# Path Loss Exponent Estimation in Large Wireless Networks

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# Motivation

- Large-scale path loss law: signal strength attenuates with distance $d$ as $d^{\gamma}$

$$S \propto \left(\frac{d}{d_0}\right)^{-\gamma}.$$

- Though it is typically assumed in analysis and design problems that the path loss exponent (PLE) is known a priori, it is often not the case.
- The PLE has a strong impact on the quality of links, and therefore needs to be accurately estimated for the efficient design and operation of systems.

# Motivation (contd.)

- **Example 1:** The information-theoretic capacity of large random ad hoc networks scales as *

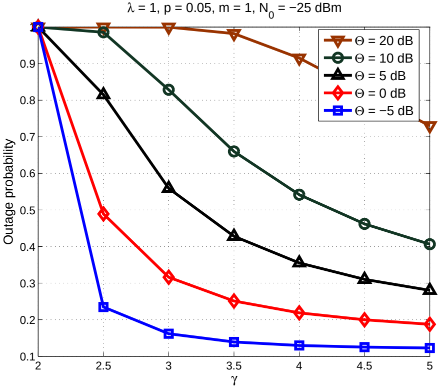
$$\begin{array}{lll} n^{2-\gamma/2} & \text{for} & 2 \leqslant \gamma < 3 \\ \sqrt{n} & \text{for} & \gamma \geqslant 3. \end{array}$$

Depending on the value of $\gamma$, different routing strategies are required to be implemented.

*A. Özgür, O. Lévêque and D. Tse, "Hierarchical Cooperation Achieves Optimal Capacity Scaling in Ad Hoc Networks," *IEEE Trans. Info. Th.*, 2007.

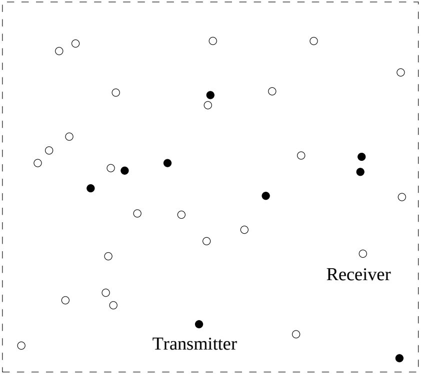
# Motivation (contd.)

- **Example 2:** Outage probability in a planar Poisson point process with Rayleigh fading.




The system performance critically depends on $\gamma$.

# System Model




Filled circles: transmitters.
Empty circles: receivers.

- An infinite Poisson point process (PPP) on $\mathbb{R}^2$ with density $\lambda$.
- Channel access scheme is ALOHA.
- $p$ is the ALOHA contention parameter. Therefore, the set of transmitters forms a PPP with density $\lambda p$.
- No synchronization.

# System Model (contd.)

- Attenuation in the channel: product of
  - large-scale path loss, with PLE $\gamma$.
  - small-scale fading ($m$-Nakagami).
    $m = 1$: Rayleigh fading ; $m \to \infty$: no fading.
- Noise is AWGN with variance $N_0$.
- All the transmit powers are equal to unity (no power control).

**Problem:** How do you accurately estimate the PLE at each node in the network in a completely distributed manner?

# What Makes Estimating the PLE Complicated?

- The large-scale path loss is commonly taken to be **deterministic** while the small-scale fading is modeled as a **stochastic** process.
- This distinction, however, does not hold when the nodes themselves are randomly arranged. So, we **need to consider the distance and fading ambiguities jointly**.
- Moreover, PLE estimation needs to be performed during the initialization of the network. During this phase, **the system is typically interference-limited** due to the presence of uncoordinated transmissions.

Purely RSS-based estimators cannot be used in these situations.

# Overview

- Propose three distributed algorithms for estimating the PLE in large random wireless networks that explicitly take into account
  - the uncertainty in the locations of the nodes.
  - the uncertainty in the fading gains across links.
  - the interference in the network.
- Provide simulation results to demonstrate the performance of the algorithms and quantify the estimation errors.

# The Big Picture

- The PLE estimation problem is essentially tackled by **equating the empirical (observed) values** of certain network characteristics **to their theoretically established values**.
- By obtaining measurements over several time slots, the PLE can be estimated at each node in a distributed fashion.
- The three PLE algorithms are **each based on a specific network characteristic**:
  - the mean interference.
  - the outage probability.
  - connectivity properties of a node.

# Simulation Details

- We use $50,000$ different realizations of the PPP to analyze the mean error performance of the algorithms, which is characterized using the 'relative' MSE, defined as $\mathbb{E}\left[(\hat{\gamma}-\gamma)^2\right]/\gamma$.
- We used $\mathrm{p}=0.05$ since it was suitable. Note the tradeoffs.
  - **Large** $\mathrm{p}$: results in few quasi-different realizations of the transmitter PPP.
  - **Small** $\mathrm{p}$: takes long for the algorithms to convergence.

# Algo. 1: Using the Mean Interference

- This algorithm assumes that the network density $\lambda$ is known.
- In theory, the mean interference is given by [†]

$$\mu = 2\pi\lambda p A_0^{2-\gamma} / (\gamma - 2) , \tag{1}$$

where $A_0$ is the near-field radius of the antenna.

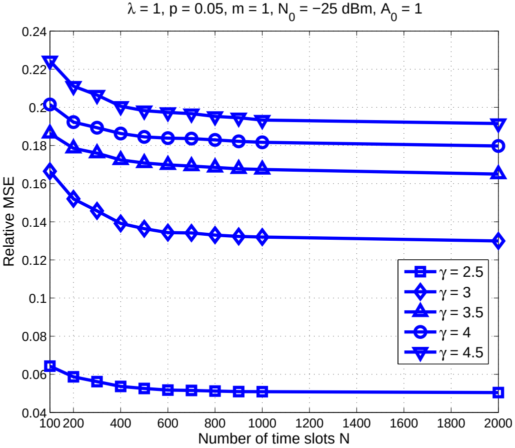
**Implementation**

- Nodes simply need to record the mean strength of the received power, $\mu'$, averaged over several time slots.
- Equating $\mu$ to $\mu'$, and using the known values of $p$, $A_0$, and $\lambda$, $\hat{\gamma}$ is found from a look-up table.

[†] J. Venkataraman, M. Haenggi and O. Collins, "Shot Noise Models for Outage and Throughput Analyses in Wireless Ad Hoc Networks," *MILCOM*, 2006.

# Algo. 1: Using the Mean Interference (contd.)

Relative MSE of $\hat{\gamma}$ versus the number of time slots.




The estimates are fairly accurate over a wide range of parameters.

# Algo 2: Based on (Virtual) Outage Probabilities

- This algorithm does not require the knowledge of $\lambda$ or $m$.
- In a PPP, when the signal power is exponentially distributed, the probability of a successful transmission $p_s$ is

$$p_s = \mathbb{P}(\mathsf{SIR} > \Theta) = \exp(-c\Theta^{2/\gamma}), \tag{2}$$

where $c = \lambda p \pi \Gamma\left(m + \frac{2}{\gamma}\right)\Gamma\left(1 - \frac{2}{\gamma}\right) \Big/ \left(\Gamma(m) m^{2/\gamma}\right)$.

- Nodes can determine the SIR, and consequently $p_s$ by computing the ratio of the power of the signal (which arrives from a virtual transmitter, and is assumed to be exponentially distributed) and the total received power.

# Algo 2: Based on (Virtual) Outage Prob. (contd.)

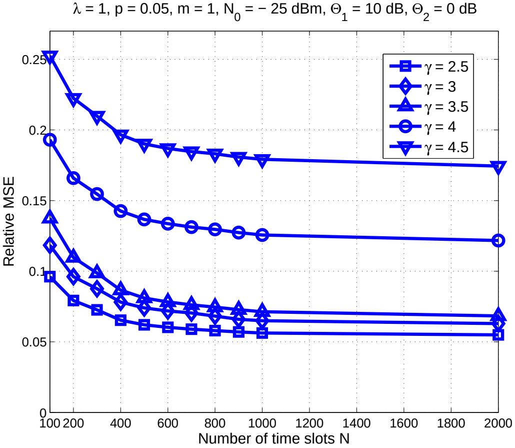
**Implementation**: A 'differential' method.

- Obtain a histogram of the observed SIR values measured over several time slots.
- The empirical success probabilities ($p_{s,i} = \mathbb{P}(\text{SIR} > \Theta_i)$, $i = 1, 2$) are obtained at two different threshold values.
- Inverting (2), an estimate of $\gamma$ is obtained as

$$\hat{\gamma} = \frac{2 \ln(\Theta_1/\Theta_2)}{\ln\left(\ln p_{s,1}/\ln p_{s,2}\right)}. \quad (3)$$

# Algo 2: Based on (Virtual) Outage Prob. (contd.)

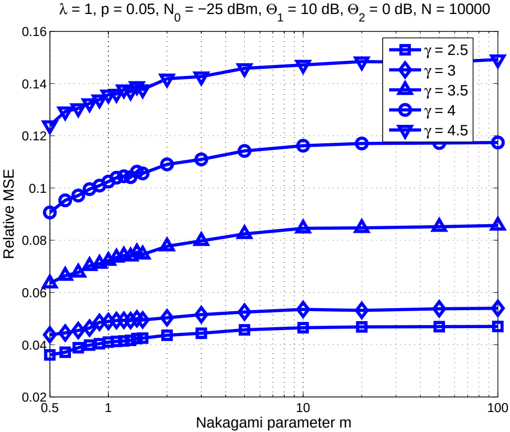
Relative MSE of $\hat{\gamma}$ versus the number of time slots.




The estimation error increases with larger $\gamma$.

# Algo 2: Based on (Virtual) Outage Prob. (contd.)

Relative MSE of $\hat{\gamma}$ versus the Nakagami parameter.




This algorithm performs more accurately at lower values of $m$.

# Algo 3: Based on the Cardinality of the Tx Set

- This algorithm also does not require to know $m$ or $\lambda$.
- Transmitter node $y$ is in receiver node $x$'s *transmitting set*, $T(x)$ if they are connected, i.e., the SIR at $x$ due to $y$'s signal is $> \Theta$.
- We prove that under the conditions of $m \in \mathbb{N}$,

$$\mathbb{E}|T(x)| = \bar{N}_T = \frac{\Gamma(m)\left(1 - \left(\frac{2}{\gamma}\right)^m\right)}{\Gamma(m + \frac{2}{\gamma})\Gamma(2 - \frac{2}{\gamma})\Theta^{2/\gamma}}. \quad (4)$$

- We see that $\bar{N}_T$ is inversely proportional to $\Theta^{2/\gamma}$, and surmise that this behavior holds at arbitrary $m \in \mathbb{R}^+$.

# Algo 3: Based on the Card. of the Tx Set (contd.)

**Implementation**

- For a known threshold $\Theta_1 \geqslant 1$, at time slot $i$, $1 \leqslant i \leqslant N$, set

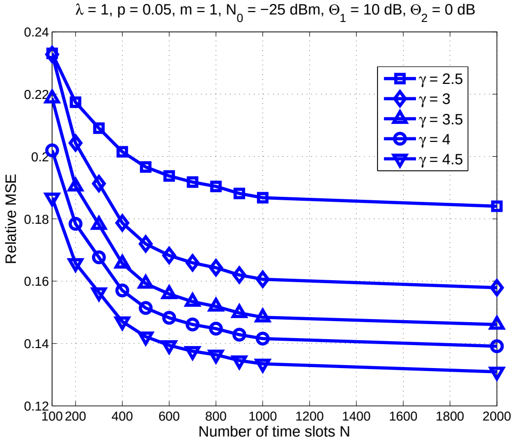
$$N_{T,1}(i) = \begin{cases} 1 & \text{if the node can decode a packet} \\ 0 & \text{otherwise.} \end{cases}$$

- Evaluate $\bar{N}_{T,1}$ and $\bar{N}_{T,2}$ at two different threshold values $\Theta_1$ and $\Theta_2$ respectively.
- In theory, we obtain $\bar{N}_{T,1}/\bar{N}_{T,2} = (\Theta_2/\Theta_1)^{2/\gamma}$.
- Inverting this, we have

$$\hat{\gamma} = (2 \ln(\Theta_2/\Theta_1)) \,/\, \ln(\bar{N}_{T,1}/\bar{N}_{T,2}). \tag{5}$$

# Algo 3: Based on the Card. of the Tx Set (contd.)

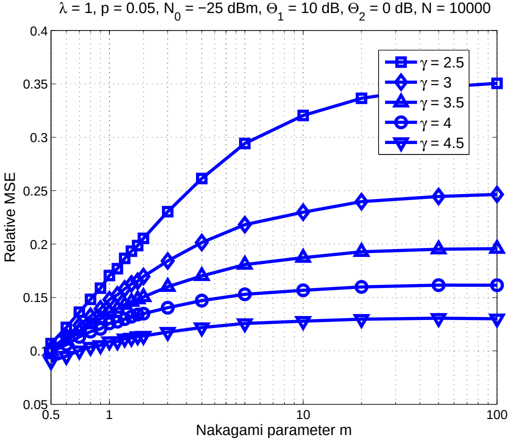
Relative MSE of $\hat{\gamma}$ versus the number of time slots.




In contrast to Algo. 1 and 2, the relative MSE decreases with $\gamma$.

# Algo 3: Based on the Card. of the Tx Set (contd.)

Relative MSE of $\hat{\gamma}$ versus the Nakagami parameter.




The estimates are more accurate at lower $m$.

# Summary and Discussion

- We have addressed the PLE estimation problem in the presence of node location uncertainties, $m$-Nakagami fading and most importantly, interference!
- Each of the algorithms are fully distributed and do not require any information on the location of other nodes or the value of $m$.
- Based on the relative MSE values, we conclude that at low values of $\gamma$, Algo. 1 performs the best (though it requires the density to be known), while when $\gamma$ is high, Algo. 3 is preferred.

# Summary and Discussion (contd.)

- Each of our algorithms work by equating empirical values with their corresponding theoretically established ones.
- The caveat is that the theoretical results are for an "average network" while in practice, we have only a single realization of the node distribution at hand. Thus, in general, the estimates we obtain are biased.
- This also explains the fact that the performance of Algorithms 2 and 3 is better at lower $m$.
- We remark that the bias (and the MSE) can be significantly lowered if nodes have access to several independent realizations of the point process or if they are allowed to communicate.