Optimal Auction Design for Single Minded Buyers with Finite Values

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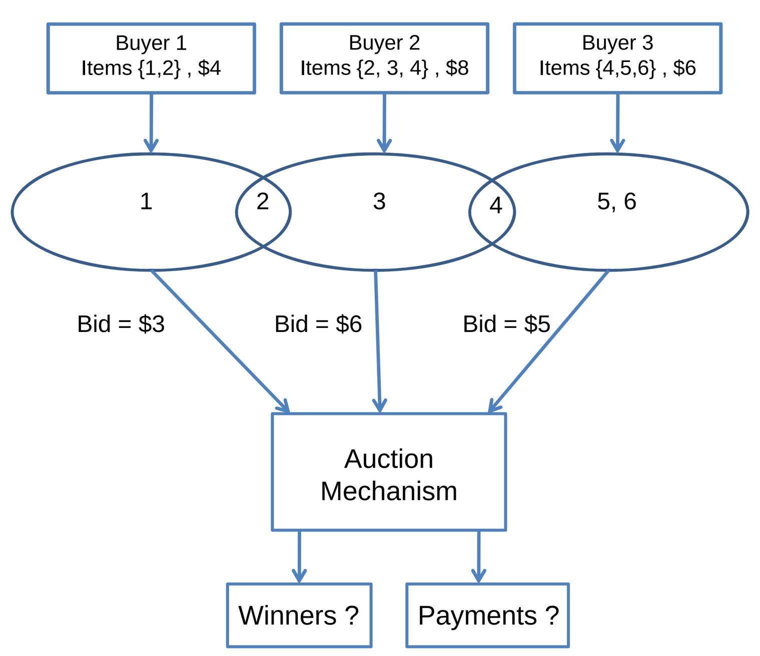
Introduction



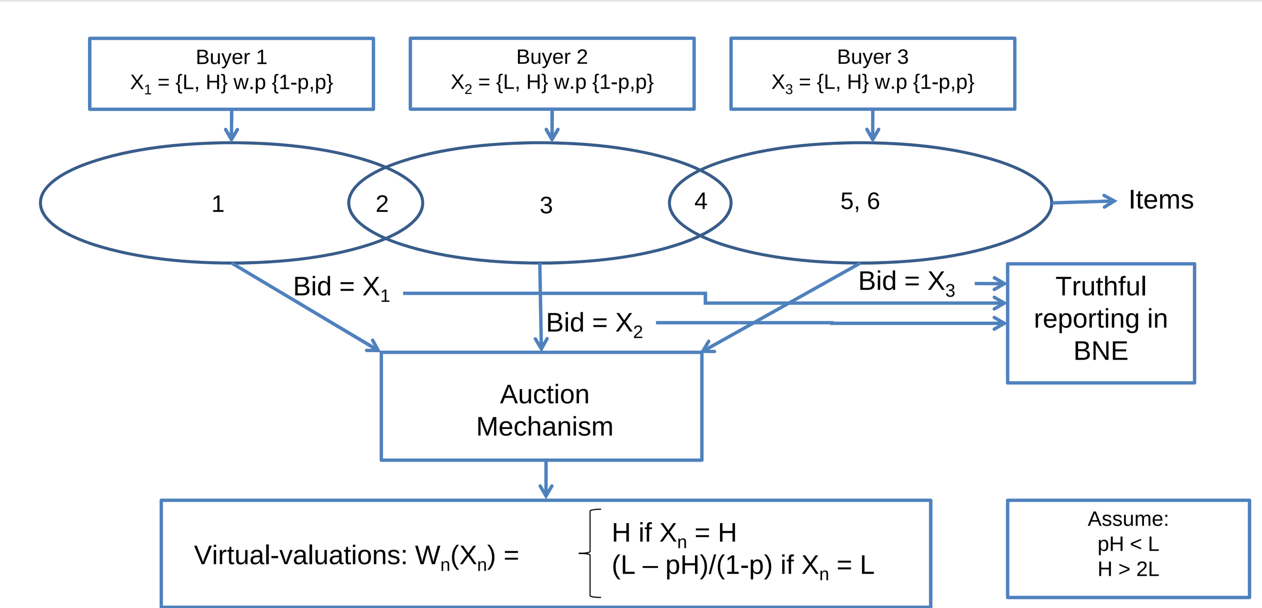
- *N* single-minded buyers competing for *S* items.
- Known bundles but bundle value is private information.
- Buyers are strategic.

Seller's problem:
Design an auction to maximize the revenue from sale

Constraints

- **Feasible allocation**: Winners must have disjoint bundles., i.e., if buyers n and m are winners then $s_n \cap s_m = \emptyset$.
- **Individual rationality**: No buyer can lose money by participating in the auction, i.e., for each buyer n, if $X_n = x_{ni}$ then $u_n(x_{ni}) \geq 0$.
- **Incentive compatibility**: Truth-telling is an equilibrium strategy, i.e., for a buyer n, $1 \leq i \leq K$, and $\forall\, b_n$, $u_n(x_{ni}) \geq q_n(b_n)x_{ni} - m_n(b_n)$.

Example



	(X_1, X_2, X_3)							
	L,L,L	L,L,H	L,H,L	L,H,H	H,L,L	H,L,H	H,H,L	H,H,H
Winners	1 & 3	1 & 3	2	1 & 3	1 & 3	1 & 3	1 & 3	1 & 3
Pavements	(L,0,L)	(L,0,L)	(0,H,0)	(L,0,H)	(L,0,L)	(L,0,L)	(H,0,L)	(L,0,L)

Model and Approach

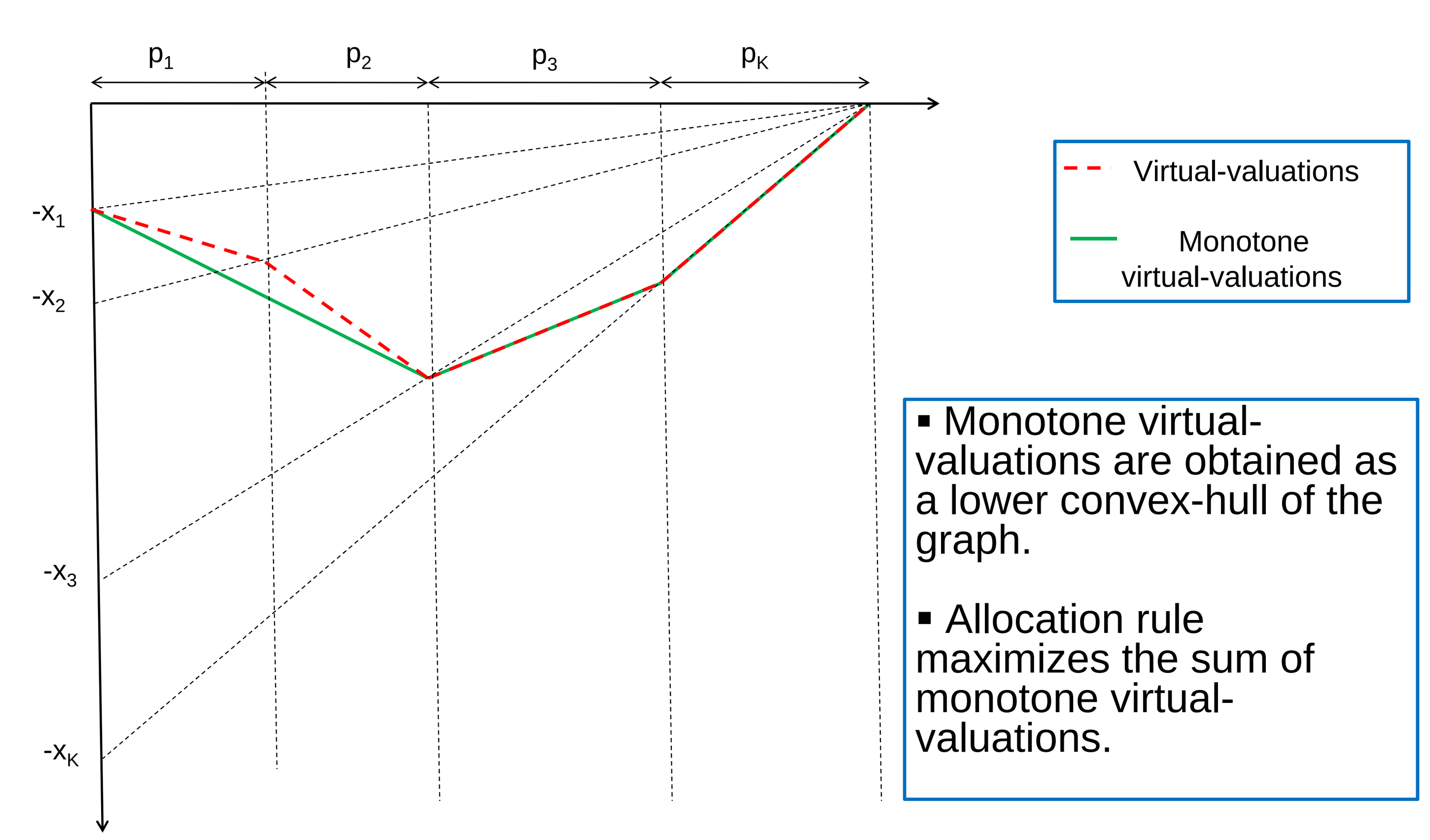
- Buyers' values are independently realized from known distributions.
- **Auction** = (allocation rule, payment rule).
- Induces a game of imperfect information among buyers.
- Constraints:
  - Feasible allocation.
  - Non-negative utility from participation.
  - Truth-telling as Bayes Nash Equilibrium (BNE).

Find an optimal allocation rule and payment rule for maximum revenue in equilibrium

Optimal Auction: Regular Case

- **Virtual-valuations**: Mapping that indicates the significance of submitted bids from revenue point of view.
- **Regularity assumption**: Virtual-valuations for each buyer is non-decreasing with respected to his value.
- **Reserve price**: Don't sell to buyers with negative virtual-valuation.
- **Winners**: The feasible allocation with maximum sum of virtual-valuations.
- **Payments**: Winners pay the minimum they need to report to still win. Losers pay nothing.

Optimal Auction: Non-Regular Case

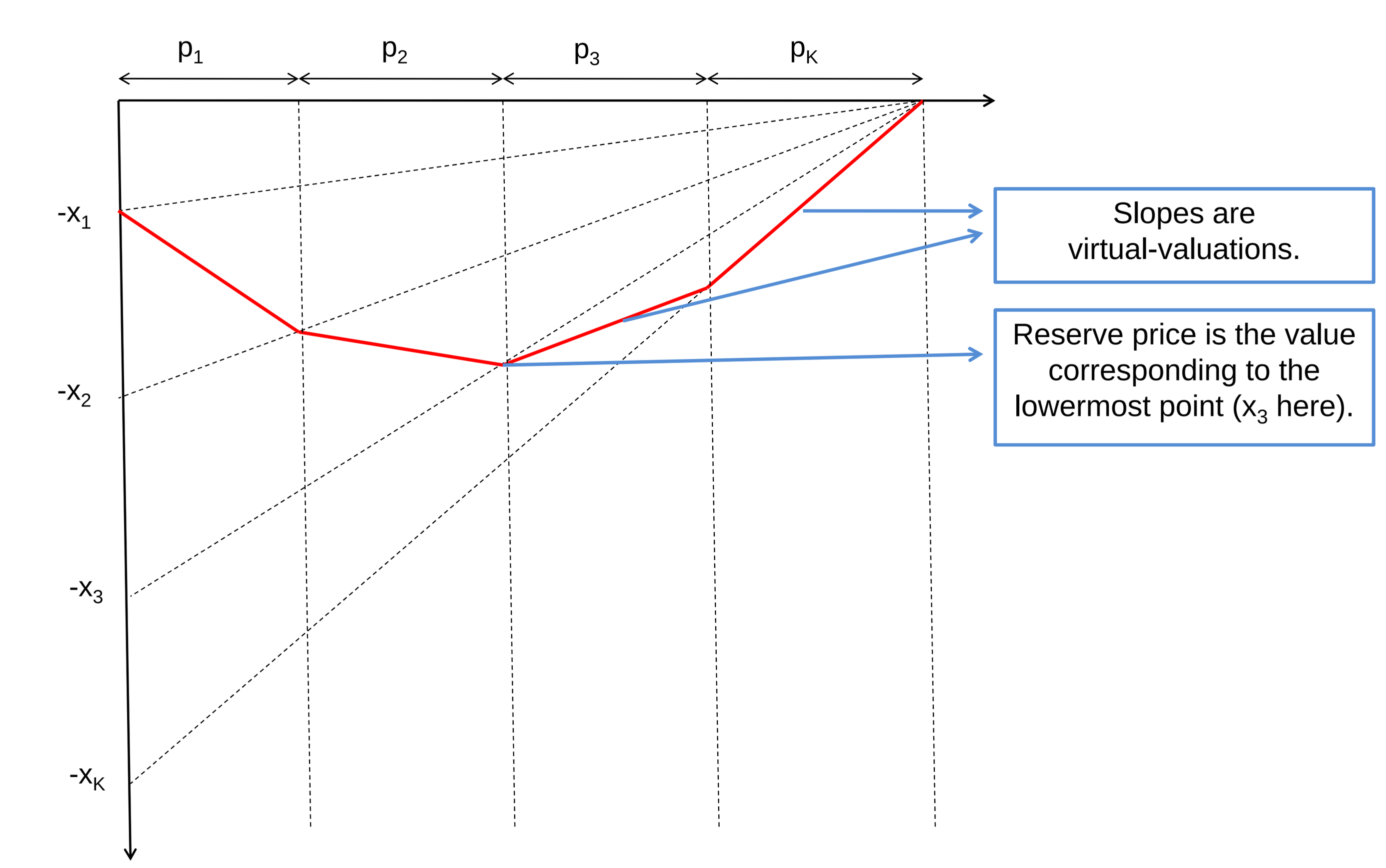


- Monotone virtual-valuations are obtained as a lower convex-hull of the graph.
- Allocation rule maximizes the sum of monotone virtual-valuations.

Notations and Definitions

- Buyer n wants the bundle s_n of items.
- Value of buyer n = random variable X_n taking K discrete values $x_{n1} \leq x_{n2} \leq \ldots \leq x_{nK}$ with probabilities $p_{n1}, p_{n2}, \ldots, p_{nK}$.
- $(Q_n(\mathbf{b}), M_n(\mathbf{b}))$ = probability that buyer n wins and the payment he makes when the reported bids are $\mathbf{b} = (b_1, b_2, \ldots, b_N)$.
- $(q_n(b_n), m_n(b_n))$ = expected values when everyone else is truthful and $X_n = b_n$.
- Expected utility from being truthful = value of outcome - payment, i.e., given that $X_n = x_{ni}$, $u_n(x_{ni}) = q_n(x_{ni})x_{ni} - m_n(x_{ni})$.

Virtual valuations: Graphical Construction



Comments & References

- Virtual-valuation of a buyer only depends on his probability distribution function.
- For the resulting auction, truthfulness about the bundle as well the value is a weakly dominant strategy for a buyer.
- The virtual-valuation approach can be used for other objectives such as maximizing sum of utilities for buyers.
- **References**:

[1] R. Myerson, "Optimal auction design," Mathematics of Operations Research, vol. 6, no. 1, pp. 58-73, 1981.