Multicasting of Digital Images over Erasure Broadcasting Channels Using Rateless Codes

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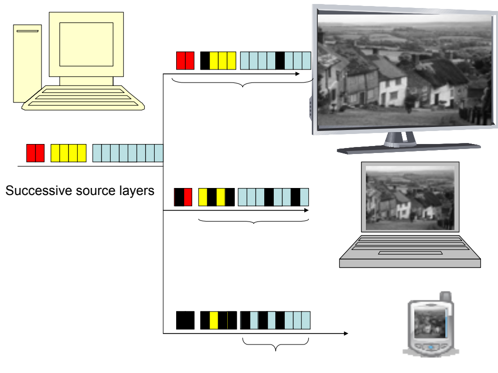


Binary Erasure Broadcast Channel (BEBC)

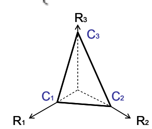
- For L users, BEBC is defined by L binary erasure channels, which has the following erasure rates and capacities:

$$\epsilon_1 \geq \ldots \geq \epsilon_L \qquad C_l = 1 - \epsilon_l$$

The BEBC belongs to the class of stochastically degraded broadcast channels.

- Capacity of the BEBC is given as follows :

$$\mathcal{C} = \left\{ (R_1, \ldots, R_L) : R_l \geq 0, \ \sum_{l=1}^{L} \frac{R_l}{C_l} \leq 1, \ l \in \{1, \ldots, L\} \right\}$$



This region can be achieved by time-sharing between individual capacities of each user.

Parallel sources

- s : Number of independent source components
  k : Block length of each source component

$$\mathbf{S} \in \mathbb{R}^{s \times k}$$

$\mathbf{S}_{(i,:)}$: i^{th} row of **S** corresponds to i^{th} source component and consists of i.i.d elements

Rate Distortion Function of **S** :

$$\mathcal{R}(D) = \frac{1}{s} \min \sum_{i=1}^{s} r_i(d_i), \ \text{s.t.} \ \sum_{i=1}^{s} \frac{v_i}{s} d_i = D$$

Reconstruction of i^{th} source component at l^{th} user

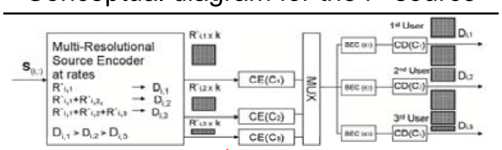
$$D_{i,l}^{(k)} = \frac{1}{k} \mathbb{E}[\|\mathbf{S}_{(i,:)} - \hat{\mathbf{S}}_{(i,:),l}\|^2]$$

Distortion of i^{th} source component at l^{th} user

Distortion at the l^{th} user

$$D_l^{(k)} = \frac{1}{s} \sum_{i=1}^{s} v_i D_{i,l}^{(k)}$$

Conceptual diagram for the i^{th} source



n : Number of channel uses
b : Number of coded bits per pixel

$$b = n/(ks)$$

$R'_{i,l}$: Source code rate at l^{th} layer for i^{th} source
R_l : Channel code rate at l^{th} layer

$$\frac{1}{s} \sum_{i=1}^{s} R'_{i,l} = b R_l$$

$$D_l = \mathcal{R}^{-1}\left(\frac{1}{s} \sum_{j=1}^{l} \sum_{i=1}^{s} R'_{i,l} \right) = \mathcal{R}^{-1}\left(b \sum_{j=1}^{l} R_j \right)$$

Optimization problems

Min Bandwidth (MB): For given target distortions $\Delta_1 \geq \cdots \geq \Delta_L$, min b

Min Weighted Total Distortion (MWTD): For given b minimize $\sum_{l=1}^{L} w_l D_l$

$$\text{minimize} \quad \sum_{l=1}^{L} w_l \left(\frac{1}{s} \sum_{i=1}^{s} v_i D_{i,l} \right) \quad \text{subject to} \quad \sum_{l=1}^{L} \frac{R_l}{C_l} \leq 1, \quad R_l \geq 0, \ \forall l$$

$$\mathcal{R}\left(\frac{1}{s} \sum_{i=1}^{s} v_i D_{i,l} \right) \leq b \sum_{j=1}^{l} R_j, \ \forall l$$

Min-Max Distortion Penalty (MMDP): minimize $\max_{l \in \mathcal{L}} D_l / D_l^{\text{opt}}$

$$D_l^{\text{opt}} = \mathcal{R}^{-1}(bC_l)$$

$$\text{minimize} \quad \alpha \quad \text{subject to} \quad \sum_{l=1}^{L} \frac{R_l}{C_l} \leq 1, \quad R_l \geq 0, \ \forall l$$

$$\mathcal{R}\left(\frac{1}{s} \sum_{i=1}^{s} v_i D_{i,l} \right) \leq b \sum_{j=1}^{l} R_j, \ \forall l$$

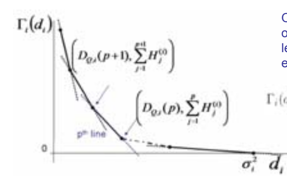
$$\frac{1}{s} \sum_{i=1}^{s} v_i D_{i,l} \leq \alpha D_l^{\text{opt}}, \ \forall l$$

Consider finite length-delay issues

Using Raptor Codes with overhead θ

$$\mathcal{C}_{ach}(\theta) = \left\{ (R_1, \ldots, R_L) : R_l \geq 0, \ \sum_{l=1}^{L} \frac{R_l}{C_l/(1+\theta)} \leq 1, \ l \in \{1, \ldots, L\} \right\}$$

Using Embedded Scalar Quantizers



Operational rate-distortion curve obtained by quantizer distortion levels and quantization indices entropies

$$\Gamma_i(d_i) = \max_{p \in \{1, \ldots, P\}} [m_{i,p} d_i + q_{i,p}]$$

Optimization problems for finite length

$$\mathcal{R}_{op}(D) = \frac{1}{s} \min_{\{d_i\},\{\gamma_i\}} \sum_{i=1}^{s} \gamma_i \quad \text{s.t.} \quad \sum_{i=1}^{s} \frac{v_i}{s} d_i \leq D,$$

$$0 \leq \gamma_i \leq \sum_{j=1}^{P} H_j^{(i)} \ \forall i$$

$$D_{Q,i}(P) \leq d_i \leq \sigma_i^2 \ \forall i$$

$$\gamma_i \geq m_{i,p} d_i + n_{i,p} \ \forall i, p$$

$$\text{minimize} \quad \alpha$$

$$\text{subject to} \quad \sum_{l=1}^{L} \frac{R_l}{C_l} \leq 1, \quad R_l \geq 0, \ \forall l$$

$$D_{Q,i}(P) \leq D_{i,l} \leq \sigma_i^2, \ \forall i, l$$

$$\gamma_{i,l} \geq m_{i,k} D_{i,l} + n_{i,l} \quad \forall i, l, k$$

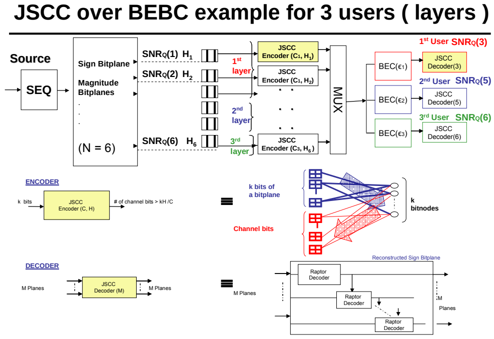
$$0 \leq \gamma_{i,l} \leq \sum_{i=1}^{P} H_j^{(i)} \ \forall i, l$$

$$\frac{1}{s} \sum_{i=1}^{s} \gamma_{i,l} \leq b \sum_{j=1}^{l} R_j, \ \forall l,$$

$$\frac{1}{s} \sum_{i=1}^{s} v_i D_{i,l} \leq \alpha D_l^{\text{opt}}, \ \forall l$$

MMDP

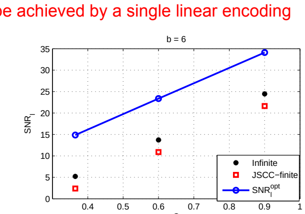
JSCC over BEBC example for 3 users (layers)



Each bit plane is encoded separately and decoded sequentially by using information from reconstructed bitplanes .

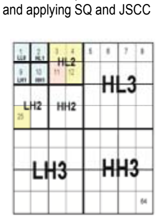
We bypass explicit entropy coding and map directly the redundant quantizer bit planes onto channel –encoded symbol using a single linear encoding operation.

Also, the limiting performance of ideal lossless compression and channel coding can be achieved by a single linear encoding stage.

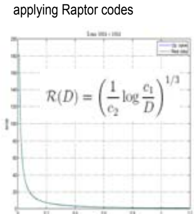


Two different techniques to multicast digital images

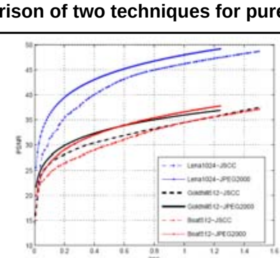
1 - Considering image subbands as parallel sources and applying SQ and JSCC



2 - Considering image as a single source whose R(D) is given by JPEG2000 achievable curve and applying Raptor codes



Comparison of two techniques for pure compression



First scheme does not make use of memory between subband values while second scheme is based on JPEG2000 and uses "context modelling" . Hence the first scheme is simpler but has a performance loss due to memory . Memory can be also exploited in the first scheme, as a future work.