Control Formulation for the Multiple Access Channel with Feedback

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History on AWGN-MAC with Feedback

- Ozarow (84) characterized the capacity region of 2-user Gaussian MAC with feedback

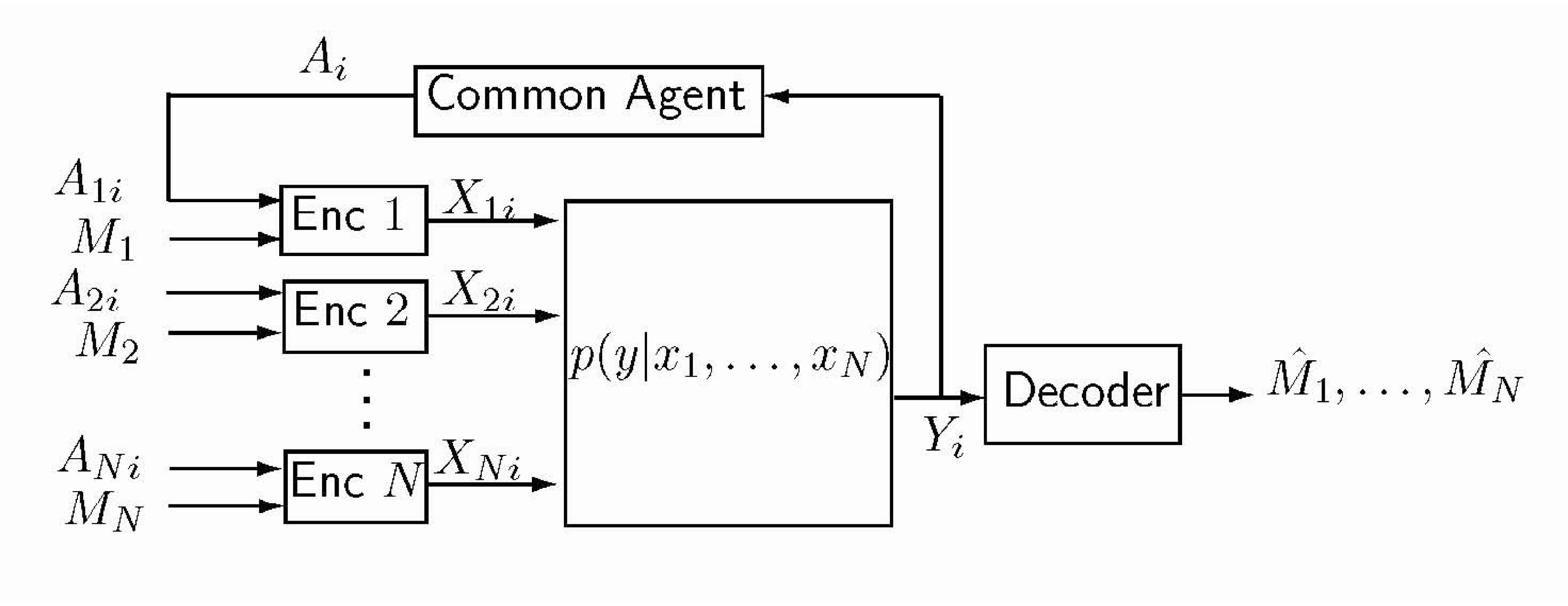
- Kramer (02) generalized Ozarow's conding scheme and provided a coding scheme (Fourier- MEC) for N-user AWGN-MAC

- Kramer and Gastpar (06), using dependence balance bound, proved that under the equal per-symbol power constraint, Fourier- MEC is optimal

- Is Ozarow-Kramer coding scheme optimal for a general block power constraint? WE DO NOT KNOW

- What techniques other than dependence balance can be used to limit cooperation and capture the causality?

Markov Decision Problem



- State : $S_i = F_{M_1 \ldots M_N}(\cdot \mid y^{i-1}, a^{i-1})$

- Action (Markov): $(A_{1i},\ldots,A_{Ni}) = \pi_i(S_i)$

$$x_{ki} = a_{ki}(m_k)$$

Upper Bound on Sum rate of AWGN-MAC with feedback

- AWGN $Y_i = X_{1i} + \cdots + X_{Ni} + Z_i,\ Z_i$ i.i.d. N(0,1)

- Power constraints $\sum_{i=1}^{n} E(X_{ki}^2) \le nP_k$

- Average expected reward under policy π

$$J(\pi) = \liminf_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} E\big(V(S_i, A_i)\big)$$

- Instantaneous reward is

$$V(s,a) = I(M_1,\ldots,M_N; Y_i \mid y^{i-1}, a^{i-1}) + \sum_{k=1}^{N} \lambda_k (P_k - E(X_{ki}^2))$$

- $\sup_\pi J(\pi)$ is an upper bound on the sum rate

Optimality Equation

Theorem[Araposthathis et al. 93]
If there exists a J^*, a bounded function $g(s)$ and a stationary policy π^* such that for all S

$$J^* + g(s) = \sup_a \left(V(s,a) + \int g(x)P(x \mid s,a)dx\right)$$

then $J^* = \sup_\pi J(\pi)$ and the stationary policy π^* attains the supremum

If we set $g(s) = 0$ then we get the full cooperation bound which is loose

Key Point: Find the right $g(s)$ which limits the cooperation

Sum Capacity for Equal Power

- One useful way of limiting cooperation is the dependence balance bound

- Using Lagrange multiplier we can define a new reward function as

$$\tilde{V}(s,a) = V(s,a) + \gamma \cdot D(s,a)$$

Where $D(s,a)$ is the dependence balance term

- Key Point: Find the right γ and λ_k

- New Result: For equal block power constraint a tight upper bound can be derived which shows Ozarow-Kramer coding scheme is optimal

- Details will be presented at Allerton in September

Conclusion

- We formulated MAC with feedback as a decentralized control problem which can be further transformed to a centralized problem with partial observation. The average cost optimality equation (ACOE) potentially can be used to prove upper bound on the sum rate

- Using dependence balance and optimization techniques, we proved that Ozarow-Kramer coding scheme for Gaussian MAC with feedback under equal block power constraint is optimal

- What is the relationship between dependence balance bound and ACOE?

- Can we always find an optimal linear controller for our MDP problem?