Two Hop Interference Networks

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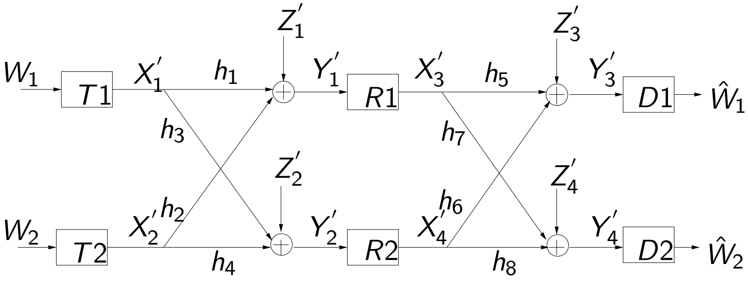
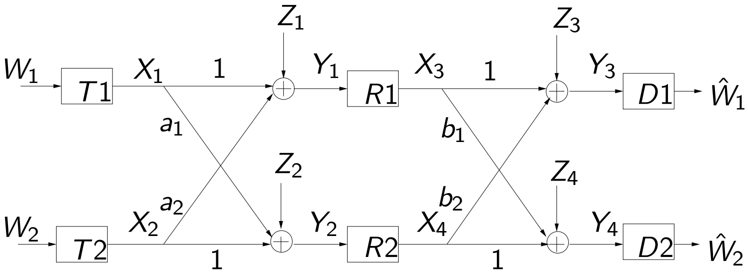
General Channel Model and Related Work



- O. Simeone, O.Somekh, Y. Bar-Ness, H. V. Poor, and S. Shamai, 2007 (explored the coherent combining gain of decode-and-forward scheme)
- C. Thejaswi, A. Bennatan, J. Zhang, R. Calderbank, and D. Cochran, 2008 (focused on the second hop and explored the possibilities of utilizing the common messages from the first hop)

Standard Form



Consider Symmetric Channel Parameters

$$a_1 = a_2 \triangleq a$$
$$b_1 = b_2 \triangleq b$$
$$P_{11} = P_{12} \triangleq P_1$$
$$P_{21} = P_{22} \triangleq P_2$$

Case I: $0 < a < 1, 0 < b < 1$

- The first hop: Han and Kobayashi's Scheme

$$R_p^{(1)} = \gamma\left(\frac{\alpha P_1}{1+a^2\alpha P_1}\right)$$
$$R_c^{(1)} = \min\left\{\gamma\left(\frac{a^2\bar{\alpha}P_1}{\sigma_1^2}\right), \frac{1}{2}\gamma\left(\frac{(1+a^2)\bar{\alpha}P_1}{\sigma_1^2}\right)\right\}$$

- The second hop: Combine DPC and MAC schemes
  - *DPC Scheme*

$$R_{p,DPC}^{(2)} = \gamma\left(\frac{\beta P_2}{1+b^2\beta P_2}\right)$$
$$R_{c,DPC}^{(2)} = \frac{1}{2}\gamma\left(\frac{(1-b^2)^2\bar{\beta}^2P_2^2}{\sigma_2^4} + \frac{2(1+b^2)\bar{\beta}P_2}{\sigma_2^2}\right)$$

  - *MAC Scheme*

$$R_{p,MAC}^{(2)} = \max_{\alpha}\left\{\min\left[\gamma\left(\frac{b^2\bar{\alpha}\beta P_2}{\sigma_3^2}\right), \frac{1}{2}\gamma\left(\frac{(1+b^2)\bar{\alpha}\beta P_2}{\sigma_3^2}\right)\right] + \gamma\left(\frac{\alpha\beta P_2}{1+b^2\alpha\beta P_2}\right)\right\}$$
$$R_{c,MAC}^{(2)} = \frac{1}{2}\gamma\left(\frac{(1+b)^2\bar{\beta}P_2}{1+(1+b^2)\beta P_2}\right)$$

Case II: $a > 1, b > 1$

A natural solution:

- The first hop: the capacity region of the strong interference channel is known as

$$R^{(1)} = \min\left(\gamma(P_1), \frac{1}{2}\gamma((1+a^2)P_1)\right)$$

- The second hop: the capacity of the Gaussian vector broadcast channel is known as

$$R^{(2)} = \frac{1}{2}\gamma((b^2-1)^2P_2^2 + 2P_2(1+b^2)).$$

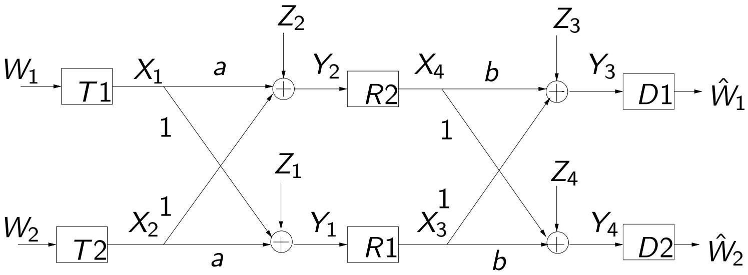
- The overall transmission rate is

$$R = \min\{R^{(1)}, R^{(2)}\}.$$

Optimality in each hop does not guarantee optimality of the entire network!

Case II: $a > 1, b > 1$

After switching the roles of the two relays, the model becomes



- Both hops are converted to weak interference
- We can apply the transmission schemes proposed in Case I
- The resulting rates always include the “natural solution” as a subset

Amplify and Forward

For in-phase relaying, set

$$X_3 = \sqrt{\frac{P_2}{(1+a^2)P_1+1}}Y_1, X_4 = \sqrt{\frac{P_2}{(1+a^2)P_1+1}}Y_2$$

The overall channel becomes

$$\begin{aligned} Y_3 &= (1+ab)X_1 + (a+b)X_2 + Z_3' \\ Y_4 &= (a+b)X_1 + (1+ab)X_2 + Z_4' \end{aligned}$$

For out-of-phase relaying, set

$$X_3 = -\sqrt{\frac{P_2}{(1+a^2)P_1+1}}Y_1, X_4 = \sqrt{\frac{P_2}{(1+a^2)P_1+1}}Y_2$$

The overall channel becomes

$$\begin{aligned} Y_3 &= (ab-1)X_1 + (b-a)X_2 + Z_3' \\ Y_4 &= (a-b)X_1 + (1-ab)X_2 + Z_4' \end{aligned}$$

Out-of-phase Relaying When $a = b$, $P_1 = P_2$

- When $a = b$, the overall channel becomes two parallel AWGN channel

$$R = \gamma\left(\frac{(1-a^2)^2 P_1}{1+a^2+1/c^2}\right) = \gamma\left(\frac{(1-a^2)^2 P_1 P_2}{(1+a^2)(P_1+P_2)+1}\right)$$

- If $a = b < 1$, the sum rate capacity of the interference channel is known at the condition of 'noisy interference'

$$a(a^2 P_1 + 1) \leq \frac{1}{2}, \text{ i.e., } P_1 \leq \frac{1}{a^2}\left(\frac{1}{2a} - 1\right)$$

  and the corresponding symmetric capacity is

$$C_1 = \gamma\left(\frac{P_1}{1+a^2 P_1}\right)$$

- The overall rate exceeds the capacity of both hops when

$$\frac{1+4a^2-a^4+\sqrt{(1+4a^2-a^4)^2+4a^2(1-a^2)^2}}{2a^2(1-a^2)^2} < P_1 < \frac{1}{a^2}\left(\frac{1}{2a}-1\right)$$

  For example, when $a = 0.15$, $51.6 < P_1 < 103.7$.

Numerical Examples

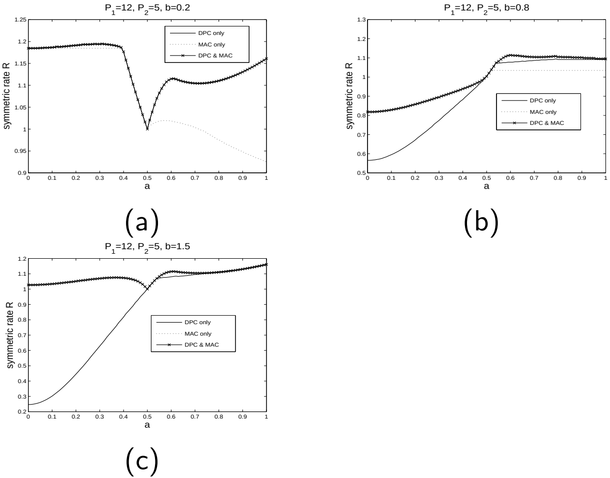
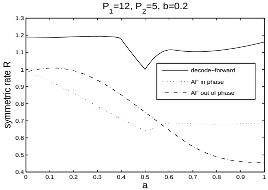
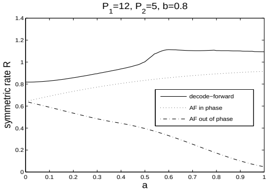


Figure: Comparison of DPC scheme and MAC scheme in the second hop for the decode-and-forward relaying.

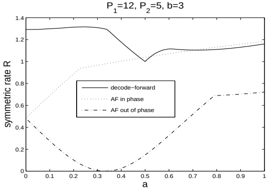
Numerical Examples



(a)



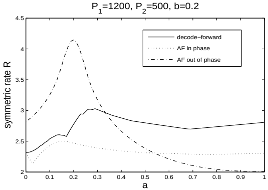
(b)



(c)

Figure: Comparison of decode-and-forward relaying and amplify-and-forward relaying in low SNR regime

Numerical Examples



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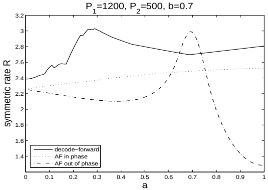
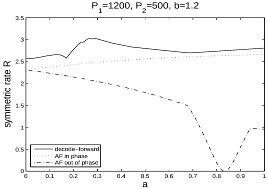


(b)



(c)

Figure: Comparison of decode-and-forward relaying and amplify-and-forward relaying in high SNR regime

Conclusion

- We investigated and compared coding schemes for the two hop interference network under various channel parameters regimes.
- If the first hop has strong interference, i.e., $a > 1$, it is always beneficial to switch the roles of the two relays so that the channel is converted to a weak interference channel with interference gain of $1/a$, and the strength of the second hop is also changed accordingly.
- For the decode-and-forward relaying, the DPC scheme and MAC scheme are both needed for the second hop. Generally however, DPC scheme dominates when b is small and MAC scheme dominates when b is large.
- DF always has better performance than AF except when a is close to b in the high SNR regime.
- When $a = b$, the overall transmission rate can exceed the capacity of both hops.