Hybrid Digital-Analog Joint Source-Channel Codes for Broadcasting Correlated Gaussian Sources

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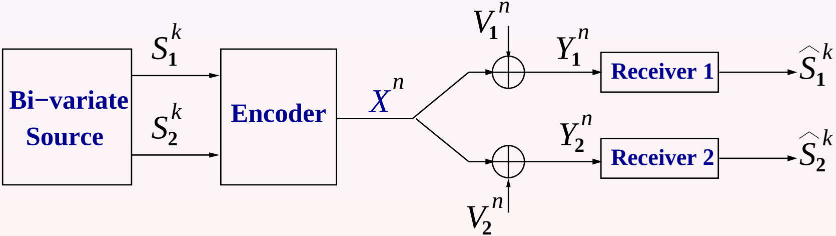
Aug 13, 2009

Outline

System Model

Broadcasting k samples of a bivariate Gaussian source in $n = \lambda k$ uses of a power-limited broadcast channel to two users



Bandwidth compression/expansion ratio: $\lambda = \frac{n}{k}$. We consider

- Matched bandwidth ($\lambda = 1$)

Problem Formulation

- Sources: $S_1(t)$ and $S_2(t)$ have zero mean and variance $\sigma_{S_1}^2$ and $\sigma_{S_2}^2$, respectively, and correlation coefficient $\rho \in (-1, 1)$.
- Joint Encoder: $X^n = \varphi\left(S_1^k, S_2^k\right)$, $\varphi : \mathbb{R}^k \times \mathbb{R}^k \to \mathbb{R}^n$
- Averaged power-limited transmitted sequence: $\frac{1}{n}\sum_{t=1}^{n} E\left[|X(t)|^2\right] \leq P$.
- Gaussian Broadcast Channel:
  - Receiver i observes $Y_i(t) = X(t) + V_i(t), \quad i = 1, 2$
  - $V_i(t) \sim \mathcal{N}\left(0, \sigma_i^2\right)$ are independently distributed over i and t, and are independent of the $X(t)$.
  - User 1: Weak User, User 2: Strong User

Problem Formulation (Cont.)

- Based on its channel output Y_i^n, user i provides an estimate $\widehat{S}_i^k = \psi_i\left(Y_i^n\right)$, where $\psi_i : \mathbb{R}^n \to \mathbb{R}^k$ is a decoding function.
- Fidelity Measure: Average squared error distortion
  $\Delta_i = \frac{1}{k} E[\sum_{t=1}^{k} |S_i(t) - \widehat{S}_i(t)|^2]$
- For a particular coding scheme $(\varphi, \psi_1, \psi_2)$, the performance is determined by the channel power constraint P and the incurred distortions Δ_1 and Δ_2 at the receivers.
- For any given power constraint $P > 0$, the distortion region $\mathcal{D}$ is defined as the convex closure of the set of simultaneously achievable distortion pairs at two users.

Our Goal

Goal

We aim to determine **achievable distortion regions** using **hybrid digital-analog (HDA) coding schemes** for two cases;
1) the source bandwidth equals the channel bandwidth,
2) broadcasting with bandwidth compression.

Note that the source-channel separation theorem does not hold in broadcasting correlated sources.

Distortion Regions with Matched Bandwidth

- Analog (Uncoded) Transmission
- Layering with Analog and Costa Coding
- Layering with Analog, Superposition and Costa Coding

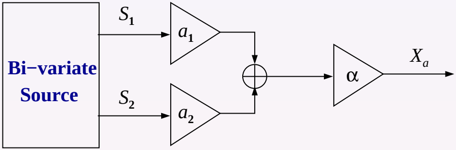
1. Analog (Uncoded) Transmission

- Scaling the encoder input subject to the channel power constraint and transmitting it without explicit channel coding.

Why Uncoded?

- For a point-to-point transmission of a Gaussian source through an AWGN channel, uncoded is optimal (Goblick, 1965)
- For broadcasting a bivariate Gaussian source, below a certain SNR threshold, uncoded transmission is optimal (Bross, Lapidoth and Tinguely, 2008).
- In broadcasting a single Gaussian source to two users, uncoded achieves simultaneously the optimal distortion for both users (Chen and Wornell, 1998).
- For sending a bivariate Gaussian source over a Gaussian MAC, below an SNR threshold, uncoded is optimal

Analog Transmission: Analysis



- **Encoder**: $X_a(t) = \tilde{\alpha} \sum_{i=1}^{2} a_i S_i(t)$,

  $\tilde{\alpha} = \sqrt{\frac{P}{a_1^2\sigma_{S_1}^2 + a_2^2\sigma_{S_2}^2 + 2a_1a_2\rho\sigma_{S_1}\sigma_{S_2}}}$, $a_i \geq 0$.
- **Decoder**: MMSE estimator
- **Distortion**: The set of simultaneously achievable distortion pairs at two users:

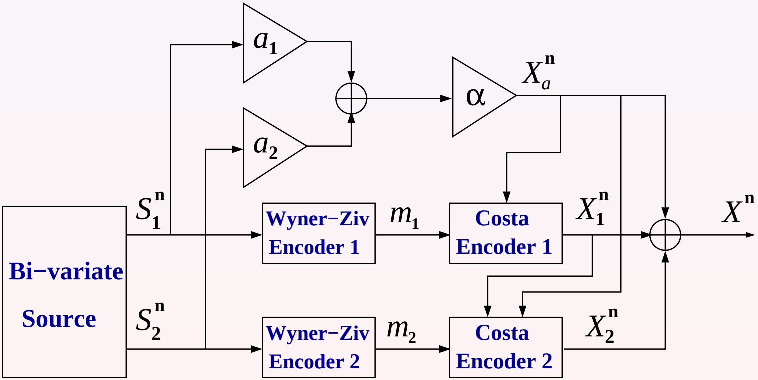
$$D_i = \sigma_{S_i}^2 - \frac{\tilde{\alpha}^2(a_i\sigma_{S_i}^2 + a_j\rho\sigma_{S_i}\sigma_{S_j})^2}{P + \sigma_i^2}, \quad i,j = 1,2, \; j \neq i$$

Joint Source-Channel Coding Schemes

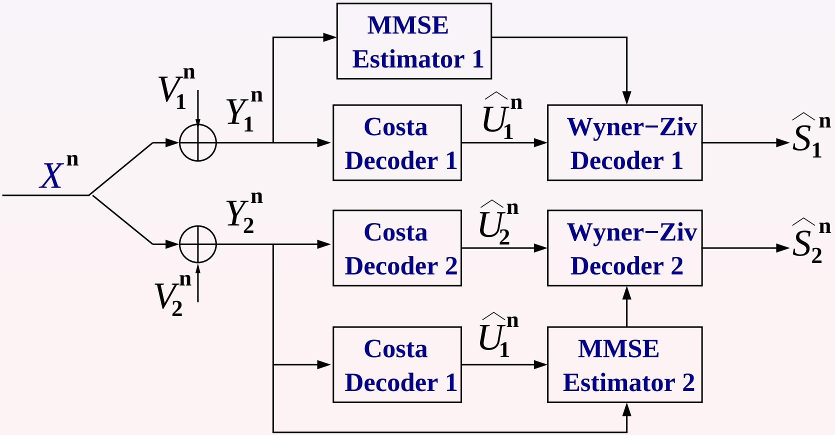
In order to exploit the advantages of both analog transmission and digital techniques, various HDA schemes have been introduced in the literature.

- Broadcasting a single memoryless Gaussian source under bandwidth mismatch, Mittal et al (02), Reznic et al (06)
- Broadcasting a Gaussian source with memory, Prabhakaran et al (05)
- Uncoded transmission for broadcasting correlated Gaussian sources, Bross et al (08)
- Inner and outer bounds for the distortion region in broadcasting a Gaussian mixture source, Reznic et al (06)

2. Layering with Analog and Costa Coding: Encoder



Layering with Analog and Costa Coding: Decoder



Layering with Analog and Costa Coding: Encoding

- **1st layer**: Uncoded transmission, X_a, of Power P_a (meant for both strong and weak users)
- Now fix P_1 and P_2 to satisfy $P = P_a + P_1 + P_2$.
- **2nd layer**: S_1 is first Wyner-Ziv coded assuming an estimate of S_1 at the receiver as side information.
- The Wyner-Ziv index is then Costa encoded treating X_a as an interference.
- $X_1^n = U_1^n - \alpha_1 X_a^n$ of power P_1 is then transmitted (meant to be decoded by the weak user).
- Costa scaling (inflation) factor α_1 is set to be $\frac{P_1}{P_1+P_2+\sigma_1^2}$.
- **3rd layer**: S_2 is also Wyner Ziv coded. The Wyner-Ziv index is then Costa encoded. $X_2^n = U_2^n - \alpha_2(X_a^n + X_1^n)$ of power $P_2 = P - P_a - P_1$ is transmitted. Note: $\alpha_2 = \frac{P_2}{P_2+\sigma_2^2}$. We merge all three layers and transmit $X^n = X_a^n + X_1^n + X_2^n$.

Layering with Analog and Costa Coding: Decoding

An achievable distortion-region can be obtained by varying P_a, P_1 and P_2 subject to $P = P_a + P_1 + P_2$. For a given P_a, P_1 and P_2, the achievable pairs of distortion can be computed as follows.

- MMSE estimation of S_1^n from analog layer
- First, Wyner-Ziv bits are decoded from the 2nd layer by Costa decoding procedure.
- The side information is used in refining the estimate of S_1^n for the weak user using the decoded Wyner-Ziv bits.
- From Wyner-Ziv distortion-rate function, $D_1 = D_1^* 2^{-2R_1^{'}}$, where D_1^* is the is the MMSE from the received Y_1^n.

$$D_1 = D_1^* \left(1 + \frac{P_1}{P_2 + \sigma_1^2}\right)^{-1}, \quad D_1^* = \sigma_{S_1}^2 - \frac{\alpha^2 (a_1 \sigma_{S_1}^2 + a_2 \rho \sigma_{S_1} \sigma_{S_2})^2}{P_a + P_1 + P_2 + \sigma_1^2}.$$

Layering with Analog and Costa Coding: Decoding (2)

- **Strong user**: First, an estimate of S_2^n can be determined from the first and the second layers (as side information)
- Based on $R_2^{'}$ decoded Wyner-Ziv bits and side information, using the Wyner-Ziv distortion-rate function, $D_2 = D_2^* 2^{-2R_2^{'}}$.
- D_2^* is the MMSE from the received Y_2^n and the decoded U_1^n, i.e., $D_2^* = \sigma_{S_2}^2 - \Gamma_2^T \Upsilon_2^{-1} \Gamma_2$,

$$\Gamma_2 = \begin{bmatrix} \alpha(a_2\sigma_{S_2}^2 + a_1\rho\sigma_{S_1}\sigma_{S_2}) \\ \alpha_1\alpha(a_2\sigma_{S_2}^2 + a_1\rho\sigma_{S_1}\sigma_{S_2}) \end{bmatrix},$$

$$\Upsilon_2 = \begin{bmatrix} P_a + P_1 + P_2 + \sigma_2^2 & P_1 + \alpha_1 P_a \\ P_1 + \alpha_1 P_a & P_1 + \alpha_1^2 P_a \end{bmatrix}.$$

3. Layering with Analog, Superposition and Costa Coding: Encoding

- This scheme also has three coding layers: analog, superposition, and Costa coding.
- In the second layer, we have two merged streams where S_1^n is broadcasted to two users.
- The first source encoder is an optimal Wyner-Ziv encoder with rate $R_1'' = \frac{1}{2}\log(1 + \frac{(1-\lambda)P_1}{\lambda P_1 + P_a + P_2 + \sigma_1^2})$, and the second source encoder is an optimal Wyner-Ziv encoder for the residual error of the first encoder with rate $R_2'' - R_1'' = \frac{1}{2}\log(1 + \frac{\lambda P_1}{P_a + P_2 + \sigma_2^2})$.
- Then, we encode the Wyner-Ziv bits with capacity-achieving channel codes and transmit with powers $(1-\lambda)P_1$ and λP_1, respectively.

Layering with Analog, Superposition and Costa Coding: Decoding

- The final distortion in estimating S_1^n at the weak user is

$$D_1 = D_1^* 2^{-2R_1''} = D_1^*(1 + \frac{(1-\lambda)P_1}{\lambda P_1 + P_a + P_2 + \sigma_1^2})^{-1}.$$

- At the strong user, first an estimate of S_1^n can be obtained

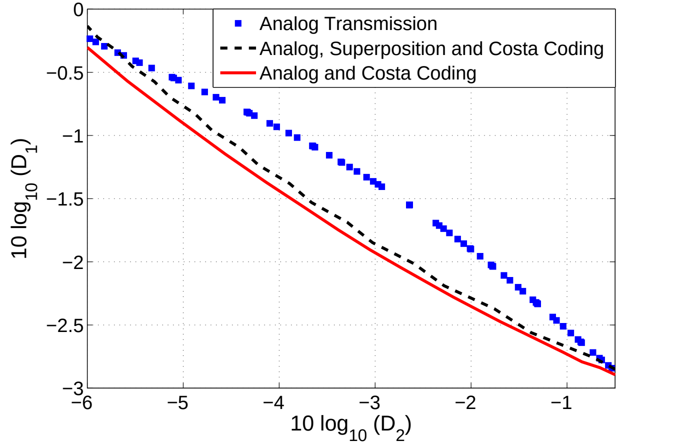
$$D_{12}^* = D_1^* 2^{-2R_2''} = D_1(1 + \frac{\lambda P_1}{P_a + P_2 + \sigma_2^2})^{-1}.$$

- Then we obtain an estimate of S_2^n from the above estimate of S_1^n with distortion: $D_2^* = \sigma_{S_2}^2 \left(1 - \rho^2 \left(1 - \frac{D_{12}^*}{\sigma_{S_1}^2}\right)\right).$
- This estimate of S_2^n acts as side information in refining the estimate of S_2^n using the decoded Wyner-Ziv bits. Thus $D_2 = D_2^* \left(1 + \frac{P_2}{\sigma_2^2}\right)^{-1}.$

Numerical Result (Matched Bandwidth)

- We transmit *n* samples of a bivariate Gaussian source with the covariance matrix $\Lambda = \begin{bmatrix} 1 & 0.2 \\ 0.2 & 1 \end{bmatrix}$ in *n* uses of a power-limited broadcast channel to two users with observation noise variances $\sigma_1^2 = -5\,\mathrm{dB}$ and $\sigma_2^2 = 0\,\mathrm{dB}$, respectively.
- The two-user broadcast channel has the power constraint $P = 0\,\mathrm{dB}$.
- The boundaries of the distortion regions for the schemes presented are shown in this figure.
- We observe that the layering with analog transmission and Costa coding outperforms all other schemes, including analog transmission.

Numerical Result (Matched Bandwidth)



Distortion regions in broadcasting a bivariate source with the correlation coefficient $\rho = 0.2$.

Conclusion

- We considered broadcasting a bivariate correlated Gaussian source to two users.
- Layered JSCC schemes for this problem were analyzed under matched bandwidth assumptions.
- We provided achievable distortion regions for different three-layered HDA coding schemes.
- Using numerical examples, we demonstrated that the layering with analog and Costa coding performs best.