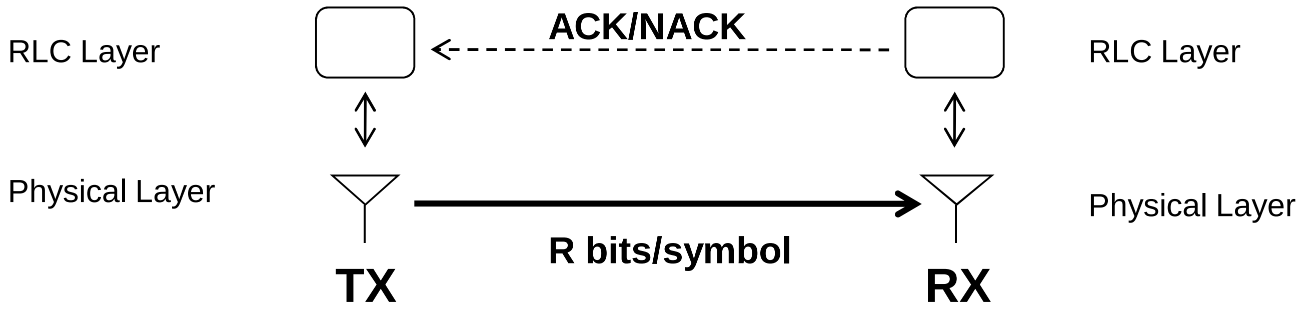
Coding vs. ARQ in Fading Channels: How reliable should the PHY be?

Peng Wu and Nihar Jindal

University of Minnesota
Department of Electrical & Computer Engineering

Introduction



- Two different and competing reliability mechanisms:
  - Coding: Reducing transmitted rate R (bits/sym) reduces packet error probability ε
  - ARQ: Retransmit packets not received correctly
- What is the optimal balance for reliable communication?

Main Assumptions

- Basic assumptions:
  - Perfect CSI at receiver, transmitter only knows channel statistics (high mobility scenario)
  - Packets retransmitted until successfully decoded
  - Error free ACK/NACK
  - Perfect error detection
  - Errors independent across packet transmissions
  - At PHY, either of HARQ and no HARQ can be used

- Important performance measure: long-term avg. successful rate

$$\eta = \frac{R}{\mathrm{E}[X]} \quad \text{bits/symbol}$$

  - X: number of PHY transmission attempts for each packet
  - η is also referred as goodput/successful throughput

PHY Reliability without PHY HARQ

- Number of packet TX’s ~ Geometric (1-ε) :

$$E[\#\ \text{of packet TX's}] = \frac{1}{1-\varepsilon}$$

- Successful throughput/goodput :

$$R(1-\varepsilon)\ \text{bits/sym}$$

  - Error prob. increases in TX rate R
  - A reliable PHY layer requires the sacrifice of TX rate
- Objective: Characterize ε^* as a function of channel parameters (SNR, time/frequency selectivity)

$$\boxed{\varepsilon^* = \arg\max_{\varepsilon}\ R(1-\varepsilon)}$$

  - Same issue arises for optimization of application layer fountain codes

Channel Model without PHY HARQ

- Fast fading: channel varies too rapidly to be tracked at TX
- Block-fading (order L time/freq selectivity): $y_t = \sqrt{SNR}\, h_k x_t + z_t$
  - h_k : channel in k-th fading block, iid across blocks
  - Channels are complex Gaussian (Rayleigh fading), known at RX
  - SNR: average received signal-to-noise ratio
- With infinite blocklength and strong channel code, successfully decode iff:

$$\frac{1}{L}\sum_{i=1}^{L} \log_2 (1+|h_i|^2\, SNR) > \mathrm{R}$$

  - Errors due to bad channel realizations
- Packet error probability = mutual information outage probability

$$\varepsilon = \mathrm{P}\left[\frac{1}{L}\sum_{i=1}^{L} \log_2 (1+|h_i|^2\, SNR) < \mathrm{R}\right]$$

Gaussian Approximation

- Approximate RX mutual information by Gaussian:

$$\varepsilon = \mathrm{P}\left[\underbrace{\frac{1}{L}\sum_{i=1}^{L} \log_2\left(1+|h_i|^2\ SNR\right)}_{\sim\ \mathrm{N}\left(\mu(SNR),\ \sigma^2(SNR)/L\right)} < \mathrm{R} \right] \approx \mathrm{Q}\left(\frac{\mu(SNR) - \mathrm{R}}{\sigma(SNR)/\sqrt{\mathrm{L}}} \right)$$

$$\Rightarrow\ R \approx \mu(SNR) - Q^{-1}(\varepsilon)\frac{\sigma(SNR)}{\sqrt{L}}$$

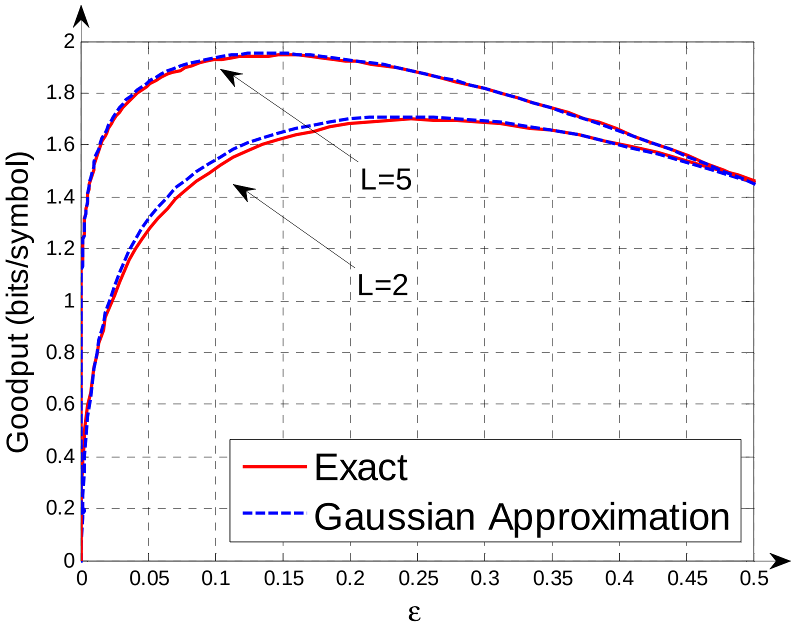
- Goodput approximation:

$$\eta_g = \mu(SNR)\left(1 - \kappa \cdot Q^{-1}(\varepsilon)\right)\left(1 - \varepsilon\right)$$

  - κ: μ-normalized standard deviation

$$\kappa = \frac{\sigma(SNR)}{\mu(SNR)/\sqrt{L}}$$

  - κ decreases in both SNR and
  - η_g is strictly concave in ε



SNR=10 dB

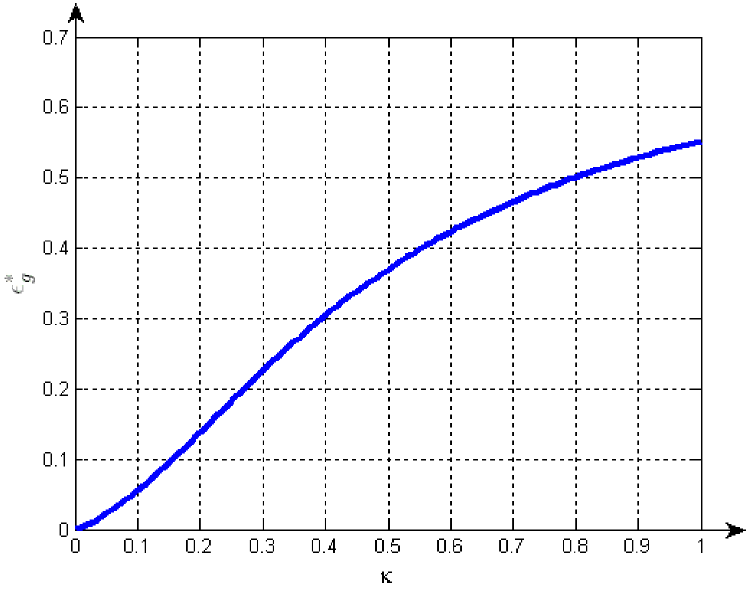
Goodput Optimization

- η_g - maximizing error probability:

$$\boxed{\varepsilon_g^*(SNR, L) = \arg\max_{\varepsilon} \left(1-\kappa\cdot Q^{-1}(\varepsilon)\right)(1-\varepsilon)}$$

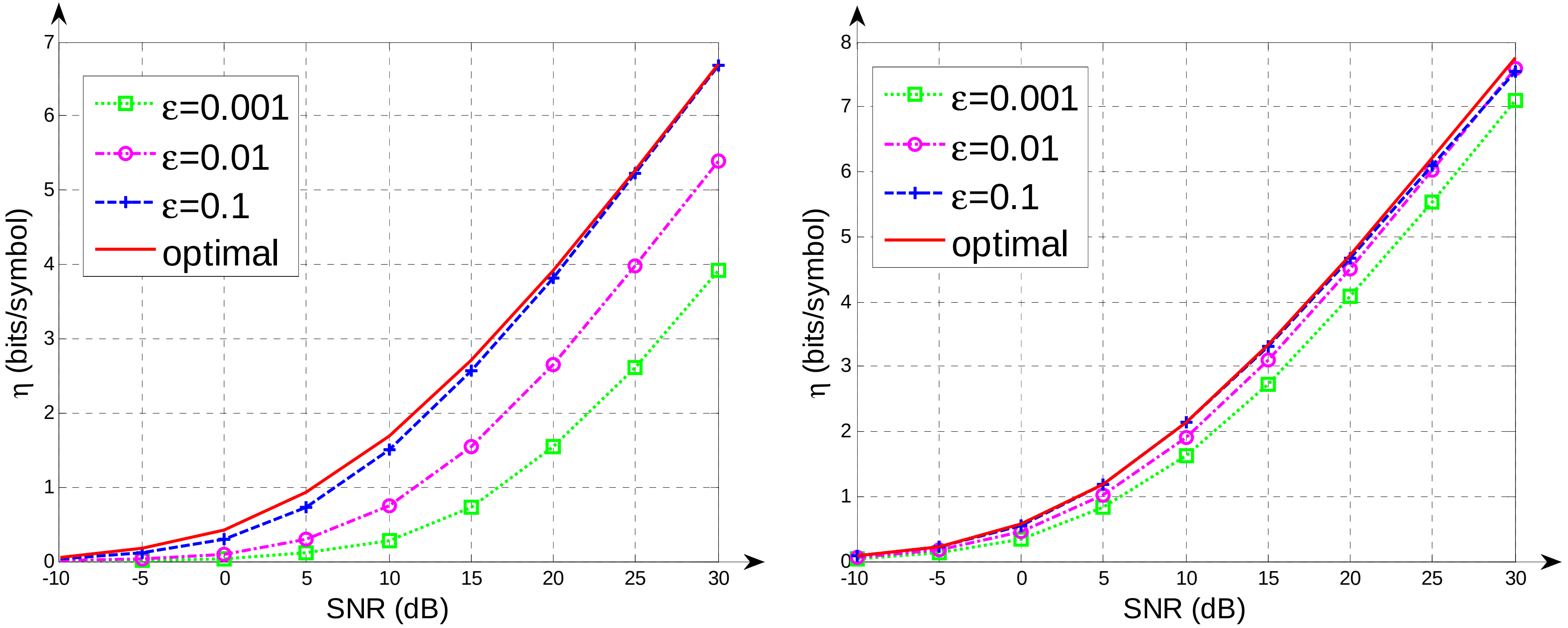
- ε_g^* satisfies

$$\left(Q^{-1}(\varepsilon_g^*) - (1-\varepsilon_g^*)\frac{dQ^{-1}(\varepsilon)}{d\varepsilon}\bigg|_{\varepsilon=\varepsilon_g^*} \right)^{-1} = \kappa$$



- ε_g^* is only determined by κ
  - Basic analysis can be easily extended to MIMO and other fading distributions
  - ε_g^* monotonically increases in κ (decreases in SNR and L)

Goodput vs. SNR



- 10% near optimal for wide range of selectivity (L)
- Making PHY too reliable incurs significant penalty in open loop systems

Incremental Redundancy (IR)

- Outage if cannot decode after L IR rounds-> triggers RLC ARQ retransmission
- Frequency-flat channel during each IR round, i.i.d. fading across rounds
- If initial transmission rate is RL bits/sym, # of IR rounds is smallest $\hat{x}$ s.t.

$$\sum_{i=1}^{\hat{x}} \log_2(1+|h_i|^2\ SNR) > \mathrm{RL}$$

- $\hat{X}$: random variable describing # of IR rounds

$$\mathrm{P}\left[\hat{X} \le k\right] = \begin{cases} \mathrm{P}\left[\sum_{i=1}^{k} \log_2(1+|h_i|^2\ SNR) > \mathrm{RL}\right] & \mathrm{k} = 1,.., \ \mathrm{L} - 1 \\ 1 & \mathrm{k} = \mathrm{L} \end{cases}$$

  - Error probability identical to expression without IR
  - Relationship between R and ε unaffected by IR

$$\varepsilon = \mathrm{P}\left[\sum_{i=1}^{L} \log_2(1+|h_i|^2\ SNR) < \mathrm{RL}\right]$$

  - Expectation of $\hat{X}$ is $\mathrm{E}[\hat{X}] = 1 + \sum_{k=1}^{L-1} \mathrm{P}\left[\sum_{i=1}^{k} \log_2(1+|h_i|^2\ SNR) < \mathrm{RL}\right]$

Goodput Optimization with IR

- Successful throughput (i.e., goodput):

$$\frac{RL}{\mathrm{E}[\hat{X}]/(1-\varepsilon)} = \underbrace{\frac{RL}{\mathrm{E}[\hat{X}]}}_{\text{PHY Transmitted Rate}}(1-\varepsilon) = \underbrace{R(1-\varepsilon)}_{\text{Goodput without IR}}\frac{L}{\mathrm{E}[\hat{X}]}$$

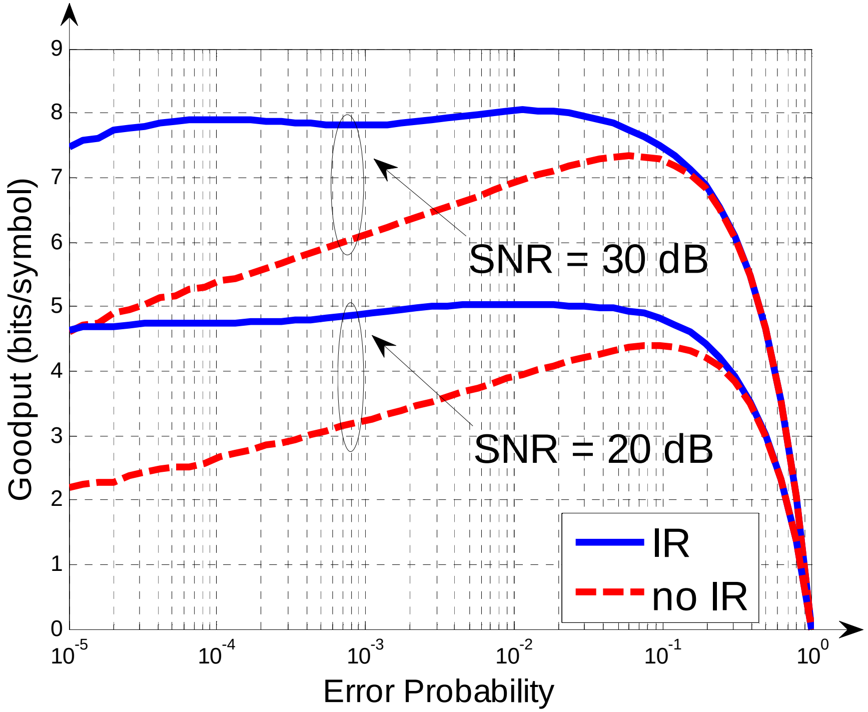
- Optimal operating point:

$$\boxed{\varepsilon^{*}_{IR}(SNR, L) = \arg\max_{\varepsilon} R(1-\varepsilon)\frac{L}{\mathrm{E}[\hat{X}]}}$$

  - $\mathrm{E}[\hat{X}]$: an increasing function of R and ε
- Result: The optimizing error probability with IR is less than or equal to the optimizing error probability without IR:

$$\varepsilon^{*}_{IR} \le \varepsilon^{*}_{no-IR}$$

Goodput (IR) vs. Error Prob.



- If error prob. (or TX rate) too large, no early termination
- If error prob. (or TX rate) too small, always terminate in one IR round
- IR makes goodput less sensitive to the choice of error probability if parameters are carefully chosen

Conclusion and Future Work

- Don’t make PHY reliable if it is difficult (e.g., lacking diversity) to do so, let ARQ achieve reliability instead
  - Increase PHY reliability only when it is easy (e.g., plenty of diversity, closed loop system) to do so
  - Same argument applies to rateless coding

- HARQ can achieve lower optimum PHY error probability
  - Insensitivity of goodput to choice of reliability makes PHY optimization almost pointless

- Future work:
  - The effect of HARQ feedback on the goodput/optimal reliabilty
  - Taking RLC delay into account