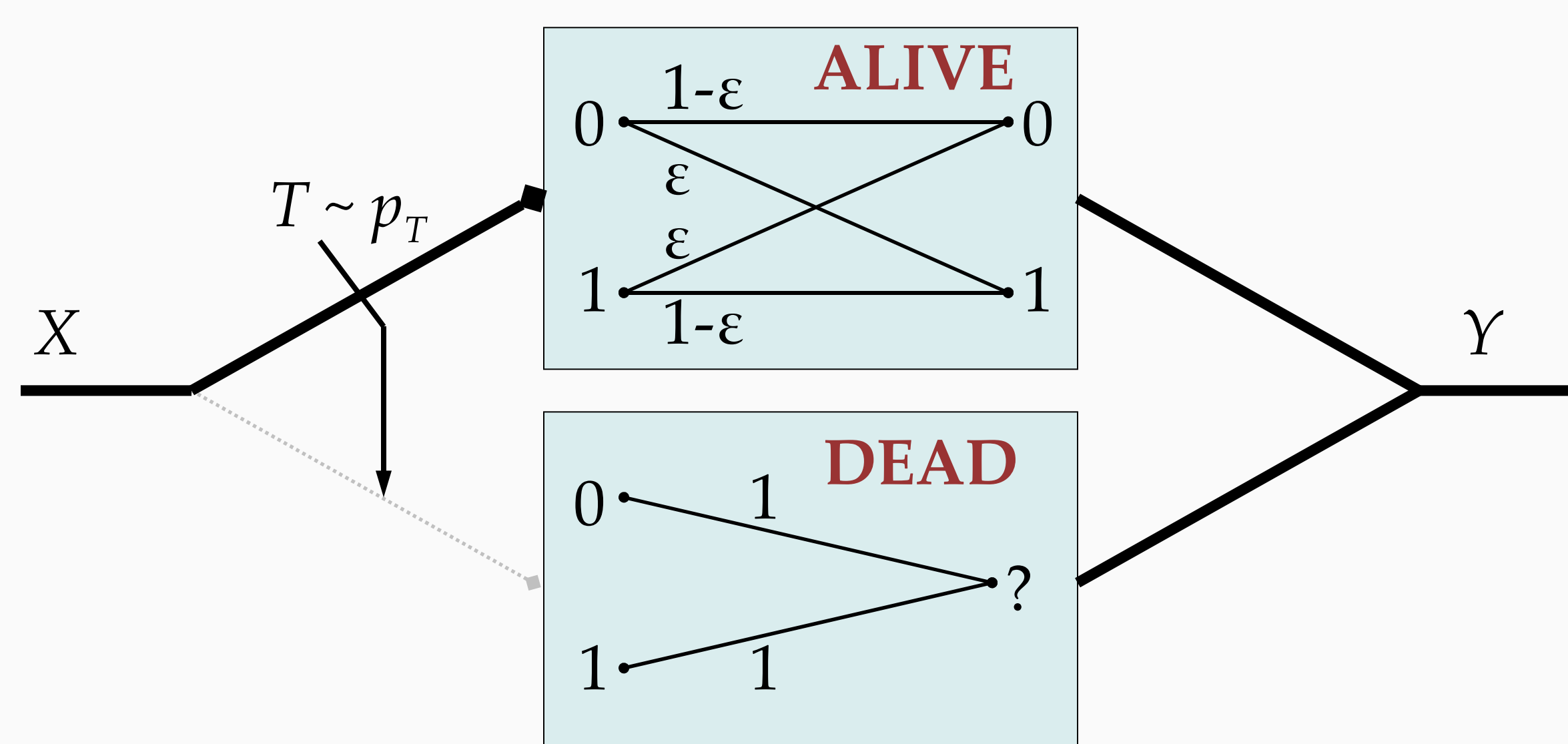


Background and Motivation

- Physical communication systems may fail catastrophically at random times
 - Terminals that may run out of energy
 - Wires that may be cut or destroyed
 - Synthetic biology systems that may die
- Communication over noisy channels when randomly stepping back from infinity
 - Brings notion of finite blocklength to the fore
- Outage in wireless communication is similar to channel death, but outage is not permanent

Channel Model



- Binary input alphabet $\mathcal{X} = \{0, 1\}$
- Ternary output alphabet $\mathcal{Y} = \{0, 1, ?\}$
- In alive state a , acts as BSC with crossover ε
- In dead state d , acts as eraser channel
- Channel starts in state $s = a$ and transitions to state $s = d$ at some random time T , where it remains for all time thereafter
- Death time distribution is denoted $p_T(t)$

Channel Properties

- Channel is a finite-state semi-Markov channel
 - Not specifically studied in information theory
- Embedded Markov chain of channel state process is irreducible

Proposition Shannon channel capacity is zero.

Proof Idea Channel is irreducible, since there is unique absorbing channel state. Capacity of eraser channel is zero, and since this lower capacity equals the upper capacity, result follows.

Communicating over a Channel that Dies

- An information stream that can be grouped into a sequence of k messages
 - (W_1, W_2, \dots, W_k) : each W_i drawn from $\mathcal{W}_i = \{1, 2, \dots, M_i\}$
- Each message is encoded into a channel input codeword $X_1^{n_i}(W_i)$
 - These *individual message codewords* are transmitted in sequence over channel
- Received channel output symbols decoded into messages (Z_1, Z_2, \dots, Z_k)
 - If channel is alive for complete transmission epoch i , Z_i is drawn from \mathcal{W}_i
 - If channel is dead for even part of transmission epoch i , Z_i is declared as Θ

Communication Performance Criteria

- The *maximal error probability* of an $(M_i, n_i)_{i=1}^k$ code is:

$$\lambda^{\max} = \max_{i \in \{1, \dots, k\}} \max_{w \in \mathcal{W}_i} \Pr[Z_i \neq w | W_i = w, Z_i \neq \Theta]$$

- The *transmission time* of an $(M_i, n_i)_{i=1}^k$ code is:

$$N = \sum_{i=1}^k n_i$$

- The *expected transmission volume* of an $(M_i, n_i)_{i=1}^k$ code is:

$$V = E_T \left\{ \sum_{i \in \{1, \dots, k\} | Z_i \neq \Theta} \log_2 M_i \right\}$$

- Given $0 \leq \eta < 1$, a pair of numbers (N_η, V_η) is said to be an *achievable transmission time-volume at η -reliability* if there exists, for some k , an $(M_i, n_i)_{i=1}^k$ code such that $\lambda^{\max} \leq \eta$, $N \leq N_\eta$ and $V \geq V_\eta$. Moreover, (N_η, V_η) is said to be an *achievable transmission time-volume at Shannon reliability* if it is an achievable transmission time-volume at η -reliability for all $0 < \eta < 1$.

Limits of Communication Performance

Proposition Positive volume is not achievable at Shannon reliability.

Proof Idea Show that easiest problem is transmitting a single individual repetition code, and that this is not possible at Shannon reliability.

Proposition The transmission time-volume

$$\left(N = \sum_{i=1}^k n_i, V = \sum_{i=1}^k R_T(e_i) \log M^*(n_i, \eta) \right)$$

is achievable at η -reliability for any sequence $(n_i)_{i=1}^k$ of individual message codeword lengths, where $e_1 = n_1$, $e_2 = n_1 + n_2$, \dots , $e_k = \sum n_k$, $R_T(t) = \Pr[T > t]$, and $M^*(n, \eta)$ is the size of an optimal length- n channel code that achieves block error probability η for a BSC(ε).

Optimizing the Signaling Scheme

- Use dynamic program to choose ordered integer partition $n_1 + \dots + n_k = N$ that yields largest V

Proposition Channel state feedback does not improve performance.

Proof Idea Solution to dynamic program does not require channel state.