Optimal Spectrum Allocation in Gaussian Interference Channels

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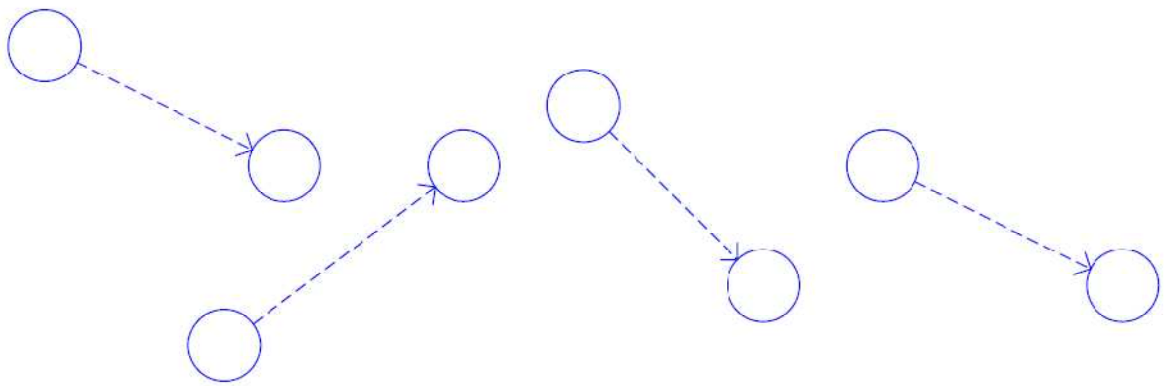
Motivation

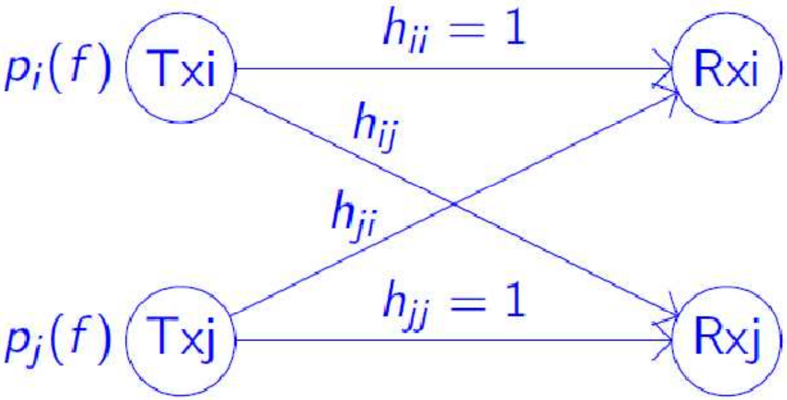
- Efficient resource allocation is critical for wireless networks.
- Interference management is needed to enable multiple users to share a common frequency band.
- Allocating power spectrum density is a key technique.

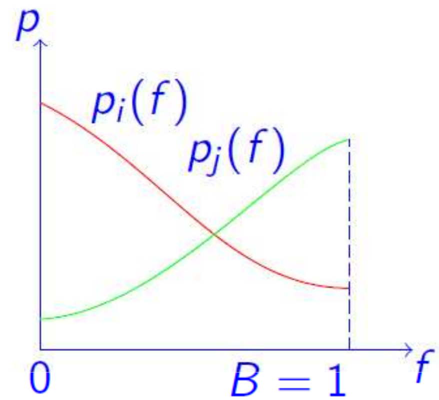
Model

- Gaussian Interference Channels (optimal performance unknown)
  - Flat fading
  - Interference treated as noise



Problem Formulation (I)

- 



- $$R_i = \int_0^1 \log\left(1 + \frac{p_i(f)}{N_0 + \sum_{j \neq i} h_{ji} p_j(f)}\right) df$$

  Power constraint: $\int_0^1 p_i(f) df \leq P_i, \; p_i(f) \geq 0$

Problem Formulation (II)

- Sum-rate maximization subject to individual user power constraints

- $$\max_{\{p_i(f)\}_{i\in\mathcal{M}}} \sum_{i\in\mathcal{M}} \int_0^B \log\left(1+\frac{h_{ii}p_i(f)}{N_0+\sum_{j\neq i} h_{ji}p_j(f)}\right) df$$
  $$\text{s.t.} \quad \int_0^B p_i(f) \le P_i, \quad p_i(f) \ge 0, \quad i \in \mathcal{M}.$$

Preliminaries

- For a system with M users, there exists a optimal solution in which each users employs a piece-wise constant power density function in at most M+1 disjoint frequency bands.
- Each user’s power constraint must be tight at optimal solution.

Formulation Simplification

Let $\alpha_k = B_k / B$,

$$\max \sum_{k=1}^{K} \alpha_k \sum_i \log\left(1 + \frac{p_i^k}{N_0\alpha_k + \sum_{j\neq i} h_{ji} p_j^k}\right),$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \alpha_k = 1,\ \alpha_k > 0;\ \sum_{k=1}^{K} p_i^k \le P_i,\ p_i^k \ge 0.$$

$$x_i^k := \frac{p_i^k}{N_0\alpha_k} \ (SNR_i^k)$$

$$\max \sum_{k=1}^{K} \alpha_k F(\mathbf{x}^k),$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \alpha_k = 1,\ \alpha_k > 0;\ \sum_{k=1}^{K} \alpha_k x_i^k \le \frac{P_i}{N_0},\ x_i^k \ge 0.$$

$$\mathbf{x}^k := (x_1^k, \ldots, x_M^k)$$

$$F(\mathbf{x}) := \sum_i \log\left(1 + \frac{x_i}{1+\sum_{j\neq i} h_{ji} x_j}\right)$$

Main Results

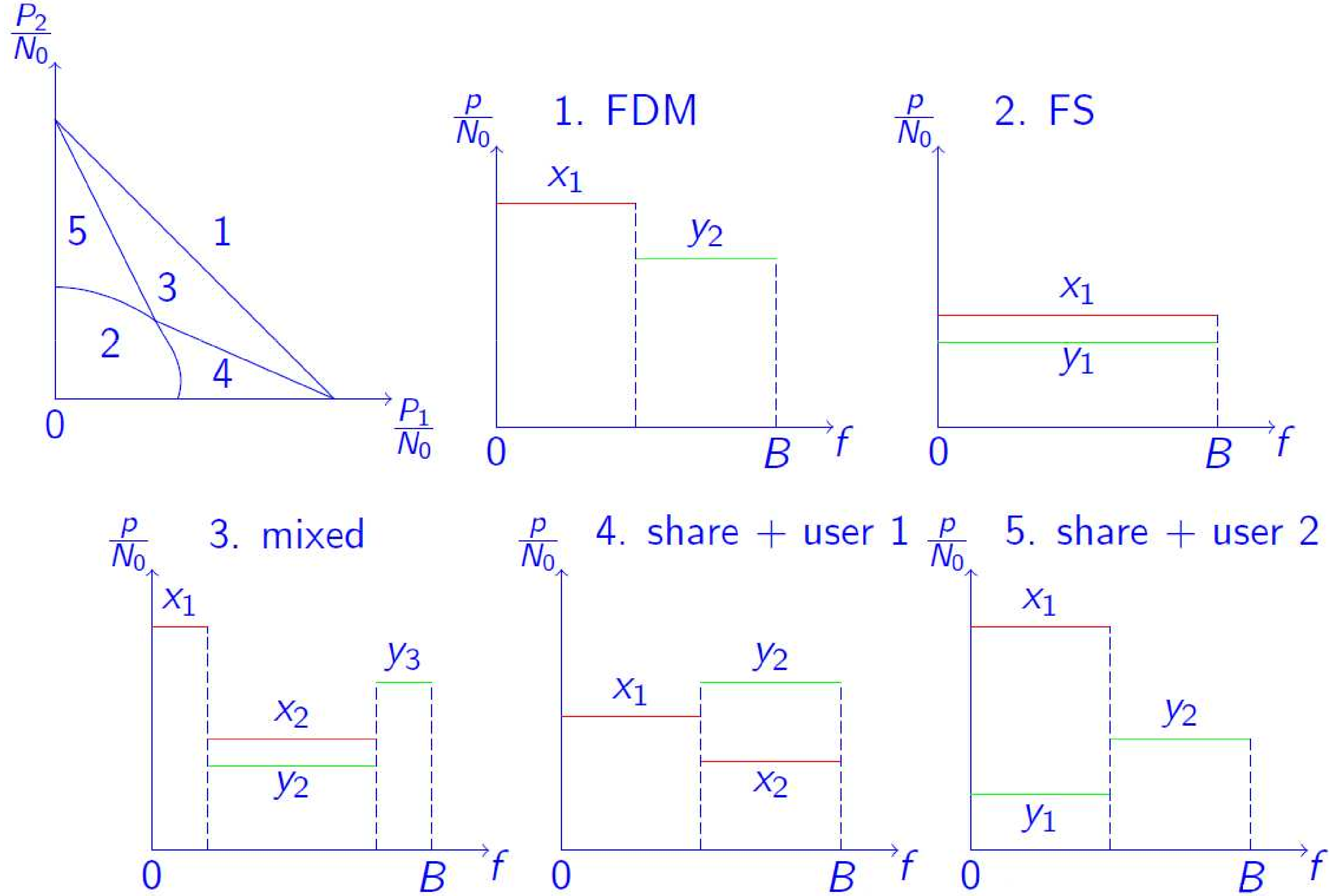
- To find a convex combination of SNR vectors on each band, which satisfies power constraint and maximizes the corresponding convex combination of the sum-rate on each band.

- Complete characterization of solutions for two users case
  - At most one shared band.
  - Optimality criteria are obtained.

Two Users Case

- Number of sub-bands $K \leq 3$
- SNR pair for sub-band k: $x_k = \frac{p_1^k}{N_0 \alpha_k}$, $y_k = \frac{p_2^k}{N_0 \alpha_k}$
- $F(x, y) := \log\left(1 + \frac{x}{1+ay}\right) + \log\left(1 + \frac{y}{1+bx}\right)$
  - F(x,y) is non-concave, non-convex in general

Optimal Spectrum Allocation (Two Users)



Proof Sketches

- $F^*(\frac{P_1}{N_0}, \frac{P_2}{N_0})$: optimal convex combination of F

Equivalent to finding F^* satisfying

1. $F^* \geq F$;
2. F^* is concave; $(1 + 2 \Rightarrow$ upper bound$)$
3. F^* is feasible everywhere.

- Ideas
  - The optimum from FS/FDM, F_0, provides a lower bound.
  - Construct a concave upper bound G from F_0.
  - Derive the optimal sum-rate F^* from G.

Multiple Users (M>2)

- Tight power constraint result still holds.
- Preceding analysis method is difficult to apply to multiple users cases.
- A *alternating bandwidth and power* (ABP) update algorithm can be used to find local optimal solutions.

Conclusions and Future Work

- Completely characterized the optimal spectrum allocation maximizing sum-rate for a two-user flat Gaussian interference channel, treating interference as noise.
- Generalization to multiple-user cases is a challenging future work.