Efficient Interior-point Algorithms for Linear Programming Decoding

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STATUS QUO

- Prior work on LP decoding of LDPC codes attempts mainly to enhance the *speed* and *word error rate* of the decoder.
- Interior-point methods offer the following advantages over other LP methods for decoding
  - polynomial-time convergence
  - rounding to the optimal codeword in intermediate stages
  - less sensitivity to degenerate vertices
- Estimating the solution of the set of linear equations instead of completely solving them in each iteration can enhance the speed of the algorithm greatly.

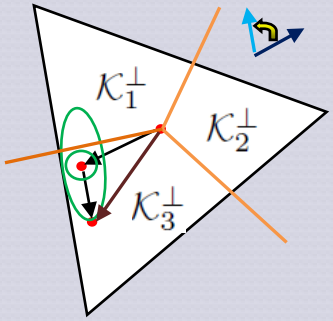
NEW INSIGHTS

- The *min-sum* algorithm can be used for approximating the solution of the set of linear equations in each iteration
  - requires a convex decomposition which can be accomplished by exploiting the block-wise structure of the matrix of constraints.
- The adaptive rule for adjusting the cost function in each iteration can be used in order to enhance the speed of the algorithm.

ACHIEVEMENT

MAIN ACHIEVEMENT:

- **New classes of interior-point algorithms which adaptively adjust the cost-function in each iteration in order to improve the speed of convergence are introduced. The adaptive rule can be based on**
  - the position of the point inside the fundamental polytope in the previous step.
  - the direction of the movement with respect to an already defined reference point in the previous step.
  - both of the above, in other words a combinatorial rule.



- **The adaptive rule must satisfy**

$$f(-\mathcal{K}_i^\perp) \subseteq -\mathcal{K}_i^\perp$$

HOW IT WORKS:

In each iteration of the ellipsoid method the algorithm applies the adaptive function to find a new cost function for the LP. Afterward, instead of solving the set of linear equations, it uses several iterations of conjugate gradients or min-sum algorithm only to find a point close to the optimal solution. In this way, the algorithm manages to follow the central path and converge to the optimal solution.

IMPACT

- **The algorithm can significantly decrease the amount of computations required for decoding each codeword.**
- **The adaptive LP methods introduced by this work can be used in different fields in order to improve different aspects of the optimization.**
- **The convex decomposition introduced for the decoding problem is always valid and does not depend on any constraint.**

NEXT-PHASE GOALS

- Develop an adaptive rule which also improve the *word error rate* (WER) of the decoder.
- Augment other WER enhancing methods to the algorithm.
- Apply the adaptive cost function methods to dynamic problems such as online resource allocation.

Adaptive algorithms using approximate solutions at each iteration can significantly increase the speed of decoding!