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Superposition codes for Degraded Broadcast Binary-Symmetric Channels

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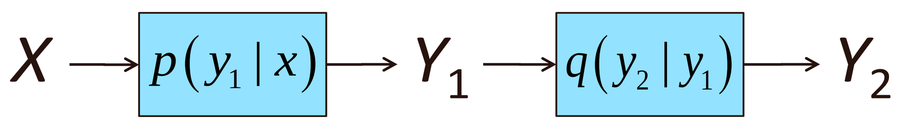
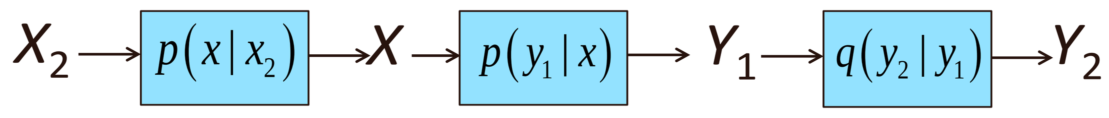
Degraded Broadcast Channels

- Physically Degraded Broadcast Channels
  - The worse channel $p(y_2|x)$ is a degraded version of the better channel $p(y_1|x)$.



- Stochastically Degraded Broadcast Channels
  - It has the same marginal transmission probabilities as a physically degraded broadcast channel.
  - It can be considered as a physically degraded broadcast channel.

- Capacity Region [Cover72][Bergmans73][Gallager74]

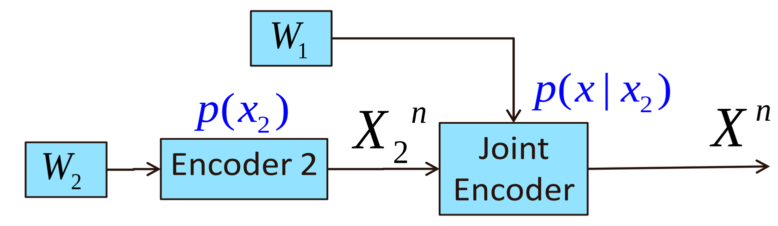


  - The capacity region is the convex hull of the closure of all rate pairs (R_1, R_2) satisfying

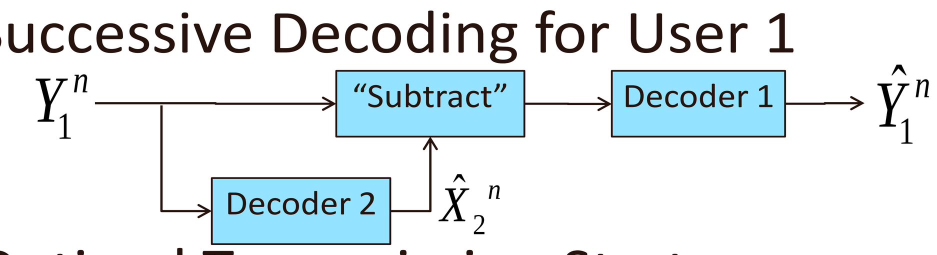
$$R_1 \le I(X;Y_1|X_2),$$
$$R_2 \le I(X_2;Y_2),$$

  for some joint distribution $p(x_2)p(x|x_2)p(y_1,y_2|x)$
  - Joint encoding and successive decoding are used to achieve the capacity region.

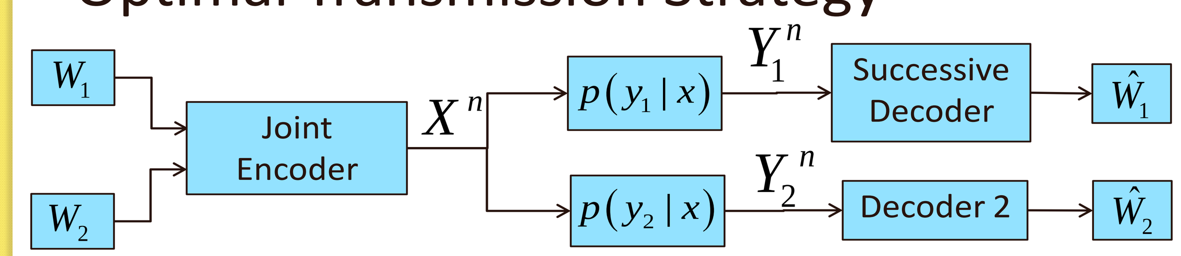
- Joint Encoding



- Successive Decoding for User 1

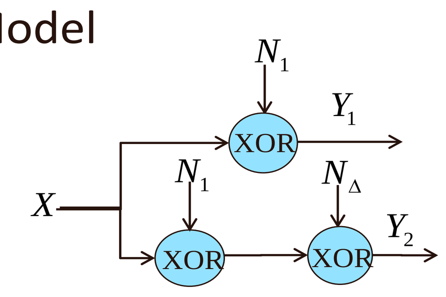


- Optimal Transmission Strategy

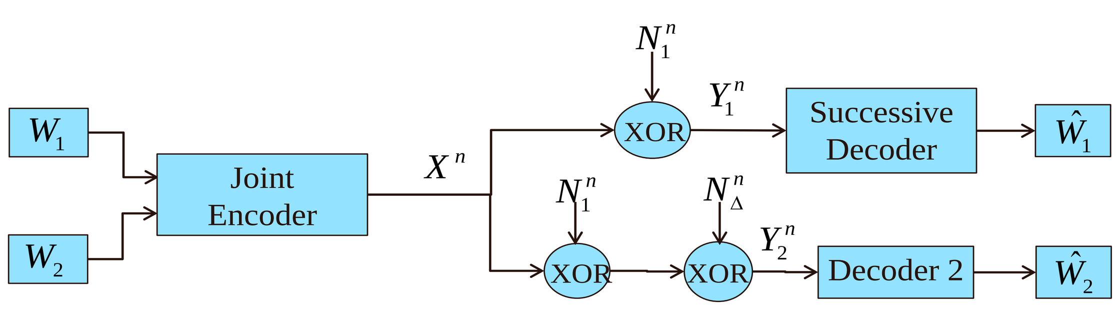


Broadcast Binary-Symmetric Channels

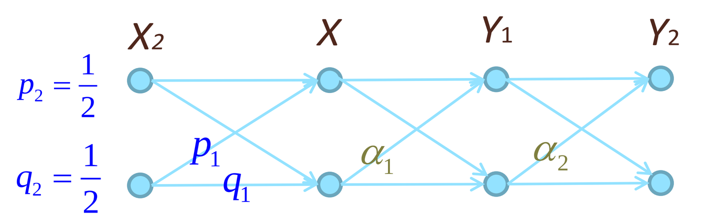
- Broadcast Binary-Symmetric Channels
  - Channel Model



  - Optimal Transmission Strategy [Cover72][Bergmans73]



- Implicit expression of the capacity region



$$p_1 + q_1 = p_2 + q_2 = 1$$

  - The capacity region is the convex hull of the closure of all rate pairs (R_1,R_2) satisfying

$$R_1 \le I(X;Y_1|X_2)|_{p_1}$$
$$R_2 \le I(X_2;Y_2)|_{p_1}$$

  for some probabilities p_1

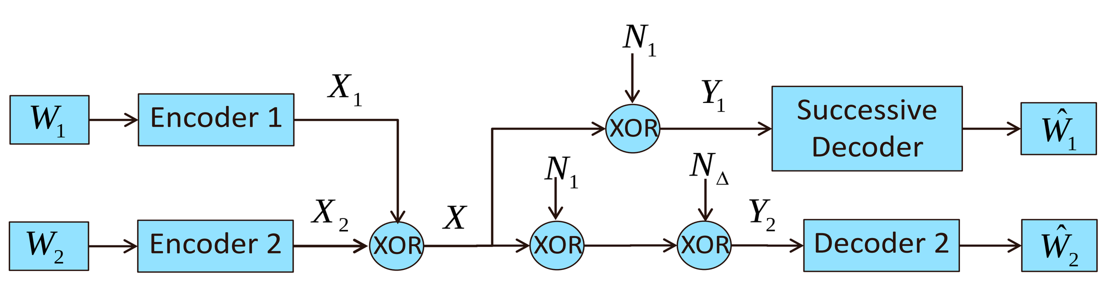
- The capacity of Broadcast Binary-Symmetric Channel is

$$R_1 \le I(X;Y|X_2) = h_n(T_{YX}p_1) - h_n(T_{YX}e_1) \qquad R_2 \le I(X_2;Z) = h_m(T_{ZX}u) - h_m(T_{ZX}p_1)$$

- p_1 is the distribution of X_1
- u is the uniform distribution
- T_{YX} and T_{ZX} are the transition probability matrices

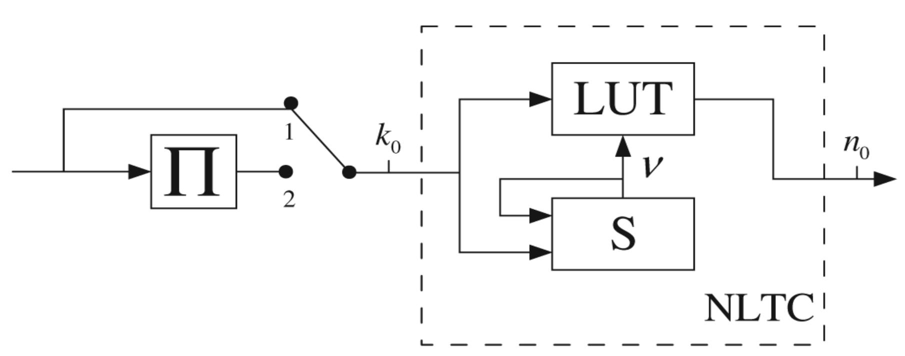
$$T_{YZ} = \begin{bmatrix} 1-\alpha_1 & \alpha_1 \\ \alpha_1 & 1-\alpha_1 \end{bmatrix} \qquad T_{XZ} = \begin{bmatrix} 1-\alpha_2 & \alpha_2 \\ \alpha_2 & 1-\alpha_2 \end{bmatrix}\begin{bmatrix} 1-\alpha_1 & \alpha_1 \\ \alpha_1 & 1-\alpha_1 \end{bmatrix}$$

Superposition coding with Non-linear turbo codes



- It is an independent encoding scheme.
- The one's densities of X_1 and X_2 are p_1 and p_2 respectively.
- The broadcast signal X is the XOR of X_1 and X_2.
- User 2 with the worse channel decodes the message W_2 directly.
- User 1 with the better channel needs a successive decoder.

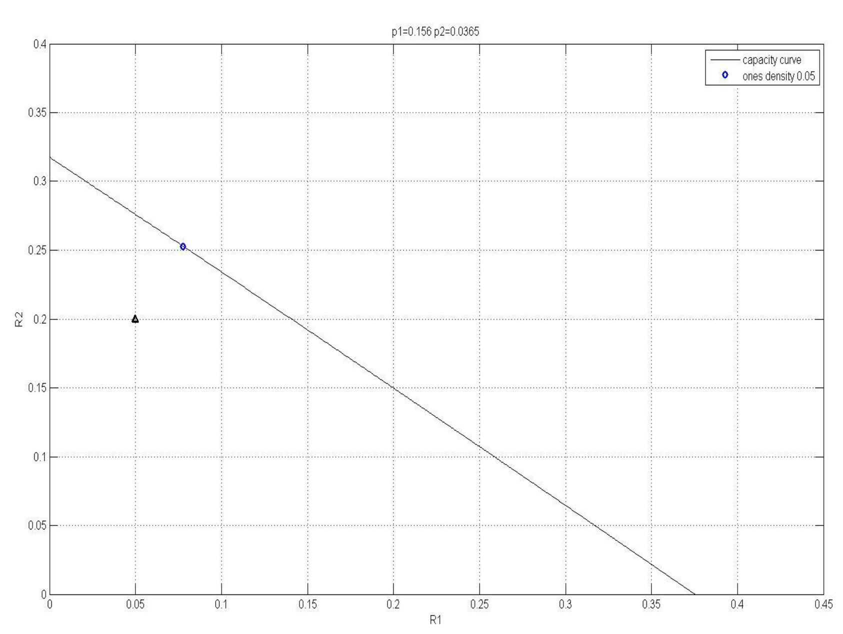
- Nonlinear turbo codes can provide a controlled distribution of ones and zeros. [Griot 06]
- Nonlinear turbo codes designed for Binary-Symmetric channels are used.
- Encoding structure of nonlinear turbo



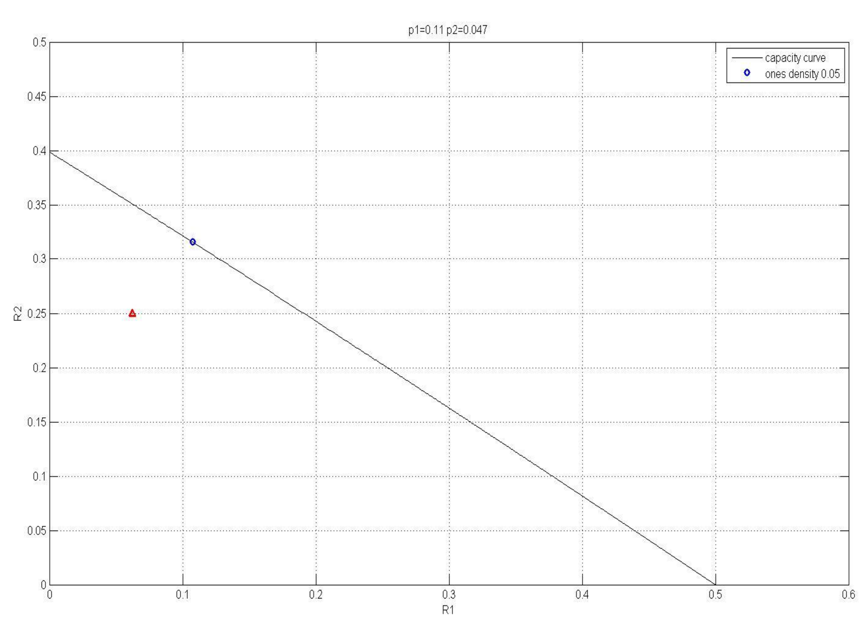
- In order to design a good non-linear turbo code, we need a good interleaver, designed one's position for given ones' density and corresponding decoder.
- An extended spread interleaver is used. [Fragouli 01]
- One's positions are determined with following criteria:
  - Hamming distance of different output from the same state is maximized.
  - Hamming distance of different output to the same state is maximized.
  - The two criteria above can be extended to desired number of successive branches.

Simulation Results

- p1=0.156, p2=0.0365, ones' density=0.05
- R1 is 0.027 away from capacity
- R2 is 0.05 away from capacity



- p1=0.11, p2=0.047, ones' density=0.0625
- R1 is 0.05 away from capacity
- R2 is 0.06 away from capacity



- References:

[1] B. Xie and R. D. Wesel, "A Mutual Information Invariance Approach to Symmetry in Discrete Memoryless Channels". ITA 2008, San Diego, USA, Jan. 27 - Feb. 1, 2008.
[2] M. Griot, A. I. Vila Casado, W.-Y. Weng, H. Chan and R. D. Wesel, "Nonlinear Trellis Codes for Binary-Input Binary-Output Multiple Access Channels With Single-User Decoding," Accepted in IEEE Transactions on Communications.
[3] C. Fragouli and R. D. Wesel, "Turbo encoder design for symbol interleaved parallel concatenated trellis coded modulation," IEEE Transactions on Communications, vol. 49, no. 3, March 2001.