Optimal Transmission for Dying Channels

Meng Zeng[†], Rui Zhang[‡], Shuguang Cui[†]

[†]Wireless Networking and Communications Lab, Texas A&M University
[‡]Institute of Infocomm Research, Singapore

August 3, 2009

Application Scenarios

- In a cognitive radio network or a sensor network, consider a point-to-point communication link subject to a fatal random attack.

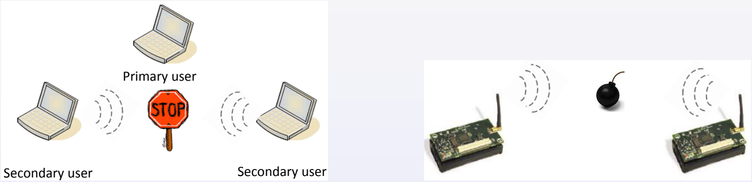

Figure: Application scenarios of dying channel

How fast and reliable is the communication over a dying channel?

Outage Capacity Definition

- Model as a K-block fading channel with a *random* delay constraint.
- Adopt the outage capacity as the metric.

Definition

$$C_{\text{out}}(P,\eta) = \max_{K} \sup_{\mathbf{P}_K : \sum_{i=1}^{K} P_k \le KP} \Bigg\{ R : \Pr\{\frac{1}{K}\sum_{i=1}^{L}\log(1+\alpha_i P_i) < R\} < \eta \Bigg\},$$

where $L = \min(\lfloor T \rfloor, K)$.

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Uniform Power Allocation

- The only variable to optimize is the number of blocks K when assuming uniform power allocation.
- We derive the lower and upper bounds of outage probability.
- When high SNR and Rayleigh fading are assumed, we obtain the closed-form solution for the optimal K.

Lower and Upper Bounds

- Lower bound:

$$Pr\left\{\sum_{i=1}^{L}\log(1+\alpha_i P) < KR\right\} \geq w_0 + \sum_{i=1}^{K-1}\left\{F\left(\frac{e^{KR/i}-1}{P}\right)\right\}^i w_i + \left\{F\left(\frac{e^{R}-1}{P}\right)\right\}^K w_K^*.$$

- Upper bound:

$$\Pr\left\{\sum_{i=1}^{L}\log(1+\alpha_i P) < KR\right\} \leq w_0 + \sum_{i=1}^{K-1}\left\{F\left(\frac{e^{KR}-1}{P}\right)\right\}^i w_i + \left\{F\left(\frac{e^{KR}-1}{P}\right)\right\}^K w_K^*.$$

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Optimal Coding Length

- For Rayleigh fading in high SNR regime, the outage probability is:

$$p_{out}(K) \approx \xi e^{KR} + \frac{1}{P^K e^{(\lambda - R)K}} + w_0,$$

where $\xi = (1 - e^{-\lambda})\frac{\beta/P}{1-\beta/P}, \ \beta = e^{-\lambda}$.

- The optimal K is:

$$K^* = \log\left[\frac{\lambda + \log P - R}{\xi R}\right]\frac{1}{\lambda + \log P}.$$

Choose $\lfloor K^* \rfloor$ or $\lceil K^* \rceil$ that gives the smaller outage probability.

Simulation Results

- Rayleigh fading and exponentially distributed random attack time.

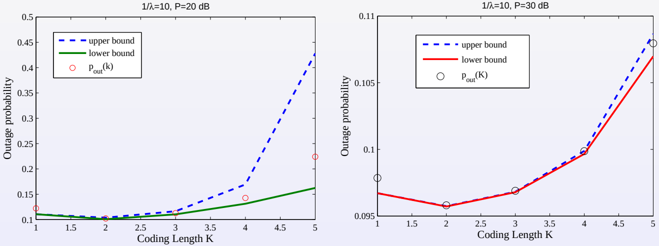

Figure: Outage probability vs. coding length, R=1 bps/Hz.

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Optimal Power Vector

- General results:
  - The optimal power allocation has a non-increasing profile, i.e., $P_1 \geq P_2 \geq \cdots \geq P_K$.
  - When the fading gains are the same, the optimal coding length is $K = 1$ and $P_1 = P$.
- Conditioned on K, the optimization problem becomes

$$\begin{aligned} \min_{\mathbf{P}_K} \quad & \Pr\left\{ \sum_{i=1}^{L} \log(1+\alpha_i P_i) < KR \right\} \\ \text{s.t.} \quad & \sum_{i=1}^{K} P_i \leq KP. \end{aligned}$$

- For some special cases, it can be cast into convex optimization problems .

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High SNR Rayleigh Fading Case

- Rayleigh fading in high SNR regime:

$$\min_{\mathbf{P}_K \in D_+} \quad w_0 + \frac{c_1}{P_1} + \frac{c_2}{P_1 P_2} + \cdots + \frac{c_K}{\prod_{i=1}^{K} P_i}$$

$$\text{s.t.} \quad \sum_{i=1}^{K} P_i \leq KP,$$

where $D_+ = \{\mathbf{P} \in \mathbb{R}_+^K : P_1 \geq P_2 \geq \cdots \geq P_K \geq 0\}$ is a convex cone.

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Log-normal Fading Case

- Log-normal fading distribution

$$
\begin{aligned}
\min_{\mathbf{P_K}} \quad & w_0 + \sum_{n=1}^{K} \frac{1}{2}\left\{1 + \operatorname{erf}\left(\frac{KR - \sum_{i=1}^{n} \log P_i}{\sqrt{2n}}\right)\right\} w_n \\
\text{s.t.} \quad & \sum_{i=1}^{K} P_i \leq KP \\
& KR - \log P_1 \leq 0 \\
& KR - \log P_1 - \log P_2 \leq 0 \\
& \cdots\cdots \\
& KR - \sum_{i=1}^{K} \log P_i \leq 0.
\end{aligned}
$$

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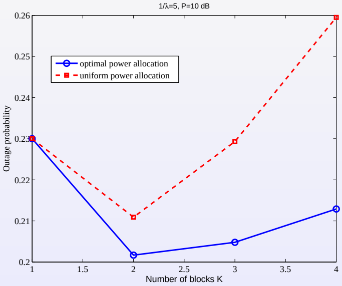
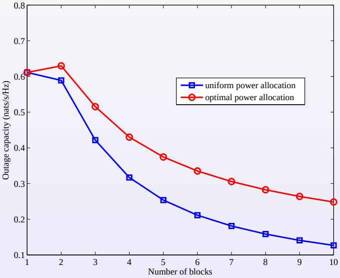
Simulation Results

- Exponentially distributed random attack time.
- Rayleigh fading and Log-normal fading.





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Outage Probability Definition

- Definition
  The outage probability in the multiple parallel channels case is give as:

$$p_{out}(R,P,N) = \Pr\{\sum_{i=1}^{N}\frac{1}{K}\sum_{k=1}^{L_i}\log(1+g_k^{(i)}P/N) < R\}$$
$$\approx \Pr\left\{\frac{1}{N}\sum_{i=1}^{N}Y_i < R/P\right\}, \qquad (1)$$

where $Y_i = \frac{1}{K}\sum_{k=1}^{L_i} g_k^{(i)}$.

Outage Probabilities

- The independent random attack case

$$p_{out}(R,P,N) \approx \Phi(\frac{R/P-\mu_Y}{\sigma_Y/\sqrt{N}}). \tag{2}$$

- The m-dependent random attack case

$$p_{out}(R,P,N) \approx \Phi(\frac{R/P-\mu_Y}{\sqrt{\upsilon_m/N}}). \tag{3}$$

where $\upsilon_m = \frac{\mu_L \sigma_g^2}{K^2} + \frac{\mu_g^2 \sigma_L^2}{K^2}(1+2m\rho)$.

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Outage Exponents

- The independent attack case

$$\Lambda(s) := \log E\left[\exp(sY_i)\right] = \log M_Y(s). \quad (4)$$

- The m-dependent attack case
  an approximate outage exponent can be quantified as:

$$\mathcal{E}_{mdp}(R/P) \approx \frac{(\mu_Y - R/P)^2}{2\upsilon_m}. \quad (5)$$

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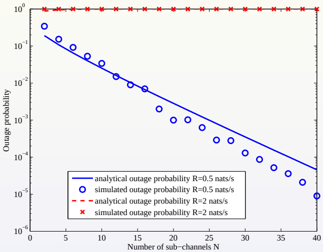
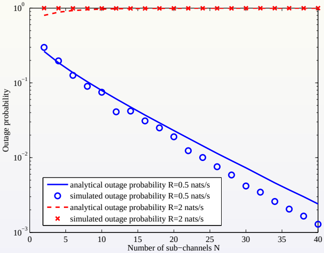
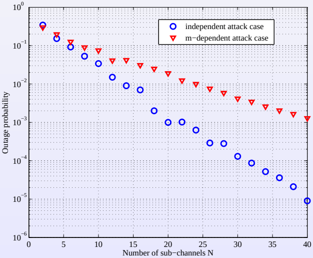
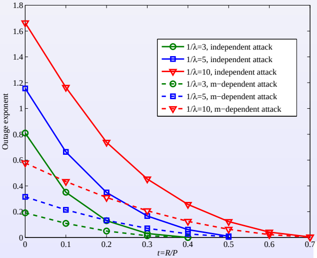
Results









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Summary

- Considered a new type of channel existing in cognitive radio and sensor networks.
- Defined the outage capacity for dying channels.
- Investigated the power allocation and the optimal coding length.
  - There exists an optimal coding length.
  - The optimal power vector is non-increasing.
  - The power allocation problem is convex for some special cases.
- Studied the asymptotic outage probability and outage exponents for parallel dying channels.