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The Optimality of Two Prices: Maximizing Revenue in a Stochastic Network

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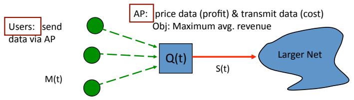
The AP Revenue Maximization Problem

System Model:


Users: send data via AP

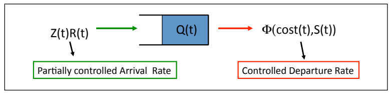
AP: price data (profit) & transmit data (cost)
Obj: Maximum avg. revenue

Model: (1) Slotted time t = {0, 1, 2, ….}
(2) Dynamic demand state M(t): models "aggregate willingness to pay"
(3) Demand function F(M(t),p(t))→E{R(t)}=E{# Arrivals in reaction to p(t),M(t)}
(4) Channel state S(t) and rate cost function Φ(cost(t), S(t)) = μ(t) (bits/slot)

Assumptions: (1) M(t), S(t): finite state space, ergodic, Markovian, distribution not known
(2) AP knows current value of M(t), S(t) on each slot t, and F(m,p)
(3) R(t): random arrivals, E{R(t)} = F(M(t), p(t)) , R(t) < Rmax, conditionally independent of past M(t), p(t)
(4) F(m,p) continuous over p for all M, F(m,p) < Rmax, else *arbitrary*

Operation: Every Time slot, the AP queues the data (Q(t) = backlog).
-Observes demand & channel state M(t), S(t).
-Decides whether or not to accept new data, i.e., Z(t)=1/0 & Picks Price p(t)
-Makes Resource Allocation decision cost(t).
-Accepts packets R(t) & Transmits at rate μ(t) = Φ(cost(t), S(t))

Goal: Max Time Avg. Profit, i.e., Avg. Z(t)R(t)p(t)-cost(t) s.t. stability



The optimum avg. profit can be obtained with exact steady state dist. Info. for M(t) and S(t)

Theorem: (Max Profit with Stability)
The max average profit Profit* is the solution to the max problem

$$\begin{aligned}
\max\quad & \mathrm{Pr}ofit = Income_{av} - Cost_{av}\\
s.t.\quad & Income_{av} = E_m\left\{\phi^{(m)}\sum_{k=1}^{\infty}\alpha_k^{(m)}F(m,p_k^{(m)})p_k^{(m)}\right\}\\
& Cost_{av} = E_S\left\{\sum_{k=1}^{\infty}\beta_k^{(s)}\cos t_k^{(s)}\right\}\\
& \lambda_{av} = E_m\left\{\phi^{(m)}\sum_{k=1}^{\infty}\alpha_k^{(m)}F(m,p_k^{(m)})\right\} \le \mu_{av} = E_S\left\{\sum_{k=1}^{\infty}\beta_k^{(s)}\Phi(\cos t_k^{(s)},S)\right\}\\
& 0 \le \phi^{(m)} \le 1\\
& 0 \le p_k^{(m)} \le p_{\max}\ \forall k,m \quad 0 \le \cos t_k^{(s)} \le C_{\max}\ \forall k,s\\
& \sum_{k=1}^{\infty}\alpha_k^{(m)} = 1 \forall m \quad \sum_{k=1}^{\infty}\beta_k^{(s)} = 1 \forall s
\end{aligned}$$

Note:

-Theorem Proved by using Caratheodory's Theorem.

-Immediate corollary: A stationary randomized STAT* algorithm achieves the optimum avg. profit.

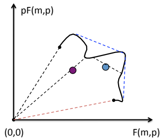
The Optimality of Two Prices Theorem

Theorem:
λ*= avg. rate of STAT*, Income*=avg. income of STAT*
For each M(t)=m, exists 2 business-price points $(Z_1(m), p_1(m))$, $(Z_2(m), p_2(m))$, and probabilities $q_1(m)$ & $q_2(m)$ s.t.

$$\lambda^* = E_m[q_1(m) F(m, p_1(m)) + q_2(m) F(m, p_2(m))]$$
$$Income^* = E_m[q_1(m) p_1(m) F(m, p_1(m)) + q_2(m) p_2(m) F(m, p_2(m))]$$

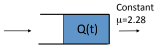
Proof : Caratheodory's thm + its extension + Continuity of F(m,p)

For any F(m,p) demand function, AP can maximize its revenue by using two prices per M(t)

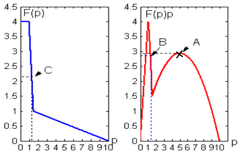


This example shows that using a single price does NOT always lead to optimal average profit, thus the number Two is *tight*

Example 1:



$$F(p) = \begin{cases} 4 & 0 \le p \le 1\\ -6p+10 & 1 < p \le 1.5\\ -\frac{2}{17}p+\frac{20}{17} & 1.5 < p \le 10 \end{cases}$$



A=(5,50/17) B=(1.2867,2.9285) C=(1.2867,2.28)

Single price with constant Z(t)=1
P*=5 at A=(5,50/17)

$\mathrm{Profit}_{\mathrm{Single}}$= 50/17≈2.9412
λ_A = 10/17 ≈0.5882<<2.28 !!

Single price with varying Z(t)
$\mathrm{Profit}_{\mathrm{single}}$≤3.42

But: use p1=31/30 & p2=5, each with prob. 0.5
Profit = 3.4339>$\mathrm{Profit}_{\mathrm{single}}$
λ = 2.1941<2.28

Our dynamic Pricing and Transmission Scheduling Algorithm PTSA achieves the optimum average profit

PTSA: For a control parameter V (affects a delay tradeoff)…

Admission Control: Observe M(t) & Q(t), choose:

$$P(t)=\arg\max_{p \text{ in } \mathcal{P}}[\,VpF(M(t),p)-2Q(t)F(M(t),p)\,]$$

Whenever the maximum is positive, Z(t)=1,announce P(t);
Else Z(t)=0,send "CLOSED".

★ Note: For F(m,p)=mF(p), the AP can choose p(t) without info of M(t)

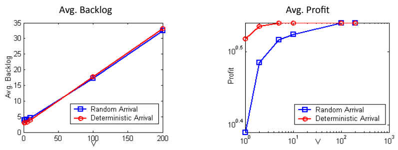
Demand-Blind Pricing!

Cost/Transmission: Observe S(t) & Q(t), choose:

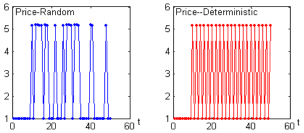
$$Cost(t)=\arg\max_{cost \text{ in } \mathcal{C}}[2Q(t)\Phi(cost,S(t)) - Vcost(t)]$$

Performance Result:
Backlog Bound: Q(t) ≤Qmax = VPmax+Rmax
Achieved Average Profit: $\mathrm{Profit}_{\mathrm{PTSA}} \ge \mathrm{Profit}^* - O(\log V/V)$

Simulation results of PTSA on Example 1


Avg. Backlog

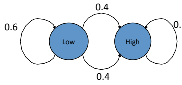
Avg. Profit



Deterministic:
R(t) = F(M(t),p(t))
Random:
R(t) = 2F(M(t),p(t)) w. prob. ½
0 w. prob. ½

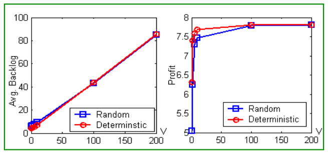
More simulation results of PTSA

Dynamic Demand states, {High, Low}
Constant service rate 2.28 with zero cost

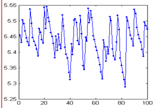


$$F^{LOW}(p) = \begin{cases} 4 & 0 \le p \le 1\\ -6p+10 & 1 < p \le 1.5\\ -\frac{2}{17}p+\frac{20}{17} & 1.5 < p \le 10 \end{cases}$$

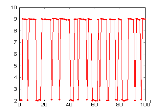
$$F^{High}(p) = \begin{cases} 10-p & 0 \le p \le 2\\ -6p+20 & 2 < p \le 3\\ -\frac{1}{7}p+\frac{17}{7} & 3 < p \le 10 \end{cases}$$



Price chosen at 'Low'



Price chosen at 'High'



Conclusion & References

This work characterizes the max avg. profit of APs in wireless mesh nets and offers a revenue maximizing algorithm PTSA.

New in our work:
(1) Stochastic Wireless Net. Environment
(2) General (non-concave) Demand Function
(3) Queueing Effect

References:
1. L. Huang and M. J. Neely, "The Optimality of Two Prices: Maximizing Revenue in a Stochastic Network," Proc. of 45th Annual Allerton Conference on Communication,Control, and Computing (invited paper), Sept. 2007
2. L. Georgiadis, M. J. Neely, L. Tassiulas, "Resource Allocation and Cross-Layer Control in Wireless Networks," Foundations and Trends in Networking, Vol. 1, no. 1, pp. 1-144, 2006.

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