Multilevel Coding and Multistage Decoding for M-ary Two-Dimensional ISI Channels

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M-ary 2-D ISI Channel Model and Multilevel Coding

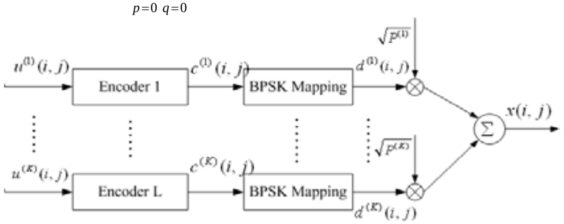
•Input Array: $x(i,j) \in \{-M+1, -M+3, \cdots, M-3, M-1\}$

•Discrete Channel Response: $h(p,q)$

•Post-detected *i.i.d.* Gaussian Noise: $n(i,j) \sim \mathrm{N}(0,\sigma^2)$

•Output Array:

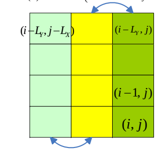
$$y(i,j) = \sum_{p=0}^{L_Y} \sum_{q=0}^{L_X} h(p,q)x(i-p, j-q) + n(i,j)$$



Multi-strip BCJR Algorithm

- $LAPPR^{(k)}(i,j)$ can't be precisely computed.
- Approximation is used to approach $LAPPR^{(k)}(i,j)$
- Define

$$L(d^{(k)}(i,j)|y(i,:)) = \log\left[\frac{P(d^{(k)}(i,j)=-1|y(i,:))}{P(d^{(k)}(i,j)=1|y(i,:))}\right]$$



- $LAPPR^{(k)}(i_0,j_0) \approx \sum_{i=i_0-L_Y}^{i_0} L(d^{(k)}(i_0,j_0)|y(i,:))$
- Apply BCJR strip wise to compute $L(d^{(k)}(i,j)|y(i,:))$
- How to Compute the step metric in the BCJR algorithm which is defined by

$$\gamma_{i,j}^{(k)}(s',s) = \log p(y(i,j)|s',s)p(s|s') \;\; ?$$

Remark: It's easy for binary but hard for M-ary. Only the data from level k+1 to K is known at stage k, but the undecoded data from level 1 to level k-1 are needed to known at this step. If full branch BCJR is applied, the computation complexity is increased to $M^{Lx \cdot Ly}$ which is only $2^{Lx \cdot Ly}$ for binary inputs.

Estimation of the Mean and the Variance

$$\hat{\mu}_{(k)}(i,j) = \mathrm{E}\{y(i,j)|d^{(k:K)}(i:i-L_y, j:j-L_x), d^{(1:k-1)}(i,j)\}$$

$$= h(0,0)\cdot x(i,j) + \sum_{p=1,q=1}^{L_x,L_y} h(p,q)x^{(k:K)}(i-p,j-q) + \frac{1}{W}\sum_{w=1}^{W}\sum_{p=1,q=1}^{L_x,L_y} h(p,q)x^{(1:k-1)}(i-p,j-q)$$

Where $x^{(k1:k2)}(i,j) = \sum_{k=k1}^{k2}\sqrt{P^{(k)}}d^{(k)}(i,j)$, and W is the number of possible combinations of $\{d^{(1:k-1)}(i-p,j-q)|p=1:L_x, q=1:L_y\}$.

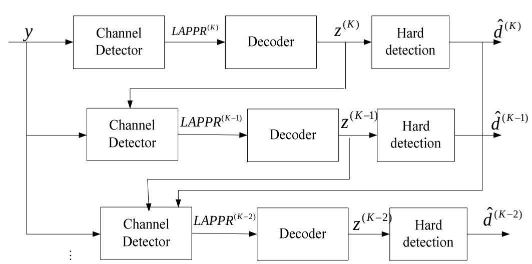
$$\hat{\sigma}^2_{(k)}(i,j) = \mathrm{E}\left\{\left|y(i,j)-\hat{\mu}_{(k)}\right|^2 \middle| \begin{matrix} d^{(k:K)}(i:i-L_y, j:j-L_x), \\ d^{(1:k-1)}(i,j) \end{matrix}\right\} = \sigma^2 + \frac{1}{W}\sum_{w=1}^{W}\left|A_0 + A_1 + A_2 - \hat{\mu}_{(k)}\right|^2$$

$$A_0 = h(0,0)\cdot x(i,j)$$

$$A_1 = \sum_{p=1,q=1}^{L_x,L_y} h(p,q)x^{(k:K)}(i-p,j-q)$$

$$A_2 = \sum_{p=1,q=1}^{L_x,L_y} h(p,q)x^{(1:k-1)}(i-p,j-q)$$

Equalization and Decoding Scheme



The *log a posteriori probability ratio* (LAPPR):

$$LAPPR^{(k)}(i,j) = \log\left[\frac{P(d^{(k)}(i,j)=-1|y)}{P(d^{(k)}(i,j)=1|y)}\right]$$

The soft output from the decoder on stage k : $z^{(k)}(i,j)$

The hard decision on stage

$$\hat{d}^{(k)}(i,j) = \begin{cases} -1 & if \quad z^{(k)}(i,j) \geq 0 \\ 1 & if \quad z^{(k)}(i,j) < 0 \end{cases}$$

Gaussian Approximation of $\gamma_{i,j}^{(k)}(s',s)$

- Taking the bits of the undecoded levels as identical independent random variables with equal probability and averaging the interference of the undecoded levels.
- $\gamma_{i,j}^{(k)}(s',s)$ is computed by Gaussian approximation as:

$$\gamma_{i,j}^{(k)}(s',s) = \log p(y(i,j)|s',s) = \log P\left\{y(i,j)\middle|d^{(k:K)}(i-L_y:i, j-L_x:j)\right\}$$

$$= \log\frac{1}{2^{k-1}}\sum_{d^{(1:k-1)}(i,j)} P\left\{y(i,j)\middle| \begin{matrix} d^{(k:K)}(i-L_y:i, j-L_x:j) \\ , d^{(1:k-1)}(i,j) \end{matrix}\right\}$$

$$\approx \log\frac{1}{2^{K-k}}\cdot\frac{1}{2\pi\hat{\sigma}_{(k)}^{\;2}}\sum_{d^{(1:k-1)}(i,j)}\exp\left\{-\frac{\left[y(i,j)-\hat{\mu}_{(k)}(i,j)\right]^2}{2\hat{\sigma}_{(k)}^{\;2}(i,j)}\right\}$$

where $\hat{\mu}_{(k)}(i,j)$ and $\hat{\sigma}^2_{(k)}(i,j)$ are the estimated mean and variance of $y(i,j)$ given $d^{(k:K)}(i-L_y:i, j-L_x:j)$.

Simulation Result

Channel response is $h = \frac{1}{\sqrt{18.25}}\begin{bmatrix} 0.5 & 1 \\ 1 & 4 \end{bmatrix}$. Semi-algebraic LDPC components codes with the code rates 0.4, 0.7 and 0.8 for level 1, level 2 and level 3 respectively are used for BER simulation.

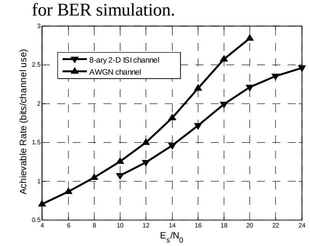


Fig. 1. the total achievable rates

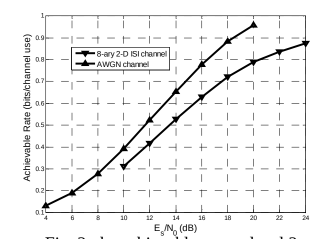


Fig. 2. the achievable rate at level 2

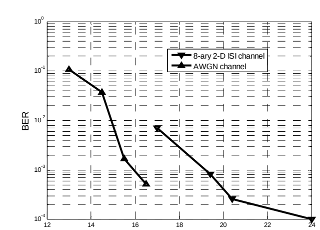


Fig. 3. average BER

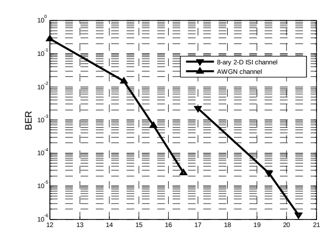


Fig. 4. BER Performance at Level 2

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