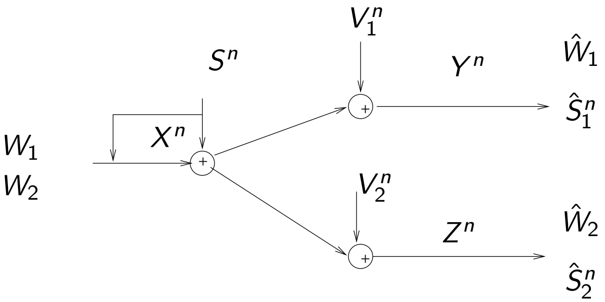
Message Transmission and State Estimation Over Gaussian Broadcast Channels

Wei Liu

Electrical Engineering and Computer Science
Syracuse University
Advisor: Dr. Biao Chen

August 12, 2009

System Model



Gaussian Broadcast Channel with States (GBCS)

- Noncausal additive state information: $S^n \in \mathcal{N}(0, QI_n)$
- Gaussian noise: $V_1^n \sim \mathcal{N}(0, N_1 I_n)$ and $V_2^n \sim \mathcal{N}(0, N_2 I_n)$, $N_1 \leq N_2$.
- Encoder: $\frac{1}{n} \sum_{i=1}^{n} E\{X_i^2(W_1, W_2, S^n)\} \leq P$
- Decoders: $(\hat{W}_1, \hat{S}_1^n) = g_1(Z^n)$ and $(\hat{W}_2, \hat{S}_2^n) = g_2(Z^n)$
- Application: Hybrid digital analog radio broadcast system.

Definitions

Decoding (Receiver $i = 1, 2$)

- Message decoding error: $P_{ei} = \frac{1}{2^{nR_i}} \sum_{j=1}^{2^{nR_i}} Pr\{\hat{W}_i \neq j | W_i = j\}$.
- State estimation error(MSE): $Ed(S^n, \hat{S}_i^n) = \frac{1}{n} \sum_{j=1}^{n} E(S_j - \hat{S}_{ij})^2$.

Achievable quadruples

Definition: A quadruple (R_1, R_2, D_1, D_2) is said to be achievable if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, n)$ codes such that the probability of errors $P_{e1} \rightarrow 0$ and $P_{e2} \rightarrow 0$ and the MSE for state estimation $Ed(S^n, \hat{S}_1^n) \leq D_1$ and $Ed(S^n, \hat{S}_2^n) \leq D_2$, as $n \rightarrow \infty$.

Achievable region

Definition: The achievable region $\mathbb{R}$ is defined as the convex hull of all the achievable quadruples.

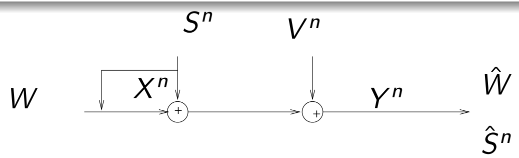
Related works

Multiple User Writing on Dirty Paper (Kim-Sutivong-Sigurjonsson, ISIT04)

Capacity region = Capacity region of GBC without states: for $\gamma \in [0,1]$,

$$\begin{aligned} R_1 &\leq \frac{1}{2}\log\left(1+\frac{\gamma P}{N_1}\right) \\ R_2 &\leq \frac{1}{2}\log\left(1+\frac{(1-\gamma)P}{\gamma P+N_2}\right) \end{aligned}$$

Channel Capacity and State Estimation (CCSE) for State Dependent Gaussian Channels(Sutivong, Chiang, Cover, and Kim 05IT)



CCSE for state dependent Gaussian channels

Optimal rate distortion tradeoff

$$R \leq \frac{1}{2}\log\left(1+\frac{\gamma P}{N}\right)$$
$$D \geq \frac{Q(N+\gamma P)}{(\sqrt{Q}+\sqrt{(1-\gamma)P})^2+N+\gamma P}$$

for some $0 \leq \gamma \leq 1$

Coding strategy

- Coding: combining Costa's dirty paper coding and state amplification

Other recent related works (only list few here)

- Merhav-Shamai, Information rates subject to state masking, 07IT.
- Kim-Sutivong-Cover, State amplification, 08IT.

Main results: an achievable region

Theorem

Inner bound $\mathbb{R}_i$: *For the state dependent Gaussian broadcast channel, the rate distortion quadruple* (R_1, R_2, D_1, D_2) *is achievable if*

$$
\begin{aligned}
R_1 &\leq \frac{1}{2}\log\left(1 + \frac{\gamma P}{N_1}\right) \\
R_2 &\leq \frac{1}{2}\log\left(1 + \frac{\beta P}{\gamma P + N_2}\right) \\
D_1 &\geq \frac{Q(N_1 + \beta P + \gamma P)}{(\sqrt{Q} + \sqrt{(1-\beta-\gamma)P})^2 + N_1 + \beta P + \gamma P} \\
D_2 &\geq \frac{Q(N_2 + \beta P + \gamma P)}{(\sqrt{Q} + \sqrt{(1-\beta-\gamma)P})^2 + N_2 + \beta P + \gamma P}
\end{aligned}
$$

where $0 \leq \beta \leq 1$ *and* $0 \leq \gamma \leq 1 - \beta$.

Proof

Sketch of proof of achievability

- Coding strategy: Combine multiuser dirty paper coding and state amplification.
- Split the power into three parts: βP, γP and $(1-\beta-\gamma)P$
- For each s^n: generate $X_s = \sqrt{\frac{(1-\beta-\gamma)P}{Q}} s^n$
- total interference: $X_s + s^n$
- Message transmission: Multiuser dirty paper coding with power βP and γP, respectively.
- Decoder: MMSE estimation of the state.

Main results: an outer bound

Theorem

Outer bound $\mathbb{R}_o$: *For the state dependent gaussian broadcast channel, the rate distortion tradeoff (R_1, R_2, D_1, D_2) should satisfy*

$$
\begin{aligned}
R_1 &\leq \frac{1}{2}\log(1+\frac{\gamma P}{N_1}) \\
R_2 &\leq \frac{1}{2}\log(1+\frac{\beta P}{\gamma P + N_2}) \\
D_1 &\geq \frac{QN_1 \exp(2(R_1+R_2))}{(\sqrt{Q}+\sqrt{(1-\beta-\gamma)P})^2 + N_1 + \beta P + \gamma P} \\
D_2 &\geq \frac{Q(N_2+\gamma P)\exp(2R_2)}{(\sqrt{Q}+\sqrt{(1-\beta-\gamma)P})^2 + N_2 + \beta P + \gamma P}
\end{aligned}
$$

for some $0 \leq \beta \leq 1$ and $0 \leq \gamma \leq 1-\beta$.

- Techniques: converse of the capacity region of Gaussian broadcast channel and rate distortion theory.

Special case: one sided distortion constraint on worse user

Corollary

One sided distortion constraint on worse user: *For the state dependent Gaussian broadcast channel with only one distortion constraint* (D_2) *at worse receiver* Z*, the rate distortion tradeoff* (R_1, R_2, D_2) *is achievable if and only if*

$$
\begin{aligned}
R_1 &\leq \frac{1}{2}\log\left(1 + \frac{\gamma P}{N_1}\right) \\
R_2 &\leq \frac{1}{2}\log\left(1 + \frac{\beta P}{\gamma P + N_2}\right) \\
D_2 &\geq \frac{Q(N_2 + \beta P + \gamma P)}{(\sqrt{Q} + \sqrt{(1-\beta-\gamma)P})^2 + N_2 + \beta P + \gamma P}
\end{aligned}
$$

where $0 \leq \beta \leq 1$ *and* $0 \leq \gamma \leq 1 - \beta$.

Special case: one sided distortion constraint on worse user

The intuition

- Multiuser Dirty Paper Coding is optimal for GBCS (Kim *et al*).
- State amplification is optimal for point to point case (Sutivong *et al*).
- Optimal coding:
  - Power allocation: βP, γP and $(1-\beta-\gamma)P$
  - Worse user: do a optimal Sutivong *et al*'s coding for point to point channel (treat good user as noise), hence achieve R_2 and D_2
  - Good user: can decode W_2 and get a clean signal, hence achieve a rate R_1
  - Each steps are optimal.

Other special cases

- Pure state estimation: $R_1 = R_2 = 0$, the bounds match.
- Symmetric Broadcast channel: noise variance $N_1 = N_2 = N$, bounds match.
- High SNR regime: two bounds match.
- Pure information transmission: reduced to the capacity of GBC.
- $R_2 = 0, D_2 = \infty$: reduced to Sutivong *et al*'s region.

Conclusion and selected references

Conclusion

- We consider the message transmission and state estimation over Gaussian broadcast channels.
- Proposed an achievable region and an outer bound for the optimal rate distortion trade off.
- Our region is tight for several special cases.

Selected References

- Y. H. Kim, A. Sutivong, and S. Sigurjonsson, Multiple writing on dirty paper, in Proc. IEEE Int. Symp. Infor.Theory, Chicago, IL, USA, Jun./Jul 2004.
- A. Sutivong, M. Chiang, T. M. Cover, and Y. H. Kim, Channel capacity and state estimation for state-dependent Gaussian channels, IEEE. Trans. Inform. Theory, vol. 51, no.4, pp. 1486-1495, April 2005.