Capacity Bound and Achievable Rates for Half-duplex Gaussian Relay Channels with Correlated Noises

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Channel Model

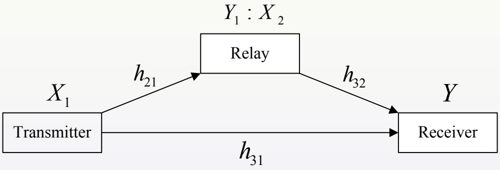

Figure 1: The Relay Channel Model

- Half duplex: Time division with parameter α.
- Correlated noises at the relay (N_1) and the receiver (N).
- Define SNRs as:

$$\gamma_{21} = \frac{h_{21}^2 P_1^{(1)}}{N_1}, \gamma_{32} = \frac{h_{32}^2 P_2}{N}, \gamma_{31}^{(1)} = \frac{h_{31}^2 P_1^{(1)}}{N}, \gamma_{31}^{(2)} = \frac{h_{31}^2 P_1^{(2)}}{N}.$$

Capacity Bound and Achievable Rates

The upper bound of the capacity:

$$C^+ = \max_{0 \leq \rho_x \leq 1} \min\{C_1^+(\rho_x), C_2^+(\rho_x)\},$$

where

$$C_1^+(\rho_x) = \alpha\Gamma\left(\gamma_{31}^{(1)}\right) + (1-\alpha)\,\Gamma\left(\gamma_{31}^{(2)} + \gamma_{32} + 2\rho_x\sqrt{\gamma_{31}^{(2)}\gamma_{32}}\right);$$

$$C_2^+(\rho_x) = \alpha\Gamma\left(\frac{\gamma_{21} + \gamma_{31}^{(1)} - 2\rho_z\sqrt{\gamma_{21}\gamma_{31}^{(1)}}}{1-\rho_z^2}\right) + (1-\alpha)\,\Gamma\left(\left(1-\rho_x^2\right)\gamma_{31}^{(2)}\right).$$

Achievable Rates

The achievable rate of Decode-and-Forward:

$$R_{DF} = \max_{0 \le \rho_x \le 1} \min\{R_1(\rho_x), R_2(\rho_x)\},$$

where

$$R_1(\rho_x) = \alpha\Gamma(\gamma_{21}) + (1-\alpha)\Gamma\left((1-\rho_x^2)\gamma_{31}^{(2)}\right);$$

$$R_2(\rho_x) = \alpha\Gamma\left(\gamma_{31}^{(1)}\right) + (1-\alpha)\Gamma\left(\gamma_{31}^{(2)} + \gamma_{32} + 2\rho_x\sqrt{\gamma_{31}^{(2)}\gamma_{32}}\right).$$

- No utilization of the correlated noises.
- DF can achieve the capacity when $\rho_z = \sqrt{\gamma_{31}^{(1)}/\gamma_{21}}$, which is equivalent to the degraded relay channel.

Achievable Rates

The achievable rate of Compress-and-Forward:

$$R_{CF} = \alpha \Gamma \left(\frac{\gamma_{21} + \gamma_{31}^{(1)} + \gamma_{31}^{(1)} N_w / N_1 - 2\rho_z \sqrt{\gamma_{21}\gamma_{31}^{(1)}}}{(1-\rho_z^2) + N_w/N_1} \right) + (1-\alpha)\,\Gamma\left(\gamma_{31}^{(2)}\right),$$

where the quantization noise power N_w is obtained by

$$N_w = N_1 \frac{(1-\rho_z^2) + \gamma_{21} + \gamma_{31}^{(1)} - 2\rho_z \sqrt{\gamma_{21}\gamma_{31}^{(1)}}}{\left(1+\gamma_{31}^{(1)}\right)\left(\left(1+\frac{\gamma_{32}}{1+\gamma_{31}^{(2)}}\right)^{\frac{1-\alpha}{\alpha}} - 1\right)}.$$

- CF can achieve the capacity when $\rho_z = \sqrt{\gamma_{21}/\gamma_{31}^{(1)}}$, which is equivalent to the reversely-degraded relay channel.

Achievable Rates

The achievable rate of Amplify-and-Forward:

$$R_{AF} = \frac{1}{2}\Gamma\left(\gamma_{31}^{(1)} + \frac{\gamma_{32}\left(\sqrt{\gamma_{21}} - \rho_z\sqrt{\gamma_{31}^{(1)}}\right)^2}{1 + \gamma_{21} + \gamma_{32}\left(1 - \rho_z^2\right)}\right).$$

- Monotonic decrease in AF rate when $\rho_z \in [-1, 0]$.

Capacity-achieving case: DF ($\alpha = 0.5$)

- $h_{21} = \frac{1}{d}, \quad h_{32} = \frac{1}{1-d}, \quad h_{31} = 1, \quad d \in (0, 1).$

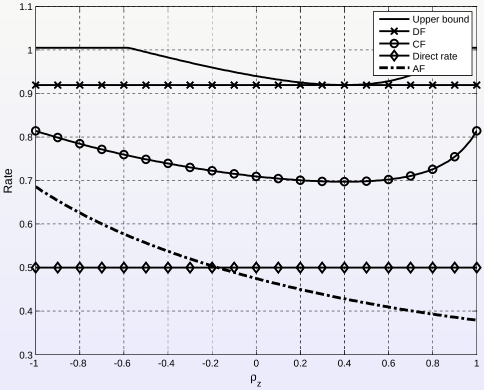

Figure 2: Rate vs. ρ_z, $d = 0.4$, $\alpha = 0.5$.

Capacity-achieving case: DF ($\alpha = 0.5$)

- $h_{21} = \frac{1}{d}, \quad h_{32} = \frac{1}{1-d}, \quad h_{31} = 1, \quad d \in (0, 1).$

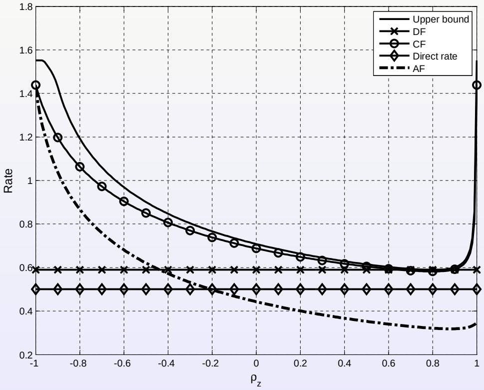

Figure 3: Rate vs. ρ_z, $d = 0.8$, $\alpha = 0.5$.

Capacity-achieving case: DF (optimal α)

- $h_{21} = \frac{1}{d}, \quad h_{32} = \frac{1}{1-d}, \quad h_{31} = 1, \quad d \in (0, 1).$

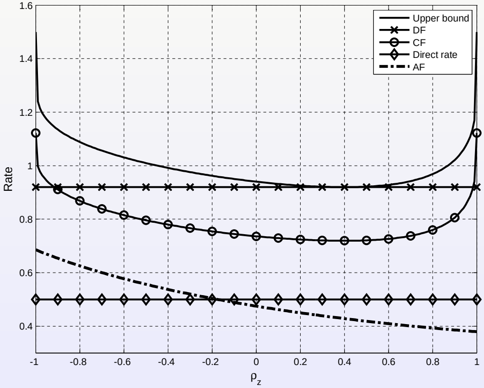

Figure 4: Rate vs. ρ_z, $d = 0.4$, at optimal α.

Capacity-achieving case: DF (optimal α)

- $h_{21} = \frac{1}{d}, \quad h_{32} = \frac{1}{1-d}, \quad h_{31} = 1, \quad d \in (0, 1).$

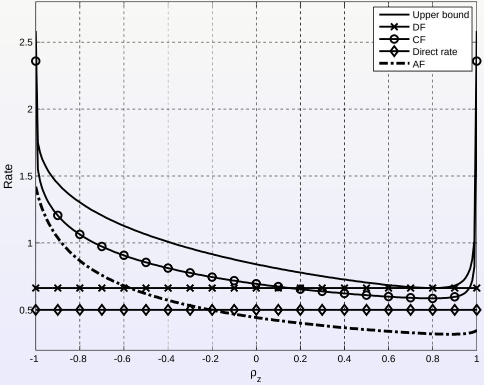

Figure 5: Rate vs. ρ_z, $d = 0.8$, at optimal α.

Capacity-achieving case: CF ($\alpha = 0.5$)

- $h_{21} = r, \quad h_{32} = 1 - r, \quad h_{31} = 1, \quad r \in (0, 1).$

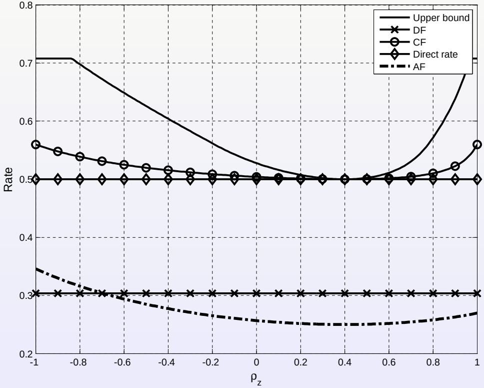

Figure 6: Rate vs. ρ_z, $r = 0.4$, $\alpha = 0.5$.

Conclusion and Future Work

1. The relay channel with correlated noises is studied.
   - Upper bound, DF, CF, AF
2. Future work:
   - Power allocation
   - Coding scheme which utilizes the noises correlation