Dependence Balance Bounds for Gaussian Networks with Feedback and Cooperation

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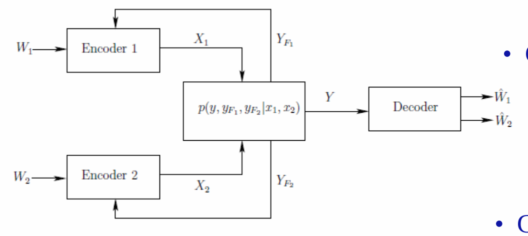
Multiple Access Channel with Generalized Feedback



- Gaussian MAC with noisy feedback.

$$Y = X_1 + X_2 + Z$$
$$Y_{F_1} = Y + Z_1$$
$$Y_{F_2} = Y + Z_2$$

- Gaussian MAC with user cooperation.

$$Y = \sqrt{h_{10}}X_1 + \sqrt{h_{20}}X_2 + Z$$
$$Y_{F_1} = \sqrt{h_{21}}X_2 + Z_1$$
$$Y_{F_2} = \sqrt{h_{12}}X_1 + Z_2$$

Drawback of the cut-set bound

- MAC with noisy feedback.
- **CS not sensitive to feedback noise.**
- MAC with user cooperation.
- **CS sensitive to cooperation noise.**
- **Not sensitive enough.**

Need new outer bounds sensitive to feedback noise

Dependence Balance Bounds for MAC-NF and MAC-UC

$$R_1 \leq I(X_1; Y, Y_{F_2}|X_2, T_2)$$
$$R_2 \leq I(X_2; Y, Y_{F_1}|X_1, T_1)$$
$$R_1 + R_2 \leq I(X_1, X_2; Y, Y_{F_1}, Y_{F_2}|T_1, T_2)$$
$$R_1 + R_2 \leq I(X_1, X_2; Y)$$

for some $p(t_1, t_2, x_1, x_2)$ such that

$$p(t_1, t_2, x_1, x_2, y, y_{F_1}, y_{F_2}) = p(t_1, t_2, x_1, x_2)p(y, y_{F_1}, y_{F_2}|x_1, x_2)$$

and such that

$$I(X_1; X_2|T_1, T_2) \leq I(X_1; X_2|Y_{F_1}, Y_{F_2}, T_1, T_2)$$

- No cardinality bounds on T1, T2.
- **Modify** the DB bound depending on channel model.
- Resulting bounds for NF, UC with **one** auxiliary random variable T.
- Cardinality of T **can be bounded**.

Outline of Proof

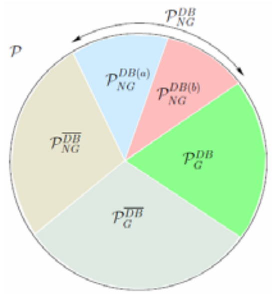
$$0 \leq I(W_1; W_2|Y_{F_1}^n, Y_{F_2}^n)$$
$$= I(W_1; W_2|Y_{F_1}^n, Y_{F_2}^n) - I(W_1; W_2)$$
$$\vdots$$
$$\leq n(I(X_1; X_2|Y_{F_1}, Y_{F_2}, T_1, T_2) - I(X_1; X_2|T_1, T_2))$$

Main steps in the proof:

- Seemingly trivial inequality $0 \leq I(W_1; W_2|Y_{F_1}^n, Y_{F_2}^n)$.
- Symmetry of $I_3(A; B; C) = I(A; B) - I(A; B|C)$.
- Encoding functions: $X_{1i} = f_{1i}(W_1, Y_{F_1}^{i-1}), X_{2i} = f_{2i}(W_2, Y_{F_2}^{i-1})$.
- Conditioning reduces entropy.

Evaluation of DB Bounds for Gaussian Channels

- Need to consider joint densities p(x1,x2,t) satisfying DB.
- **Claim: Jointly Gaussian densities satisfying DB suffice.**
- **Approach**: Partition the set of densities in 5 sets.
- Main Step : To show that for every NG density satisfying DB , there is a Gaussian density satisfying DB and yielding larger rates.



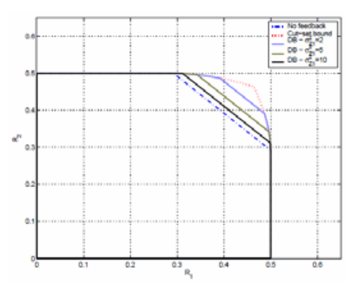
Proof of main step for noisy feedback

- Use of a multivariate generalization of Costa's EPI [Payaro, Palomar, ISIT 2008].
- Use of properties of 3×3 covariance matrices.

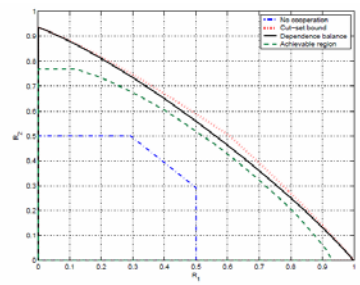
Proof of main step for user cooperation

- Identifying a conditionally independent structure among (T, X_1, X_2).
- Use of a result on Gaussian MAC with conferencing encoders [Bross, Lapidoth, Wigger, ISIT 2008].

Illustration of Bounds and Limiting Cases



$P_1 = P_2 = \sigma_Z^2 = 1$ and $\sigma_{Z_1}^2 = \sigma_{Z_2}^2 = 2, 5, 10.$



$P_1 = P_2 = \sigma_Z^2 = \sigma_{Z_1}^2 = \sigma_{Z_2}^2 = 1$ and $h_{10} = h_{20} = 1, h_{12} = 3, h_{21} = 2.$

Noisy Feedback

Cut-set bound is insensitive to the feedback noise variances $\sigma_{Z_1}^2, \sigma_{Z_2}^2$.

$\mathcal{DB}_{NF}^{MAC}$ is sensitive to feedback noise variances $\sigma_{Z_1}^2, \sigma_{Z_2}^2$.

- As $\sigma_{Z_1}^2, \sigma_{Z_2}^2 \to 0$, $\mathcal{DB}_{NF}^{MAC} \to \mathcal{CS}$ (Ozarow's capacity result).
- As $\sigma_{Z_1}^2, \sigma_{Z_2}^2 \to \infty$, $\mathcal{DB}_{NF}^{MAC} \to \mathcal{C}_{No-Feedback}$ (A new capacity result).

User Cooperation

Cut-set bound is sensitive to cooperation noise variances $\sigma_{Z_1}^2, \sigma_{Z_2}^2$.

$\mathcal{DB}_{UC}^{MAC}$ is more sensitive to feedback noise variances $\sigma_{Z_1}^2, \sigma_{Z_2}^2$.

- As $\sigma_{Z_1}^2, \sigma_{Z_2}^2 \to 0$, $\mathcal{DB}_{UC}^{MAC} \to \mathcal{CS}$ (Degenerates to the total cooperation line)
- As $\sigma_{Z_1}^2, \sigma_{Z_2}^2 \to \infty$, $\mathcal{DB}_{UC}^{MAC} \to \mathcal{C}_{No-Cooperation}$ (A new capacity result).

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