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A Cross-Layer Perspective on Rateless Coding for Wireless Channels

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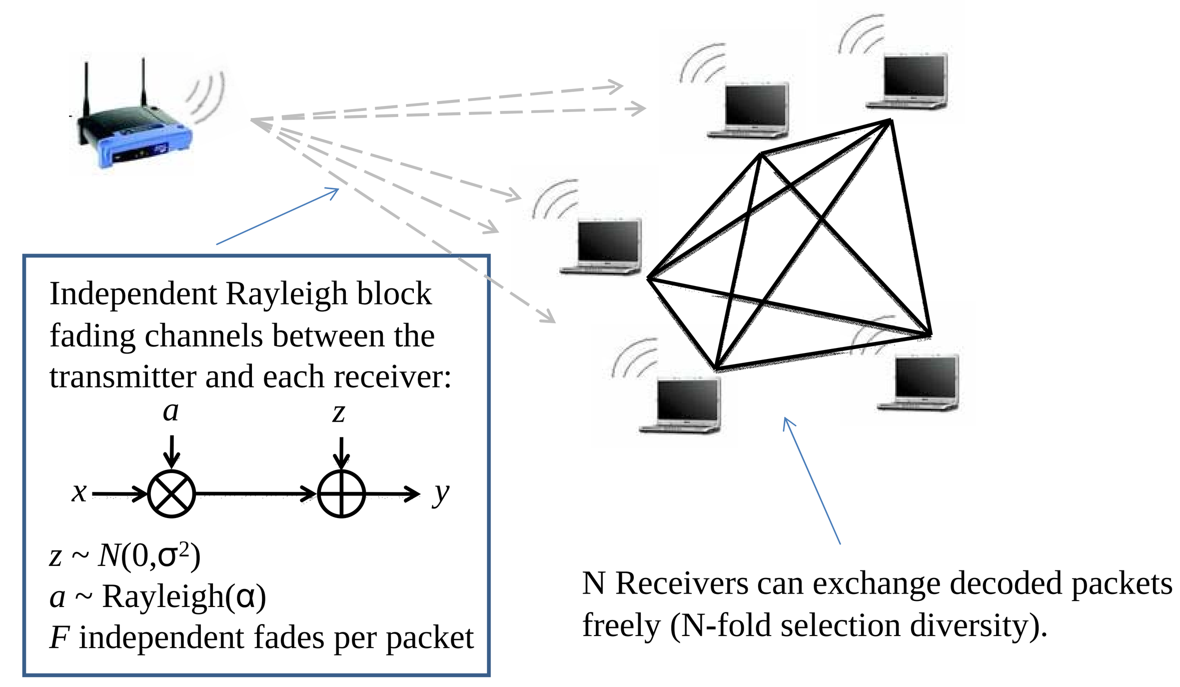
Reliable Message Delivery over Wireless Channels

•Rateless packet-level codes increase redundancy over time to facilitate content distribution in packet-based networks.

•In wireless networks, we need to consider the effects of physical- and application-layer coding jointly.

•We study the optimal cross-layer allocation of coding resources to ensure reliable transmission over the channel.

Physical System Model



•Application-Layer Rateless codes are well suited for wired networks.

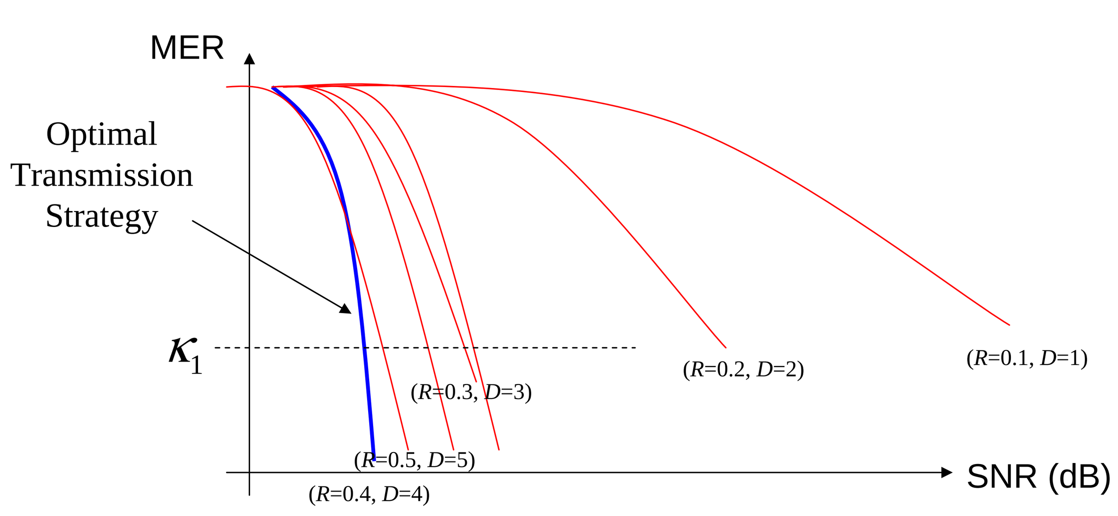
•A direct application of these codes to wireless networks could be suboptimal.

- •It reduces the amount of physical-layer coding available.
- •More physical-layer coding reduces outage probability and hence the need for erasure coding.

•The cross-layer tradeoff is complicated.

A Cross-Layer Optimization Problem

•As the code rate, R, increases, the message error rate (MER) curves get better and then worse suggesting the existence of an optimal transmission scheme.

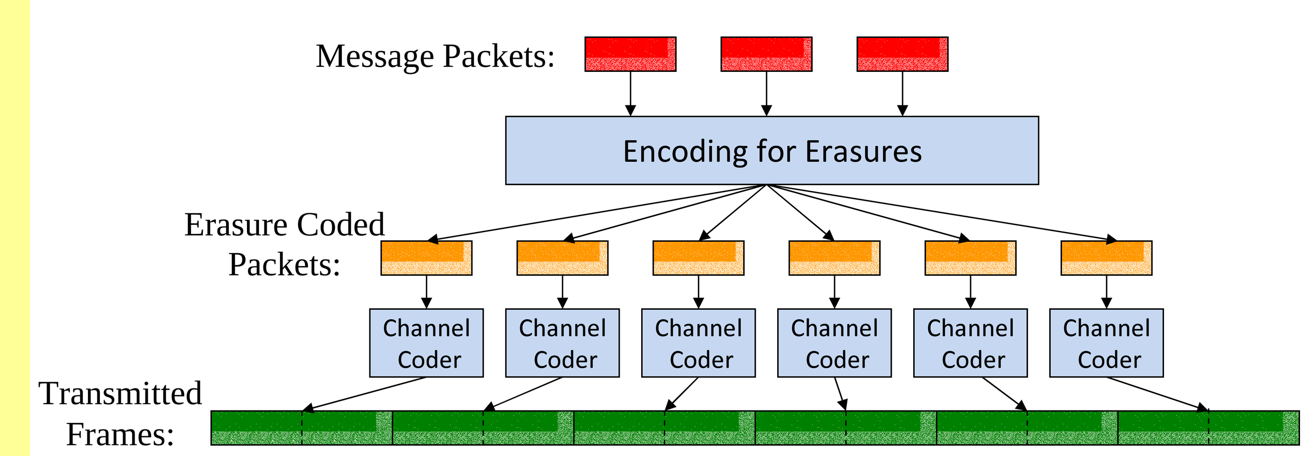


•Cross-Layer Coding is required for optimality

•Application Layer: Redundancy is added through a packet-level erasure code.

•Physical Layer: Each packet is coded for transmission using traditional physical-layer codes.

The Cross-Layer Encoder Architecture



Formulation of the Optimization Problem

Our formal problem statement: How can we allocate coding resources for best performance?

minimize: The minimum operational SNR

subject to: •Probability of Message Error $\leq \kappa_1$

•Probability of Packet Erasure = Outage Probability

•Assuming Rayleigh Fading and AWGN

Without simplification, the problem has combinatorial constraints.

maximize: λ

subject to: $\sum_{i=0}^{\hat{m}-1} \binom{RTk^{-1}}{i} \left(1-p_e^N\right)^i \left(p_e^N\right)^{RTk^{-1}-i} \leq \kappa_1$

$$p_e = \Pr\left[cFR > \sum_{i=1}^{F} \log(1+\gamma_i)\right]$$

$$\gamma_i \sim \text{Exponential}(\lambda)$$

$$\lambda = \sigma^2 / 2\alpha^2$$

Under the assumption of moderately large message length, we formulate a quasi-convex approximation.

maximize: λ

subject to: $p_e^*(R) = \Pr\left[cFR > \sum_{i=1}^{F} \log(1+\gamma_i)\right]$

$$Z^2 RTk^{-1}\left(RTk^{-1}+\kappa_2\right) - ZRTk^{-1}\left(2(\hat{m}-1)+\kappa_2\right) = -(\hat{m}-1)^2$$

$$p_e^*(R) = (1-Z)^{1/N}$$

$$\kappa_2 = \left(\Phi^{-1}(\kappa_1)\right)^2$$

Explicit Solutions for Three Cases of Interest

Slow Fading: One or fewer fades per transmitted frame.

$$p_e^*(R) = \Pr\left[e^{cR}-1 > \gamma\right] = 1 - e^{-\lambda(e^{cR}-1)}$$

$$\Rightarrow \lambda = -\frac{\log\left(1-p_e^*(R)\right)}{e^{cR}-1}$$

Fast Fading: ~15 or more fades per transmitted frame.

$$p_e^*(R) = \Phi\left(\sqrt{F}\,\frac{cR-\mu(\lambda)}{\sqrt{\upsilon(\lambda)}}\right)$$

$$\mu(\lambda) = e^{\lambda}\int_{\lambda}^{\infty} t^{-1} e^{-t}\,dt$$

$$\upsilon(\lambda) = 2e^{\lambda}\int_{\lambda}^{\infty} t^{-1}\log(t) e^{-t}\,dt - \mu(\lambda)\left(2\log(\lambda)+\mu(\lambda)\right)$$

Noisy Channel: Low average SNR.

$$\lambda = \frac{P_\Gamma^{-1}(p_e^*(R)\,|\,F,1)}{cFR}$$

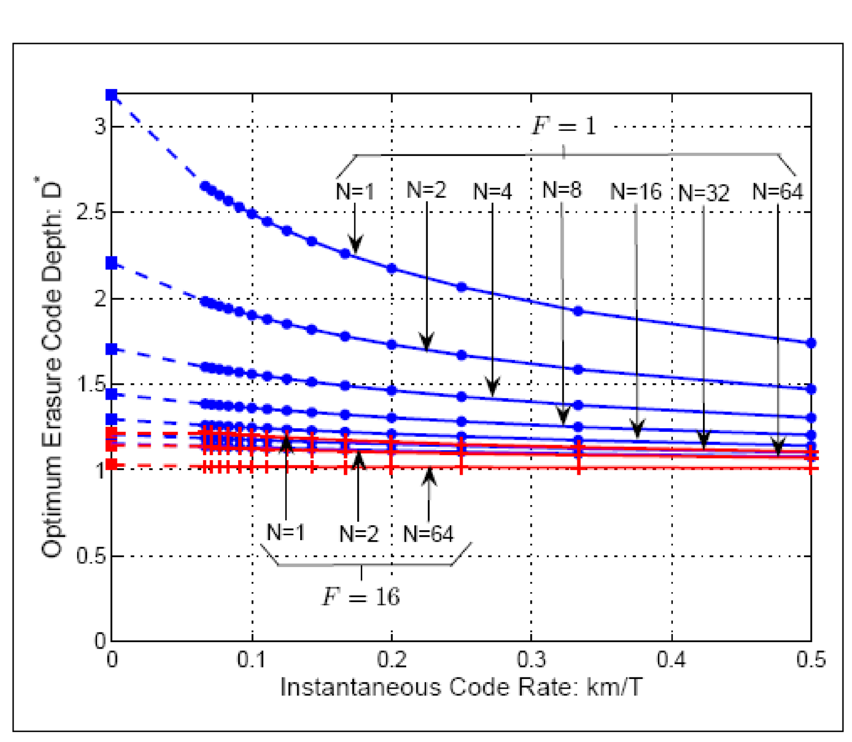
$$P_\Gamma^{-1}(p\,|\,F,1) = \left\{x : \frac{1}{(F-1)!}\int_0^x t^{F-1} e^{-\lambda t}\,dt = p\right\}$$

The Optimal Cross-Layer Transmission Strategy

•By combining the solutions of the three cases, we can observe how the optimal transmission strategy evolves as a function of instantaneous code rate.

•The optimal transmission strategy changes very little when additional diversity, through cooperative nodes or multiple fades, is available.

•We prove that a limiting strategy exists as the instantaneous code rate approaches zero. Hence, the optimal strategy is weakly dependent on rate at low average SNR's.



•We can also plot the gain achieved by using the optimal cross-layer transmission strategy.

•We observe that the gain is significant when little other diversity is available in the system. In this case, the code has the effect of adding "virtual" diversity.

•When diversity is already available in the system, through cooperative nodes or multiple fades, a pure physical-layer coding strategy is nearly optimal.