Partial Iterative Equalization and Channel Decoding

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Outline

- Background
- Turbo Codes and Partial Iterative Decoding
- Turbo Equalization
- Partial Iterative Equalization and Channel Decoding
- Channel decoding 1.) convolutional code

  2.) turbo code – decoder in serial mode or parallel mode.
- Future Work

Background

- The ‘turbo equalization’ approach has shown strong capability in mitigating ISI incurred by frequency selective channels.
- For optimal detection, complexity exponential with channel memory length
- Most low complexity algorithms incur performance loss.
  DFE, MMSE, DDFSE, etc.

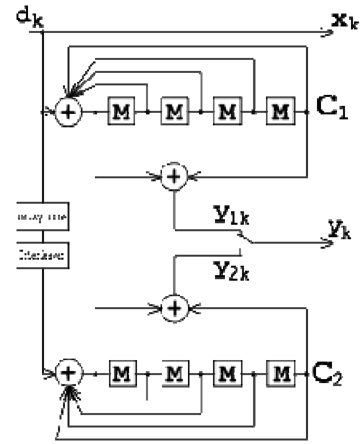
Turbo Codes and Iterative Decoding

Log-MAP algorithm:

$$L(u(n)\,|\,\mathbf{y}) = \ln\left(\frac{P(u(n)=+1\,|\,\mathbf{y})}{P(u(n)=-1\,|\,\mathbf{y})}\right)$$

$$= \ln\Big(\sum_{(S',S)\Rightarrow x(n)=+1} \exp\{A_{n-1}(S') + \Gamma_n(S',S) + B_n(S)\}\Big)$$

$$- \ln\Big(\sum_{(S',S)\Rightarrow x(n)=-1} \exp\{A_{n-1}(S') + \Gamma_n(S',S) + B_n(S)\}\Big)$$

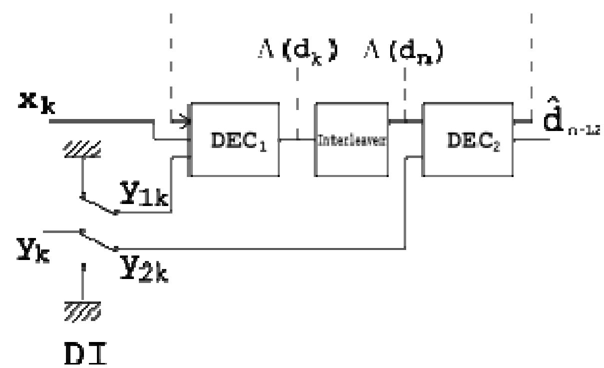


A turbo encoder

where $A_n(S) = \ln\Big(\sum_{allS'} \exp\{A_{n-1}(S') + \Gamma_n(S',S)\}\Big)$

$$B_{n-1}(S') = \ln\Big(\sum_{allS} \exp\{B_n(S) + \Gamma_n(S',S)\}\Big)$$

$$\Gamma_n(S',S) = -\frac{1}{2\sigma^2}\sum_{l=0}^{1}(y_l(n) - a_l(n)x_l(n))^2 + \ln(P(u(n)))$$



$\Rightarrow A_{n-1}(S)$ and $B_n(S)$, $n = 1, 2, \cdots, N$, are two Markov chains.

A turbo decoder

Existing iteration stopping techniques

- Frame-based stopping rules:
  - Cross-entropy J. Hagenauer, et. al., 1996
  - Risk function G. Bauch, et. al., 1997
  - Hard decision aided S. Lin et. al., 2004

- Bit-level stopping rules
  - Early detection and trellis splicing Frey & Kcshischang, 1998
  - Bit hard decision changeability Haase & Rohling, 2002

Revisiting the Markov property in Log-MAP decoding:

- If the soft state values of the boundary bits are converged & known:

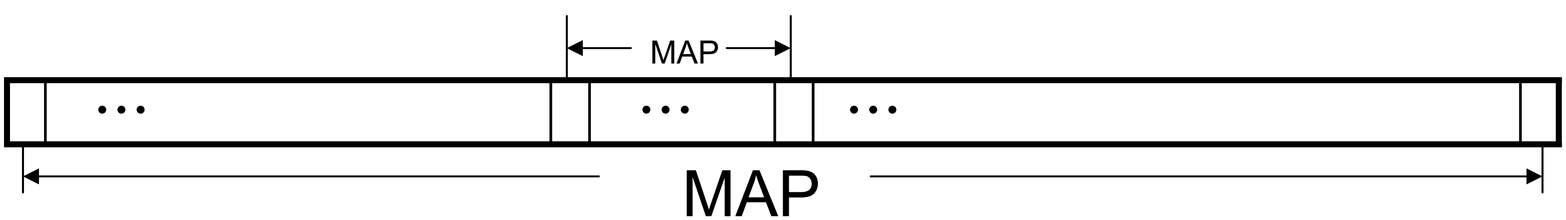
  Given the same a priori LLRs of all bits, decoding within the windowed piece performs the same as within the whole block for windowed bits.

- Decoding convergence criteria

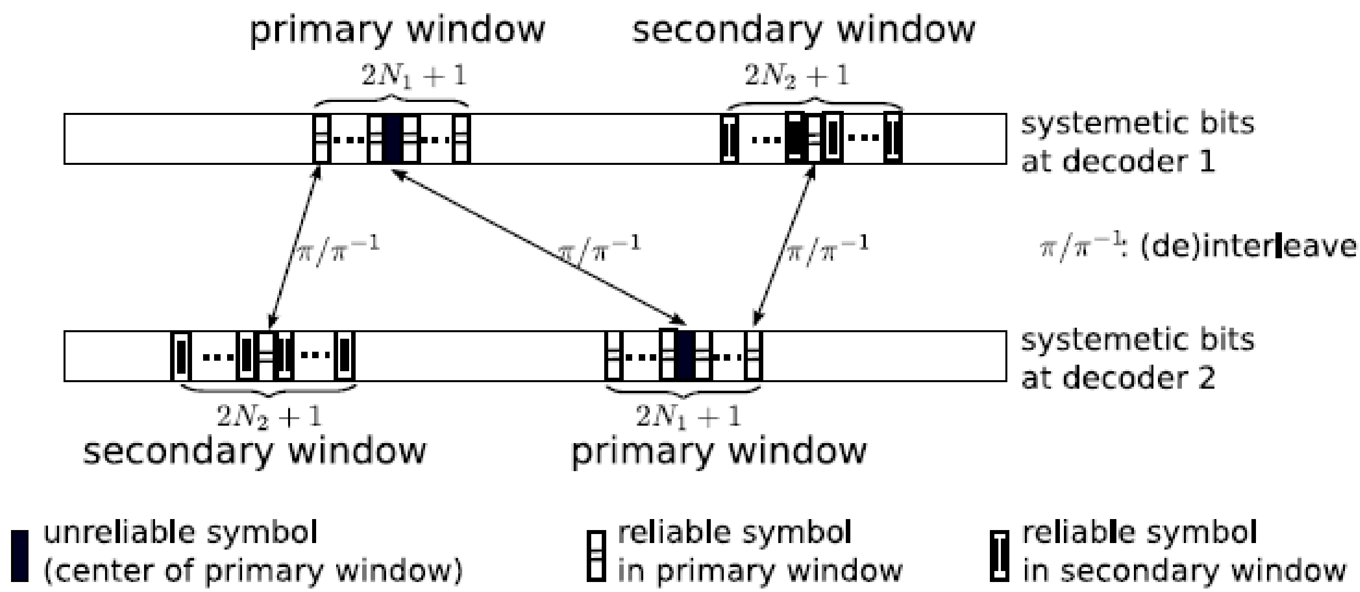
  cross-entropy

  LLR magnitudes

- Adjustment to the Log-MAP algorithm for a single decoder:



Partial iterative MAP decoding for turbo codes



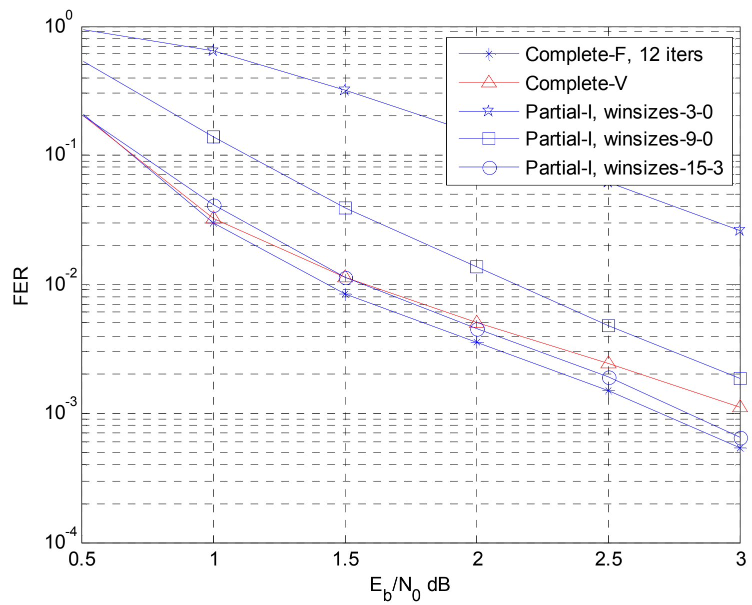
Complexity by a partial iteration: a loose upper bound

$$\frac{C_{\text{partial}}}{C_{\text{complete}}} \leq \frac{M(2N_1(2N_2+1)+2N_1+1)}{N} = \frac{M(4N_1(N_2+1)+1)}{N}$$

Simulation I – PCCC: 2 fixed initial complete iterations + partial iterations. AWGN channel, punctured rate ½ (7, 5) RSC encoders, interleaver size 1024. Primary window size 15, secondary window size 3.

Define C as an unit in complexity that is equivalent to the complexity by a single complete iteration.

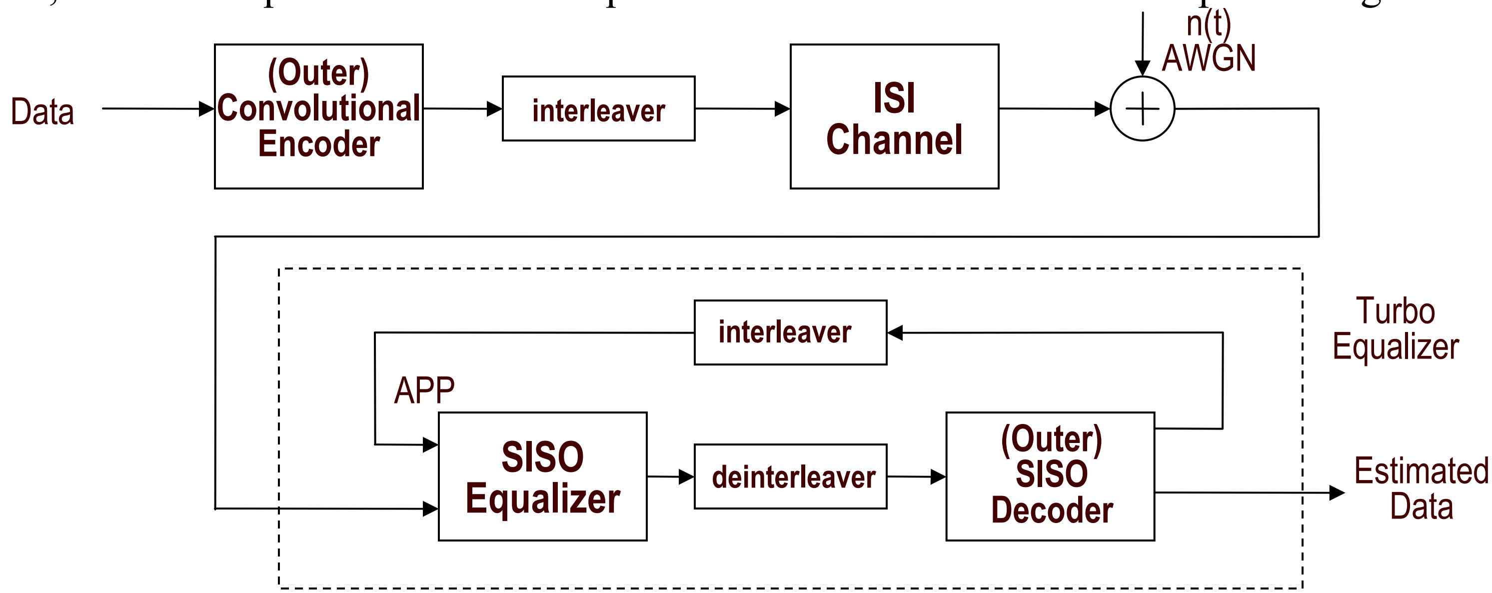
Performance and Complexity By the partial turbo decoding Algorithm.



SNR (dB)	1.0	1.5	2.0	2.5	3.0
Complexity (in C)	4.67	3.50	3.00	2.57	2.16

Turbo Equalization: Convolutional coded

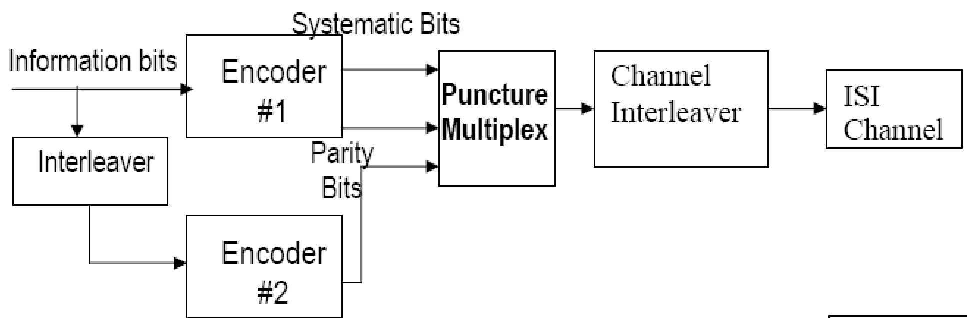
- Extend the ‘turbo principle’ to combine equalization and decoding.
  -- First introduced by Douilard et. Al., 1995.
- Similar to a serially concatenated turbo code, consider ISI channel as a rate 1 convolutional code, the turbo equalizer combines equalizer and decoder with iterative processing.



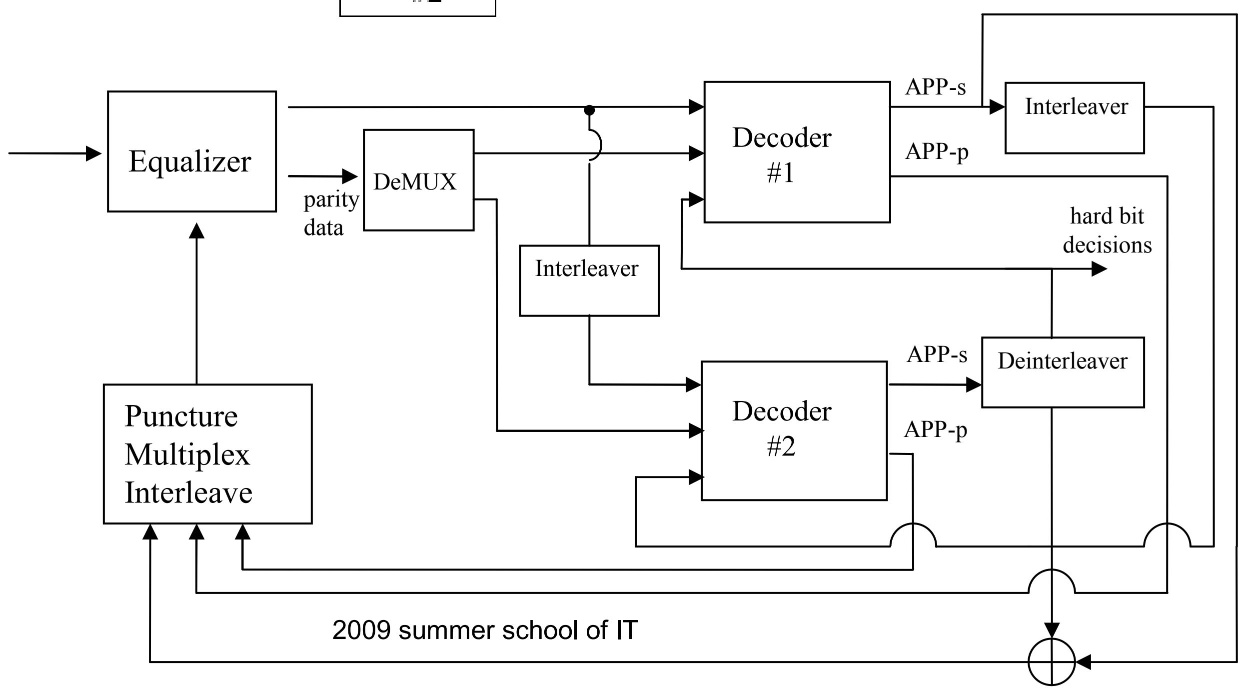
- The convolutional coding/decoding can be replaced with turbo coding/decoding for better performance.

Turbo Equalization: Turbo Coded

- Transmitter:



- Receiver:



ISI channel and MAP equalization algorithm

- System model: discrete time convolution of transmitted symbols and the ISI channel coefficients

$$y(n) = \sum_{l=0}^{L-1} h(l)x(n-l) + w(n)$$

- Forward and backward trellis metrics in MAP algorithm (for BPSK modulation):

$$A_n(S) = \ln(\sum_{allS'} \exp\{A_{n-1}(S') + \Gamma_n(S',S)\})$$

$$B_{n-1}(S') = \ln(\sum_{allS} \exp\{B_n(S) + \Gamma_n(S',S)\})$$

$$\Gamma_n(S',S) = -\frac{1}{2\sigma^2}(y(n) - \sum_{l=0}^{L-1} h(l)\hat{x}(n-l))^2 + L_{Eq}^a(n)$$

The Markov property of $A_{n-1}(S), B_n(S)$ can similarly be utilized.

Extension: Partial iterative equalization and channel decoding

Windowing processes:

1. Turbo equalization with convolutional decoding
2. Turbo equalization with turbo decoding

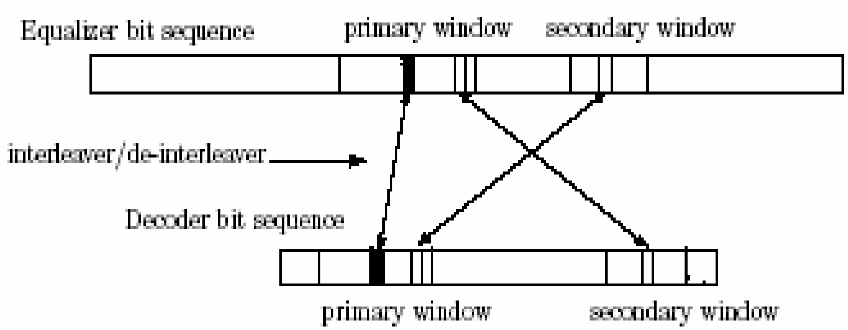


Fig. 1. Bit sequences and windows deployments for receivers with a single decoder and an equalzier.

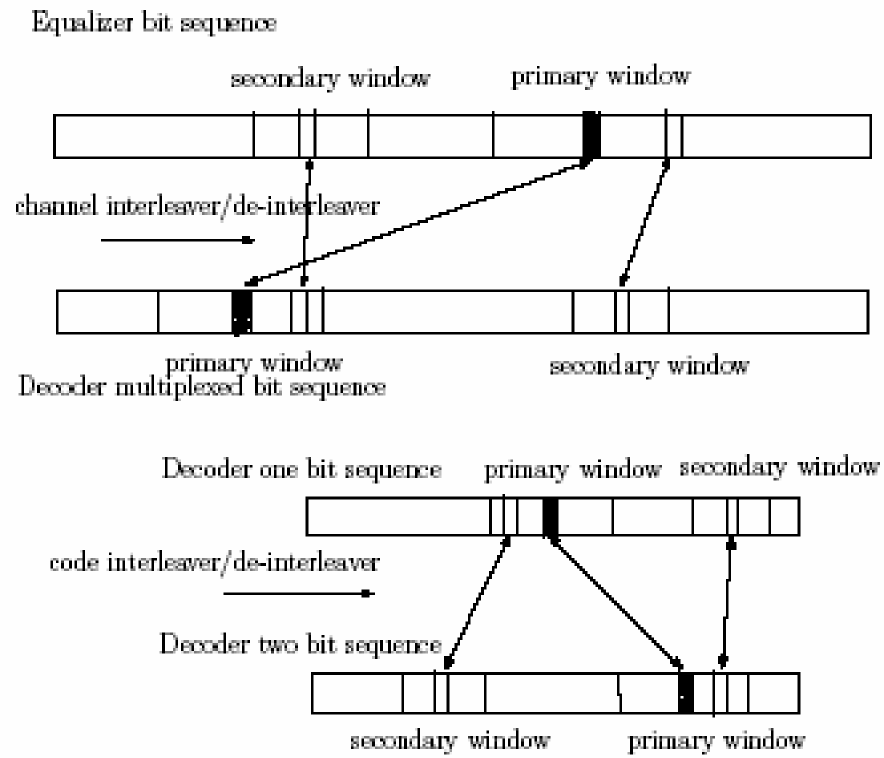


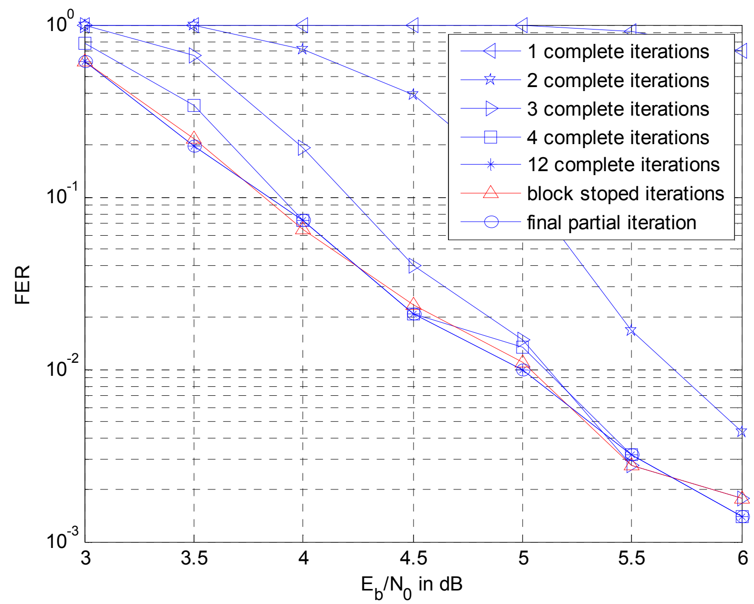
Fig. 2. Bit sequences and windows deployments for receivers with two component decoders and an equalzier.

Partial iterative equalization and decoding performance - I

▪ Transmitter: (37, 21) convolutional code;
Data frame length: 1024;
ISI channel: [0.67, 0.5, 0.39, 0.32, 0.22].

▪ The receiver applies iterative log-MAP equalization and convolutional decoding.
The performance gap between block stopped iterations and a sufficient number of complete iterations is insignificant.

▪ Applying partial iterations:
The performance is similar to block stopped iterations using cross-entropy.



E_bN_o(dB)	4.5	5	5.5	6
Partial iterations	$4.20C_{Eq}$ $4.53C_{De}^{cc}$	$3.66C_{Eq}$ $3.97C_{De}^{cc}$	$3.20C_{Eq}$ $3.47C_{De}^{cc}$	$3.03C_{Eq}$ $3.11C_{De}^{cc}$
Block Stop iterations	$3.93C_{Eq}$ $3.93C_{De}^{cc}$	$3.32C_{Eq}$ $3.32C_{De}^{cc}$	$3.05C_{Eq}$ $3.05C_{De}^{cc}$	$3.00C_{Eq}$ $3.00C_{De}^{cc}$

Partial iterative equalization and decoding performance - II

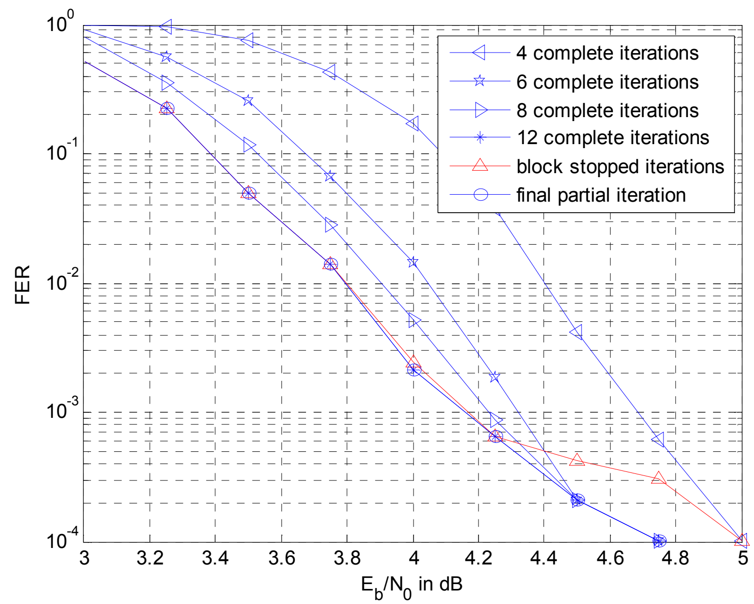
▪ Transmitter: (7, 5) turbo code; data frame length: 1024; ISI channel: [0.67, 0.5, 0.39, 0.32, 0.22].

▪ The receiver applies the serial mode for turbo decoding:

The performance gap between block stopped iterations and a sufficient number of complete iterations is more noticeable at higher SNRs.

▪ Applying partial iterations:

The optimal performance is better maintained.

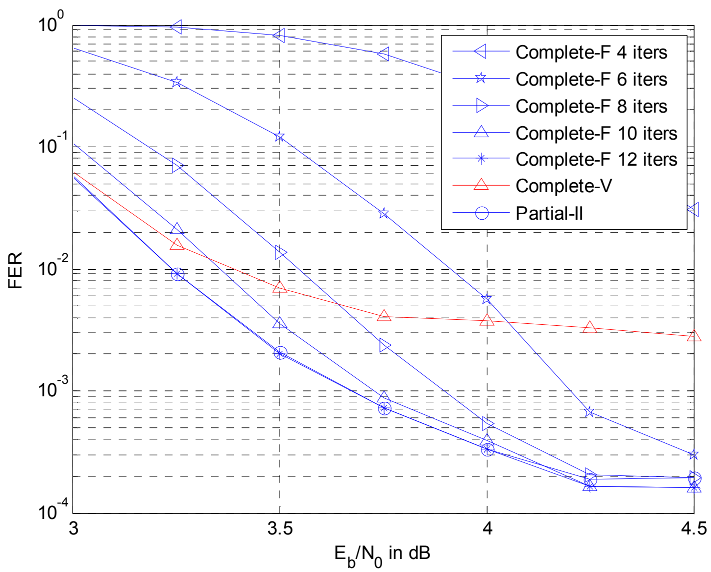


E_bN_o(dB)	3.5	4	4.5	5
Partial iterations	$7.26C_{Eq}$ $7.37C^{tc}_{De}$	$5.12C_{Eq}$ $5.22C^{tc}_{De}$	$4.12C_{Eq}$ $4.19C^{tc}_{De}$	$3.46C_{Eq}$ $3.58C^{tc}_{De}$
Block Stop iterations	$7.10C_{Eq}$ $7.10C^{tc}_{De}$	$4.97C_{Eq}$ $4.97C^{tc}_{De}$	$4.01C_{Eq}$ $4.01C^{tc}_{De}$	$3.31C_{Eq}$ $3.31C^{tc}_{De}$

Partial iterative equalization and decoding performance - III

- Transmitter: (7, 5) turbo code; data frame length: 1024; ISI channel: [0.407, 0.815, 0.407].

- The receiver applies the parallel mode for turbo decoding:
  The performance gap between block stopped iterations and a sufficient number of complete iterations become significant.



Complexity for equalization, in C_{Eq}

SNR(dB)	3.5	3.75	4.0	4.25	4.5
Complete-F	12	12	12	12	12
Complete-V	6.50	5.78	5.24	4.78	4.38
Partial-II, winsizes-15-0	7.01	6.24	5.63	5.15	4.74

Complexity for decoding, in C_{De}

SNR(dB)	3.5	3.75	4.0	4.25	4.5
Complete-F	12	12	12	12	12
Complete-V	6.50	5.78	5.24	4.78	4.38
Partial-II, winsizes-15-3	7.10	6.32	5.71	5.22	4.82

Future work

- Extension to time selective channels.
- Partial iterative MIMO ISI channel equalization and decoding.
- Delayed decision feedback sequence estimation (DDFSE) based on bit/symbol convergence. Equalization complexity reduction with less error propagation.