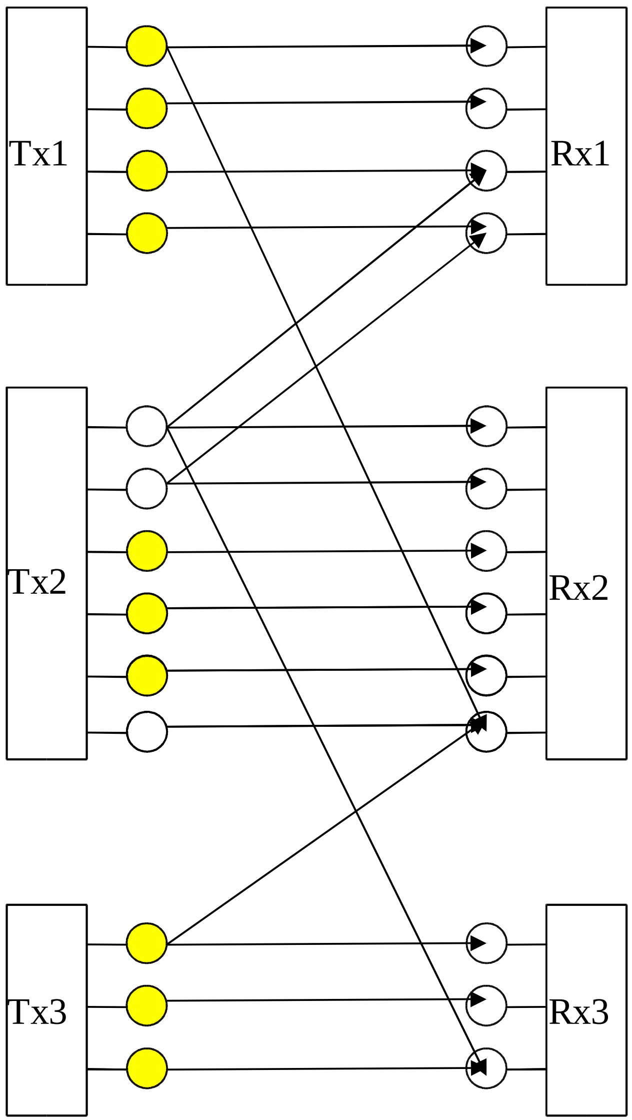
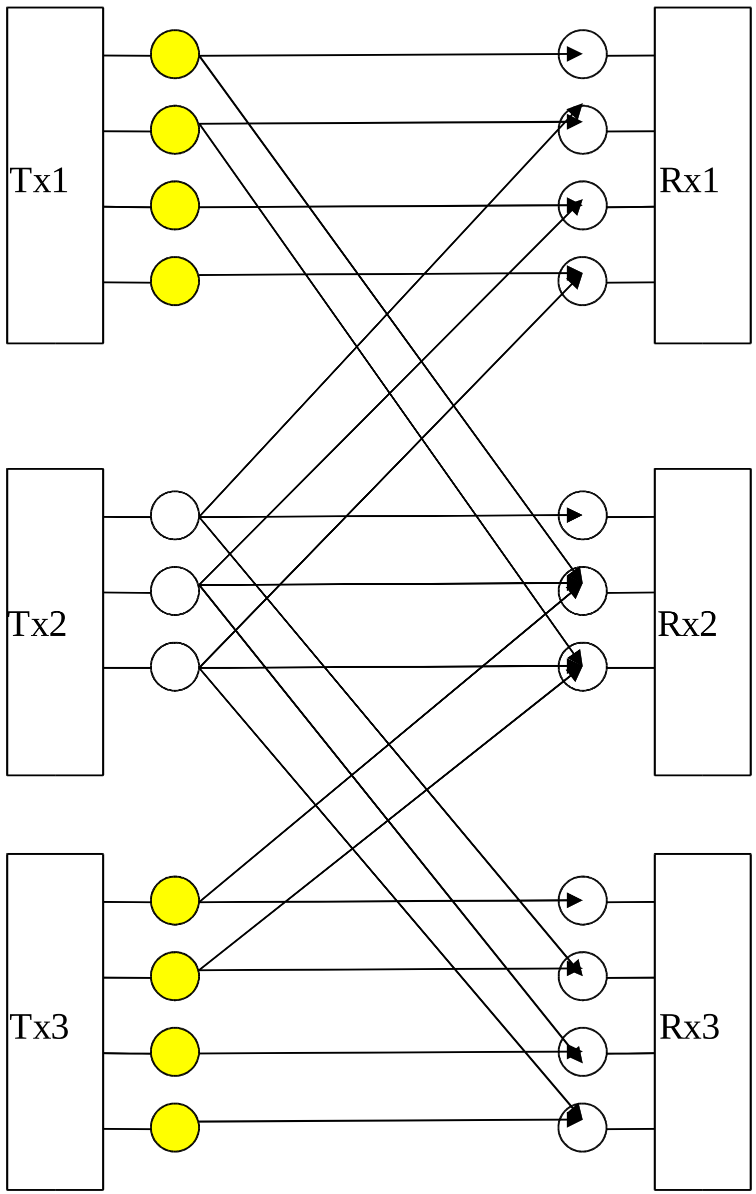


Fig 7. Low Interference Case where we can determine the sum capacity

Scenarios where bounds can be tight - II



Asome-what Symmetric Scenario

The Interference relation between user 1 and user 2 is the same as that between user 3 and user 2, i.e.,

$$n_{12} = n_{32},\ \ n_{21} = n_{23}\ ,n_{11} = n_{33}$$

If such a symmetry exists, either the upper bound (4) or the upper bound derived from 2 user sum-rates is always tight.

In this example, upper bound (4) is tight as

$$2*(R_1+R_2+R_3) \le max(n_{11,}n_{12}) + (n_{11}-n_{21})^+ + max(n_{21,}(n_{22}-n_{12})^+)$$
$$+max(n_{33,}n_{32}) + (n_{33}-n_{23})^+ + max(n_{23,}(n_{22}-n_{32})^+)$$

$$\Rightarrow R_1+R_2+R_3 \le .5*(n_{11}+n_{11}-n_{21}+n_{21}+n_{33}+n_{33}-n_{23}+n_{23})$$
$$= n_{11}+n_{33}$$

An achievable strategy is to transmit from all levels of the users 1 and 3 while user 2 shuts off

Fig 8. Asome-what Symmetric Interference Case where we can determine the sum capacity

What's Next and References

We plan to

- Characterize all the cases where the sum rate upperbounds are tight
- Extend our results to n-users
- Look at the Game Theoretic aspects of the 3 and then n user Interference games on the deterministic channel
- Use all the above results for extending to the Gaussian counterparts

References

1. Gaussian Interference Channel Capacity to Within One Bit[***Etkin, Tse, Wang '07***]
2. The Capacity Region of A Class of Deterministic Interference Channel[***El-Gamal, Costa, '82***]
3. The Approximate Capacity of the Many-to-One and One-to-Many Gaussian Interference Channels [***Bresler, Parekh, Tse, 08***]
4. Information Theoretic Games on Interference Channels[***Berry, Tse '08***]
5. A Detern=ministic Approach to Wireless Relay Networks [***Avestimehr, Diggavi, Tse '07***]
6. A New Acheivable Rate Region for the Interference Channel [***Han, Kobayashi '81***]
7. A Layered Lattice Coding Scheme for a Class of Three User Gaussian Interference Channels [***Sridharan, Jafrian, Viswanath, Jafar, Shamai '08***]
8. The Two-user Interference Channel: A Deterministic View [***Bresler, Tse '08***]